```
(*
     *** COMPUTATIONS USED IN MY 2007 PAPER - M. Q. RIECK ***
*)
```

In[196]:=

```
(* Overhead / Support Tools - Activate all of this *)

cSubs = {       (* System (1) solved for c's *)
   c1 → (r2^2 + r3^2 - D1) / (2 r2 r3),
   c2 → (r3^2 + r1^2 - D2) / (2 r3 r1),
   c3 → (r1^2 + r2^2 - D3) / (2 r1 r2)
  };

r2RSubs = {     (*  R1 = r1^2, etc.  *)
   r1 → Sqrt[R1],
   r2 → Sqrt[R2],
   r3 → Sqrt[R3]
  };

RSubs = {       (*  R1 = r1^2, etc.  *)
   R1 → (x - x1)^2 + (y - y1)^2 + Z,
   R2 → (x - x2)^2 + (y - y2)^2 + Z,
   R3 → (x - x3)^2 + (y - y3)^2 + Z
  };

DSubs = {       (*  D1 = d1^2, etc.  *)
   D1 → (x2 - x3)^2 + (y2 - y3)^2,
   D2 → (x3 - x1)^2 + (y3 - y1)^2,
   D3 → (x1 - x2)^2 + (y1 - y2)^2
  };

DRSubs = {
   D1 → Expand[D1 /. DSubs],
   D2 → Expand[D2 /. DSubs],
   D3 → Expand[D3 /. DSubs],
   R1 → Expand[R1 /. RSubs],
   R2 → Expand[R2 /. RSubs],
   R3 → Expand[R3 /. RSubs]
  };

tSubs = {   (*  Weierstrass substitutions  *)
   x1 → (1 - t1^2) / (1 + t1^2), y1 → 2 t1 / (1 + t1^2),
   x2 → (1 - t2^2) / (1 + t2^2), y2 → 2 t2 / (1 + t2^2),
   x3 → (1 - t3^2) / (1 + t3^2), y3 → 2 t3 / (1 + t3^2)
```

```
  };
tSubsSp = {x1 → (1 - t1^2)/(1 + t1^2), y1 → (2 t1)/(1 + t1^2), x2 → (1 - t2^2)/(1 + t2^2), y2 → (2 t2)/(1 + t2^2), x3 → (1 - t3^2)/(1 + t3^2), y3 → (2 t3)/(1 + t3^2),
   t1 → (t2 + t3)/(-1 + t2 t3)};  (*  t1 + t2 + t3 = t1 t2 t3 in special coordinate system  *)

untSubs = {
   t1 → y1 / (1 + x1),
   t2 → y2 / (1 + x2),
   t3 → y3 / (1 + x3)
  };

ySubs = {     (*   x1^2 + y1^2 = 1, etc.  *)
   y1^2 → 1 - x1^2,
   y1^3 → y1 (1 - x1^2),
   y1^4 → (1 - x1^2)^2,
   y1^5 → y1 (1 - x1^2)^2,
   y1^6 → (1 - x1^2)^3,
   y1^7 → y1 (1 - x1^2)^3,
   y1^8 → (1 - x1^2)^4,
   y1^9 → y1 (1 - x1^2)^4,
   y1^10 → (1 - x1^2)^5,
   y1^11 → y1 (1 - x1^2)^5,
   y1^12 → (1 - x1^2)^6,
   y1^13 → y1 (1 - x1^2)^6,
   y1^14 → (1 - x1^2)^7,
   y1^15 → y1 (1 - x1^2)^7,
   y1^16 → (1 - x1^2)^8,
   y2^2 → 1 - x2^2,
   y2^3 → y2 (1 - x2^2),
   y2^4 → (1 - x2^2)^2,
   y2^5 → y2 (1 - x2^2)^2,
   y2^6 → (1 - x2^2)^3,
   y2^7 → y2 (1 - x2^2)^3,
   y2^8 → (1 - x2^2)^4,
   y2^9 → y2 (1 - x2^2)^4,
   y2^10 → (1 - x2^2)^5,
   y2^11 → y2 (1 - x2^2)^5,
   y2^12 → (1 - x2^2)^6,
   y2^13 → y2 (1 - x2^2)^6,
   y2^14 → (1 - x2^2)^7,
   y2^15 → y2 (1 - x2^2)^7,
   y2^16 → (1 - x2^2)^8,
   y3^2 → 1 - x3^2,
   y3^3 → y3 (1 - x3^2),
   y3^4 → (1 - x3^2)^2,
   y3^5 → y3 (1 - x3^2)^2,
```

```
    y3^6 → (1 - x3^2)^3,
    y3^7 → y3 (1 - x3^2)^3,
    y3^8 → (1 - x3^2)^4,
    y3^9 → y3 (1 - x3^2)^4,
    y3^10 → (1 - x3^2)^5,
    y3^11 → y3 (1 - x3^2)^5,
    y3^12 → (1 - x3^2)^6,
    y3^13 → y3 (1 - x3^2)^6,
    y3^14 → (1 - x3^2)^7,
    y3^15 → y3 (1 - x3^2)^7,
    y3^16 → (1 - x3^2)^8
   };

special = {    (*  Valid only in special coordinate system   *)
    t3 → (t2 + t3) / (t2 t3 - 1),
    x1 x2 x3 → 1 + x1 y2 y3 + y1 x2 y3 + y1 y2 x3,
    y1 y2 y3 → y1 x2 x3 + x1 y2 x3 + x1 x2 y3,
    y2 x3 → -x2 y3 - y1,
    y2 y3 → x2 x3 - x1,
    y1 x3 → -x1 y3 - y2,
    y1 y3 → x1 x3 - x2,
    y1 x2 → -x1 y2 - y3,
    y1 y2 → x1 x2 - x3
   };

ListDepth[list_] :=
 If[ListQ[list],
  If[Length[list] == 0,
   1,
   1 + Max[Map[ListDepth, list]]],
  0
 ]

UnCoefficientList[list_, vars_] := Module[
  {depth, powers, sum, i, len, var, vars2},
  If[! ListQ[vars],
   Total[list * vars^(Range[Length[list]] - 1)],
   depth = Length[vars];
   If[depth == 0,
    list,
    If[depth == 1,
     Total[list * vars[[1]]^(Range[Length[list]] - 1)],
     sum = 0;
     len = Length[list];
     var = vars[[1]];
     vars2 = Delete[vars, 1];
```

```
      For[ i = 1, i ≤ len, i++,
       sum += UnCoefficientList[list[[i]], vars2] * var^(i - 1)
      ];
      sum
     ]
    ]
   ]
  ]
```

```
unXYΔSubs = {
    XH → x1 + x2 + x3,
    X1  →  2 x1 - x2 - x3 ,
    X2  →  2 x2 - x3 - x1 ,
    X3  →  2 x3 - x1 - x2 ,

    YH  → y1 + y2 + y3 ,
    Y1  → 2 y1 - y2 - y3 ,
    Y2  → 2 y2 - y3 - y1 ,
    Y3  → 2 y3 - y1 - y2 ,

    Δ → x1 y2 + x2 y3 + x3 y1 - x1 y3 - x2 y1 - x3 y2
  };


XYΔSubs = {
    (*
    x1→ (XH + Xp + Xm)/3,
    x2→ (XH + ω^2 Xp + ω Xm)/3,
    x3→ (XH + ω Xp + ω^2 Xm)/3,
    y1→ (YH + Yp + Ym)/3,
    y2→ (YH + ω^2 Yp + ω Ym)/3,
    y3→ (YH + ω Yp + ω^2 Ym)/3,
    *)

    x1 → (XH + X1) / 3,
    x2 → (XH + X2) / 3,
    x3 → (XH + X3) / 3,
    y1 → (YH + Y1) / 3,
    y2 → (YH + Y2) / 3,
    y3 → (YH + Y3) / 3,

    X1^2 →  ( (3 X1 - XH X1 - 3 YH Y1) + (9 - 6 XH + XH^2 - 3 YH^2) ) / 2  ,
    X1^3 →  X1 ( (3 X1 - XH X1 - 3 YH Y1) + (9 - 6 XH + XH^2 - 3 YH^2) ) / 2  ,
    X1^4 →  ( ( (3 X1 - XH X1 - 3 YH Y1) + (9 - 6 XH + XH^2 - 3 YH^2) ) / 2 )^2 ,
    X1^5 →  X1 ( ( (3 X1 - XH X1 - 3 YH Y1) + (9 - 6 XH + XH^2 - 3 YH^2) ) / 2 )^2 ,
    X1^6 →  ( ( (3 X1 - XH X1 - 3 YH Y1) + (9 - 6 XH + XH^2 - 3 YH^2 ) / 2) )^3 ,
```

```
X2^2 → ( (3 X2 – XH X2 – 3 YH Y2) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 ,
X2^3 → X2 ( (3 X2 – XH X2 – 3 YH Y2) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 ,
X2^4 → ( ( (3 X2 – XH X2 – 3 YH Y2) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 )^2 ,
X2^5 → X2 ( ( (3 X2 – XH X2 – 3 YH Y2) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 )^2 ,
X2^6 → ( ( (3 X2 – XH X2 – 3 YH Y2) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 )^3 ,

X3^2 → ( (3 X3 – XH X3 – 3 YH Y3) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 ,
X3^3 → X3 ( (3 X3 – XH X3 – 3 YH Y3) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 ,
X3^4 → ( ( (3 X3 – XH X3 – 3 YH Y3) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 )^2 ,
X3^5 → X3 ( ( (3 X3 – XH X3 – 3 YH Y3) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 )^2 ,
X3^6 → ( ( (3 X3 – XH X3 – 3 YH Y3) + (9 – 6 XH + XH^2 – 3 YH^2) ) / 2 )^3 ,

X1 X2 → ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^2 X2 → X1 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^3 X2 → X1^2 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^4 X2 → X1^3 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^5 X2 → X1^4 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X2^2 → X2 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X2^3 → X2^2 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X2^4 → X2^3 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X2^5 → X2^4 ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^2 X2^2 → ( ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^3 X2^2 → X1 ( ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^4 X2^2 → X1^2 ( ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^2 X2^3 → X2 ( ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^2 X2^4 → X2^2 ( ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^3 X2^3 → ( ( 2 (3 X3 – XH X3 – 3 YH Y3) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^3 ,

X1 X3 → ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^2 X3 → X1 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^3 X3 → X1^2 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^4 X3 → X1^3 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^5 X3 → X1^4 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X3^2 → X3 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X3^3 → X3^2 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X3^4 → X3^3 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1 X3^5 → X3^4 ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X1^2 X3^2 → ( ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^3 X3^2 → X1 ( ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^4 X3^2 → X1^2 ( ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^2 X3^3 → X3 ( ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^2 X3^4 → X3^2 ( ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^2 ,
X1^3 X3^3 → ( ( 2 (3 X2 – XH X2 – 3 YH Y2) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 )^3 ,

X2 X3 → ( 2 (3 X1 – XH X1 – 3 YH Y1) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
X2^2 X3 → X2 ( 2 (3 X1 – XH X1 – 3 YH Y1) – (9 – 6 XH + XH^2 – 3 YH^2) ) / 4 ,
```

```
X2^3 X3 → X2^2 ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 ,
X2^4 X3 → X2^3 ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 ,
X2^5 X3 → X2^4 ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 ,
X2 X3^2 → X3 ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 ,
X2 X3^3 → X3^2 ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 ,
X2 X3^4 → X3^3 ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 ,
X2 X3^5 → X3^4 ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 ,
X2^2 X3^2 → ( ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 )^2 ,
X2^3 X3^2 → X2 ( ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 )^2 ,
X2^4 X3^2 → X2^2 ( ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 )^2 ,
X2^2 X3^3 → X3 ( ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 )^2 ,
X2^2 X3^4 → X3^2 ( ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 )^2 ,
X2^3 X3^3 → ( ( 2 (3 X1 - XH X1 - 3 YH Y1) - (9 - 6 XH + XH^2 - 3 YH^2) ) / 4 )^3 ,

Y1^2 →  ( - (3 X1 + 3 XH X1 + YH Y1) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2  ,
Y1^3 →  Y1 ( - (3 X1 + 3 XH X1 + YH Y1) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2  ,
Y1^4 →  ( ( - (3 X1 + 3 XH X1 + YH Y1) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^2 ,
Y1^5 →  Y1 ( ( - (3 X1 + 3 XH X1 + YH Y1) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^2 ,
Y1^6 →  ( ( - (3 X1 + 3 XH X1 + YH Y1) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^3 ,

Y2^2 →  ( - (3 X2 + 3 XH X2 + YH Y2) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2  ,
Y2^3 →  Y2 ( - (3 X2 + 3 XH X2 + YH Y2) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2  ,
Y2^4 →  ( ( - (3 X2 + 3 XH X2 + YH Y2) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^2 ,
Y2^5 →  Y2 ( ( - (3 X2 + 3 XH X2 + YH Y2) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^2 ,
Y2^6 →  ( ( - (3 X2 + 3 XH X2 + YH Y2) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^3 ,

Y3^2 →  ( - (3 X3 + 3 XH X3 + YH Y3) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2  ,
Y3^3 →  Y3 ( - (3 X3 + 3 XH X3 + YH Y3) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2  ,
Y3^4 →  ( ( - (3 X3 + 3 XH X3 + YH Y3) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^2 ,
Y3^5 →  Y3 ( ( - (3 X3 + 3 XH X3 + YH Y3) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^2 ,
Y3^6 →  ( ( - (3 X3 + 3 XH X3 + YH Y3) + (9 + 6 XH - 3 XH^2 + YH^2) ) / 2 )^3 ,

Y1 Y2 → ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^2 Y2 → Y1 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^3 Y2 → Y1^2 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^4 Y2 → Y1^3 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^5 Y2 → Y1^4 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y2^2 → Y2 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y2^3 → Y2^2 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y2^4 → Y2^3 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y2^5 → Y2^4 ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^2 Y2^2 → ( ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^3 Y2^2 → Y1 ( ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^4 Y2^2 → Y1^2 ( ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^2 Y2^3 → Y2 ( ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^2 Y2^4 → Y2^2 ( ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^3 Y2^3 → ( ( 2 * - (3 X3 + 3 XH X3 + YH Y3) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^3 ,
```

```
Y1 Y3 → ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^2 Y3 → Y1 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^3 Y3 → Y1^2 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^4 Y3 → Y1^3 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^5 Y3 → Y1^4 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y3^2 → Y3 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y3^3 → Y3^2 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y3^4 → Y3^3 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1 Y3^5 → Y3^4 ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y1^2 Y3^2 → ( ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^3 Y3^2 → Y1 ( ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^4 Y3^2 → Y1^2 ( ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^2 Y3^3 → Y3 ( ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^2 Y3^4 → Y3^2 ( ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y1^3 Y3^3 → ( ( 2 * - (3 X2 + 3 XH X2 + YH Y2) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^3 ,

Y2 Y3 → ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2^2 Y3 → Y2 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2^3 Y3 → Y2^2 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2^4 Y3 → Y2^3 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2^5 Y3 → Y2^4 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2 Y3^2 → Y3 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2 Y3^3 → Y3^2 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2 Y3^4 → Y3^3 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2 Y3^5 → Y3^4 ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4 ,
Y2^2 Y3^2 → ( ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y2^3 Y3^2 → Y2 ( ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y2^4 Y3^2 → Y2^2 ( ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y2^2 Y3^3 → Y3 ( ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y2^2 Y3^4 → Y3^2 ( ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^2 ,
Y2^3 Y3^3 → ( ( 2 * - (3 X1 + 3 XH X1 + YH Y1) - (9 + 6 XH - 3 XH^2 + YH^2) ) / 4)^3 ,

X1 Y1 → ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1^2 Y1 → X1 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1^3 Y1 → X1^2 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1^4 Y1 → X1^3 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1^5 Y1 → X1^4 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1 Y1^2 → Y1 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1 Y1^3 → Y1^2 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1 Y1^4 → Y1^3 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1 Y1^5 → Y1^4 ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 ,
X1^2 Y1^2 → ( ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X1^3 Y1^2 → X1 ( ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X1^4 Y1^2 → X1^2 ( ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X1^2 Y1^3 → Y1 ( ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X1^2 Y1^4 → Y1^2 ( ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
```

```
X1^3 Y1^3 → ( ( (XH Y1 + YH X1 - 3 Y1) + 2 (2 XH YH + 3 YH) ) / 2 )^3 ,

X1 Y2 → ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1^2 Y2 → X1 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1^3 Y2 → X1^2 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1^4 Y2 → X1^3 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1^5 Y2 → X1^4 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1 Y2^2 → Y2 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1 Y2^3 → Y2^2 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1 Y2^4 → Y2^3 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1 Y2^5 → Y2^4 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 ,
X1^2 Y2^2 → ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^2 ,
X1^3 Y2^2 → X1 ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^2 ,
X1^4 Y2^2 → X1^2 ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^2 ,
X1^2 Y2^3 → Y2 ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^2 ,
X1^2 Y2^4 → Y2^2 ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^2 ,
X1^3 Y2^3 → ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^3 ,

X1 Y3 → ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1^2 Y3 → X1 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1^3 Y3 → X1^2 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1^4 Y3 → X1^3 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1^5 Y3 → X1^4 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1 Y3^2 → Y3 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1 Y3^3 → Y3^2 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1 Y3^4 → Y3^3 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1 Y3^5 → Y3^4 ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X1^2 Y3^2 → ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X1^3 Y3^2 → X1 ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X1^4 Y3^2 → X1^2 ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X1^2 Y3^3 → Y3 ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X1^2 Y3^4 → Y3^2 ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X1^3 Y3^3 → ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^3 ,

X2 Y1 → ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2^2 Y1 → X2 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2^3 Y1 → X2^2 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2^4 Y1 → X2^3 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2^5 Y1 → X2^4 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2 Y1^2 → Y1 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2 Y1^3 → Y1^2 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2 Y1^4 → Y1^3 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2 Y1^5 → Y1^4 ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X2^2 Y1^2 → ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X2^3 Y1^2 → X2 ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X2^4 Y1^2 → X2^2 ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X2^2 Y1^3 → Y1 ( ( (XH Y3 + YH X3 - 3 Y3) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
```

```
X2^2 Y1^4 → Y1^2 ( ( (XH Y3 + YH X3 – 3 Y3) – (2 XH YH + 3 YH)  – 3 Δ ) / 2 )^2 ,
X2^3 Y1^3 → ( ( (XH Y3 + YH X3 – 3 Y3) – (2 XH YH + 3 YH)  – 3 Δ ) / 2 )^3 ,

X2 Y2 → ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2^2 Y2 → X2 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2^3 Y2 → X2^2 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2^4 Y2 → X2^3 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2^5 Y2 → X2^4 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2 Y2^2 → Y2 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2 Y2^3 → Y2^2 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2 Y2^4 → Y2^3 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2 Y2^5 → Y2^4 ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 ,
X2^2 Y2^2 → ( ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X2^3 Y2^2 → X2 ( ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X2^4 Y2^2 → X2^2 ( ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X2^2 Y2^3 → Y2 ( ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X2^2 Y2^4 → Y2^2 ( ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X2^3 Y2^3 → ( ( (XH Y2 + YH X2 – 3 Y2) + 2 (2 XH YH + 3 YH) ) / 2 )^3 ,

X2 Y3 → ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2^2 Y3 → X2 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2^3 Y3 → X2^2 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2^4 Y3 → X2^3 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2^5 Y3 → X2^4 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2 Y3^2 → Y3 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2 Y3^3 → Y3^2 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2 Y3^4 → Y3^3 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2 Y3^5 → Y3^4 ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X2^2 Y3^2 → ( ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
X2^3 Y3^2 → X2 ( ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
X2^4 Y3^2 → X2^2 ( ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
X2^2 Y3^3 → Y3 ( ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
X2^2 Y3^4 → Y3^2 ( ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
X2^3 Y3^3 → ( ( (XH Y1 + YH X1 – 3 Y1) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^3 ,

X3 Y1 → ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3^2 Y1 → X3 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3^3 Y1 → X3^2 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3^4 Y1 → X3^3 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3^5 Y1 → X3^4 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3 Y1^2 → Y1 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3 Y1^3 → Y1^2 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3 Y1^4 → Y1^3 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3 Y1^5 → Y1^4 ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 ,
X3^2 Y1^2 → ( ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
X3^3 Y1^2 → X3 ( ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
X3^4 Y1^2 → X3^2 ( ( (XH Y2 + YH X2 – 3 Y2) – (2 XH YH + 3 YH)  + 3 Δ ) / 2 )^2 ,
```

```
X3^2 Y1^3 → Y1 ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^2 ,
X3^2 Y1^4 → Y1^2 ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^2 ,
X3^3 Y1^3 → ( ( (XH Y2 + YH X2 - 3 Y2) - (2 XH YH + 3 YH) + 3 Δ ) / 2 )^3 ,

X3 Y2 → ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3^2 Y2 → X3 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3^3 Y2 → X3^2 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3^4 Y2 → X3^3 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3^5 Y2 → X3^4 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3 Y2^2 → Y2 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3 Y2^3 → Y2^2 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3 Y2^4 → Y2^3 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3 Y2^5 → Y2^4 ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 ,
X3^2 Y2^2 → ( ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X3^3 Y2^2 → X3 ( ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X3^4 Y2^2 → X3^2 ( ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X3^2 Y2^3 → Y2 ( ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X3^2 Y2^4 → Y2^2 ( ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^2 ,
X3^3 Y2^3 → ( ( (XH Y1 + YH X1 - 3 Y1) - (2 XH YH + 3 YH) - 3 Δ ) / 2 )^3 ,

X3 Y3 → ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3^2 Y3 → X3 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3^3 Y3 → X3^2 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3^4 Y3 → X3^3 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3^5 Y3 → X3^4 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3 Y3^2 → Y3 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3 Y3^3 → Y3^2 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3 Y3^4 → Y3^3 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3 Y3^5 → Y3^4 ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 ,
X3^2 Y3^2 → ( ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X3^3 Y3^2 → X3 ( ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X3^4 Y3^2 → X3^2 ( ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X3^2 Y3^3 → Y3 ( ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X3^2 Y3^4 → Y3^2 ( ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 )^2 ,
X3^3 Y3^3 → ( ( (XH Y3 + YH X3 - 3 Y3) + 2 (2 XH YH + 3 YH) ) / 2 )^3 ,
```

$$\Delta^2 \to \frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right),$$

$$\Delta^3 \to \Delta\,\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right),$$

$$\Delta^4 \to \left(\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right)\right)^2,$$

$$\Delta^5 \to \Delta\left(\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right)\right)^2,$$

$$\Delta^6 \to \left(\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right)\right)^3,$$

$$\Delta^7 \to \Delta \left(\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right)\right)^3,$$

$$\Delta^8 \to \left(\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right)\right)^4,$$

$$\Delta^9 \to \Delta \left(\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right)\right)^4,$$

$$\Delta^{10} \to \left(\frac{1}{4}\left(27 - 18\,XH^2 + 8\,XH^3 - XH^4 - 18\,YH^2 - 24\,XH\,YH^2 - 2\,XH^2\,YH^2 - YH^4\right)\right)^5,$$

```
X1 Δ → ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1^2 Δ → X1 ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1^3 Δ → X1^2 ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1^4 Δ → X1^3 ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1^5 Δ → X1^4 ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1 Δ^2 → Δ ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1 Δ^3 → Δ^2 ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1 Δ^4 → Δ^3 ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1 Δ^5 → Δ^4 ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 ,

X1^2 Δ^2 → ( ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 )^2 ,

X1^3 Δ^2 → X1 ( ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 )^2 ,

X1^4 Δ^2 → X1^2 ( ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 )^2 ,

X1^2 Δ^3 → Δ ( ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 )^2 ,

X1^2 Δ^4 → Δ^2 ( ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 )^2 ,

X1^3 Δ^3 → ( ( 2 (3 + 2 XH) YH (X3 - X2) + (9 - 6 XH + XH^2 - 3 YH^2) (Y2 - Y3) ) / 6 )^3 ,


X2 Δ → ( 2 (3 + 2 XH) YH (X1 - X3) + (9 - 6 XH + XH^2 - 3 YH^2) (Y3 - Y1) ) / 6 ,

X2^2 Δ → X2 ( 2 (3 + 2 XH) YH (X1 - X3) + (9 - 6 XH + XH^2 - 3 YH^2) (Y3 - Y1) ) / 6 ,

X2^3 Δ → X2^2 ( 2 (3 + 2 XH) YH (X1 - X3) + (9 - 6 XH + XH^2 - 3 YH^2) (Y3 - Y1) ) / 6 ,

X2^4 Δ → X2^3 ( 2 (3 + 2 XH) YH (X1 - X3) + (9 - 6 XH + XH^2 - 3 YH^2) (Y3 - Y1) ) / 6 ,
```

```
X2^5 Δ → X2^4 ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 ,

X2 Δ^2 → Δ ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 ,

X2 Δ^3 → Δ^2 ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 ,

X2 Δ^4 → Δ^3 ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 ,

X2 Δ^5 → Δ^4 ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 ,

X2^2 Δ^2 → ( ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 )^2 ,

X2^3 Δ^2 → X2 ( ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 )^2 ,

X2^4 Δ^2 → X2^2 ( ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 )^2 ,

X2^2 Δ^3 → Δ ( ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 )^2 ,

X2^2 Δ^4 → Δ^2 ( ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 )^2 ,

X2^3 Δ^3 → ( ( 2 (3 + 2 XH) YH (X1 – X3) + (9 – 6 XH + XH^2 – 3 YH^2) (Y3 – Y1) ) / 6 )^3 ,


X3 Δ → ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3^2 Δ → X3 ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3^3 Δ → X3^2 ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3^4 Δ → X3^3 ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3^5 Δ → X3^4 ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3 Δ^2 → Δ ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3 Δ^3 → Δ^2 ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3 Δ^4 → Δ^3 ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3 Δ^5 → Δ^4 ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 ,

X3^2 Δ^2 → ( ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 )^2 ,

X3^3 Δ^2 → X3 ( ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 )^2 ,

X3^4 Δ^2 → X3^2 ( ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 )^2 ,
```

```
X3^2 Δ^3 → Δ ( ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 )^2 ,

X3^2 Δ^4 → Δ^2 ( ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 )^2 ,

X3^3 Δ^3 → ( ( 2 (3 + 2 XH) YH (X2 – X1) + (9 – 6 XH + XH^2 – 3 YH^2) (Y1 – Y2) ) / 6 )^3 ,


Y1 Δ → ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1^2 Δ → Y1 ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1^3 Δ → Y1^2 ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1^4 Δ → Y1^3 ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1^5 Δ → Y1^4 ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1 Δ^2 → Δ ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1 Δ^3 → Δ^2 ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1 Δ^4 → Δ^3 ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1 Δ^5 → Δ^4 ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 ,

Y1^2 Δ^2 → ( ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 )^2 ,

Y1^3 Δ^2 → Y1 ( ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 )^2 ,

Y1^4 Δ^2 → Y1^2 ( ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 )^2 ,

Y1^2 Δ^3 → Δ ( ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 )^2 ,

Y1^2 Δ^4 → Δ^2 ( ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 )^2 ,

Y1^3 Δ^3 → ( ( 2 (3 + 2 XH) YH (Y2 – Y3) + (9 + 6 XH – 3 XH^2 + YH^2) (X3 – X2) ) / 6 )^3 ,


Y2 Δ → ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2^2 Δ → Y2 ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2^3 Δ → Y2^2 ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2^4 Δ → Y2^3 ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,
```

```
Y2^5 Δ → Y2^4 ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2 Δ^2 → Δ ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2 Δ^3 → Δ^2 ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2 Δ^4 → Δ^3 ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2 Δ^5 → Δ^4 ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 ,

Y2^2 Δ^2 → ( ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 )^2 ,

Y2^3 Δ^2 → Y2 ( ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 )^2 ,

Y2^4 Δ^2 → Y2^2 ( ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 )^2 ,

Y2^2 Δ^3 → Δ ( ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 )^2 ,

Y2^2 Δ^4 → Δ^2 ( ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 )^2 ,

Y2^3 Δ^3 → ( ( 2 (3 + 2 XH) YH (Y3 – Y1) + (9 + 6 XH – 3 XH^2 + YH^2) (X1 – X3) ) / 6 )^3 ,


Y3 Δ → ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3^2 Δ → Y3 ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3^3 Δ → Y3^2 ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3^4 Δ → Y3^3 ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3^5 Δ → Y3^4 ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3 Δ^2 → Δ ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3 Δ^3 → Δ^2 ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3 Δ^4 → Δ^3 ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3 Δ^5 → Δ^4 ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 ,

Y3^2 Δ^2 → ( ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 )^2 ,

Y3^3 Δ^2 → Y3 ( ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 )^2 ,

Y3^4 Δ^2 → Y3^2 ( ( 2 (3 + 2 XH) YH (Y1 – Y2) + (9 + 6 XH – 3 XH^2 + YH^2) (X2 – X1) ) / 6 )^2 ,
```

```
Y3^2 Δ^3 → Δ ( ( 2 (3 + 2 XH) YH (Y1 - Y2) + (9 + 6 XH - 3 XH^2 + YH^2) (X2 - X1) ) / 6 )^2 ,

Y3^2 Δ^4 → Δ^2 ( ( 2 (3 + 2 XH) YH (Y1 - Y2) + (9 + 6 XH - 3 XH^2 + YH^2) (X2 - X1) ) / 6 )^2 ,

Y3^3 Δ^3 → ( ( 2 (3 + 2 XH) YH (Y1 - Y2) + (9 + 6 XH - 3 XH^2 + YH^2) (X2 - X1) ) / 6 )^3 ,

Xp^2 → (3 Xm - XH Xm - 3 YH Ym) / 2,
Xm^2 → (3 Xp - XH Xp - 3 YH Yp) / 2,
Xp^3 → Xp (3 Xm - XH Xm - 3 YH Ym) / 2,
Xm^3 → Xm (3 Xp - XH Xp - 3 YH Yp) / 2,
Xp^4 → ( (3 Xm - XH Xm - 3 YH Ym) / 2 )^2,
Xm^4 → ( (3 Xp - XH Xp - 3 YH Yp) / 2 )^2,
Xp^5 → Xp ((3 Xm - XH Xm - 3 YH Ym) / 2)^2,
Xm^5 → Xm ((3 Xp - XH Xp - 3 YH Yp) / 2)^2,
Xp^6 → ((3 Xm - XH Xm - 3 YH Ym) / 2)^3,
Xm^6 → ((3 Xp - XH Xp - 3 YH Yp) / 2)^3,

Yp^2 → -(3 Xm + YH Ym + 3 XH Xm) / 2,
Ym^2 → -(3 Xp + YH Yp + 3 XH Xp) / 2,
Yp^3 → Yp * -(3 Xm + YH Ym + 3 XH Xm) / 2,
Ym^3 → Ym * -(3 Xp + YH Yp + 3 XH Xp) / 2,
Yp^4 → ( (3 Xm + YH Ym + 3 XH Xm) / 2 )^2 ,
Ym^4 → ( (3 Xp + YH Yp + 3 XH Xp) / 2 )^2,
Yp^5 → Yp ( (3 Xm + YH Ym + 3 XH Xm) / 2 )^2 ,
Ym^5 → Ym ( (3 Xp + YH Yp + 3 XH Xp) / 2 )^2,
Yp^6 → - ( (3 Xm + YH Ym + 3 XH Xm) / 2 )^3 ,
Ym^6 → - ( (3 Xp + YH Yp + 3 XH Xp) / 2 )^3,

Xp Xm → (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm Xp → (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp^2 Xm → Xp (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm^2 Xp → Xm (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp Xm^2 → Xm (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm Xp^2 → Xp (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp^3 Xm → Xp^2 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm^3 Xp → Xm^2 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp Xm^3 → Xm^2 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm Xp^3 → Xp^2 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp^4 Xm → Xp^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm^4 Xp → Xm^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp Xm^4 → Xm^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm Xp^4 → Xp^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp^5 Xm → Xp^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm^5 Xp → Xm^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp Xm^5 → Xm^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xm Xp^5 → Xp^3 (9 - 6 XH + XH^2 - 3 YH^2) / 4,
Xp^2 Xm^2 → ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
```

```
Xm^2 Xp^2 → ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xp^3 Xm^2 → Xp ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xm^3 Xp^2 → Xm ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xp^2 Xm^3 → Xm ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xm^2 Xp^3 → Xp ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xp^4 Xm^2 → Xp^2 ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xm^4 Xp^2 → Xm^2 ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xp^2 Xm^4 → Xm^2 ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xm^2 Xp^4 → Xp^2 ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^2,
Xp^3 Xm^3 → ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^3,
Xm^3 Xp^3 → ( (9 - 6 XH + XH^2 - 3 YH^2) / 4 )^3,

Yp Ym → (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym Yp → (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp^2 Ym → Yp (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym^2 Yp → Ym (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp Ym^2 → Ym (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym Yp^2 → Yp (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp^3 Ym → Yp^2 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym^3 Yp → Ym^2 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp Ym^3 → Ym^2 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym Yp^3 → Yp^2 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp^4 Ym → Yp^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym^4 Yp → Ym^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp Ym^4 → Ym^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym Yp^4 → Yp^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp^5 Ym → Yp^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym^5 Yp → Ym^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp Ym^5 → Ym^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Ym Yp^5 → Yp^3 (9 + 6 XH - 3 XH^2 + YH^2) / 4,
Yp^2 Ym^2 → ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Ym^2 Yp^2 → ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Yp^3 Ym^2 → Yp ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Ym^3 Yp^2 → Ym ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Yp^2 Ym^3 → Ym ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Ym^2 Yp^3 → Yp ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Yp^4 Ym^2 → Yp^2 ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Ym^4 Yp^2 → Ym^2 ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Yp^2 Ym^4 → Ym^2 ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Ym^2 Yp^4 → Yp^2 ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^2,
Yp^3 Ym^3 → ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^3,
Ym^3 Yp^3 → ( (9 + 6 XH - 3 XH^2 + YH^2) / 4 )^3,

Xp Yp → (XH Ym + YH Xm - 3 Ym) / 2,
Yp Xp → (XH Ym + YH Xm - 3 Ym) / 2,
Xp^2 Yp → Xp (XH Ym + YH Xm - 3 Ym) / 2,
Yp^2 Xp → Yp (XH Ym + YH Xm - 3 Ym) / 2,
```

```
Xp^3 Yp → Xp^2 (XH Ym + YH Xm - 3 Ym) / 2,
Yp^3 Xp → Yp^2 (XH Ym + YH Xm - 3 Ym) / 2,
Xp^4 Yp → Xp^3 (XH Ym + YH Xm - 3 Ym) / 2,
Yp^4 Xp → Yp^3 (XH Ym + YH Xm - 3 Ym) / 2,
Xp^5 Yp → Xp^4 (XH Ym + YH Xm - 3 Ym) / 2,
Yp^5 Xp → Yp^4 (XH Ym + YH Xm - 3 Ym) / 2,
Xp Yp^2 → Yp (XH Ym + YH Xm - 3 Ym) / 2,
Yp Xp^2 → Xp (XH Ym + YH Xm - 3 Ym) / 2,
Xp Yp^3 → Yp^2 (XH Ym + YH Xm - 3 Ym) / 2,
Yp Xp^3 → Xp^2 (XH Ym + YH Xm - 3 Ym) / 2,
Xp Yp^4 → Yp^3 (XH Ym + YH Xm - 3 Ym) / 2,
Yp Xp^4 → Xp^3 (XH Ym + YH Xm - 3 Ym) / 2,
Xp Yp^5 → Yp^4 (XH Ym + YH Xm - 3 Ym) / 2,
Yp Xp^5 → Xp^4 (XH Ym + YH Xm - 3 Ym) / 2,
Xp^2 Yp^2 → ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Yp^2 Xp^2 → ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Xp^3 Yp^2 → Xp ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Yp^3 Xp^2 → Yp ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Xp^4 Yp^2 → Xp^2 ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Yp^4 Xp^2 → Yp^2 ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Xp^2 Yp^3 → Yp ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Yp^2 Xp^3 → Xp ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Xp^2 Yp^4 → Yp^2 ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Yp^2 Xp^4 → Xp^2 ( (XH Ym + YH Xm - 3 Ym) / 2 )^2,
Xp^3 Yp^3 → ( (XH Ym + YH Xm - 3 Ym) / 2 )^3,
Yp^3 Xp^3 → ( (XH Ym + YH Xm - 3 Ym) / 2 )^3,

Xm Ym → (XH Yp + YH Xp - 3 Yp) / 2,
Ym Xm → (XH Yp + YH Xp - 3 Yp) / 2,
Xm^2 Ym → Xm (XH Yp + YH Xp - 3 Yp) / 2,
Ym^2 Xm → Ym (XH Yp + YH Xp - 3 Yp) / 2,
Xm^3 Ym → Xm^2 (XH Yp + YH Xp - 3 Yp) / 2,
Ym^3 Xm → Ym^2 (XH Yp + YH Xp - 3 Yp) / 2,
Xm^4 Ym → Xm^3 (XH Yp + YH Xp - 3 Yp) / 2,
Ym^4 Xm → Ym^3 (XH Yp + YH Xp - 3 Yp) / 2,
Xm^5 Ym → Xm^4 (XH Yp + YH Xp - 3 Yp) / 2,
Ym^5 Xm → Ym^4 (XH Yp + YH Xp - 3 Yp) / 2,
Xm Ym^2 → Ym (XH Yp + YH Xp - 3 Yp) / 2,
Ym Xm^2 → Xm (XH Yp + YH Xp - 3 Yp) / 2,
Xm Ym^3 → Ym^2 (XH Yp + YH Xp - 3 Yp) / 2,
Ym Xm^3 → Xm^2 (XH Yp + YH Xp - 3 Yp) / 2,
Xm Ym^4 → Ym^3 (XH Yp + YH Xp - 3 Yp) / 2,
Ym Xm^4 → Xm^3 (XH Yp + YH Xp - 3 Yp) / 2,
Xm Ym^5 → Ym^4 (XH Yp + YH Xp - 3 Yp) / 2,
Ym Xm^5 → Xm^4 (XH Yp + YH Xp - 3 Yp) / 2,
Xm^2 Ym^2 → ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
Ym^2 Xm^2 → ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
```

```
    Xm^3 Ym^2 → Xm ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Ym^3 Xm^2 → Ym ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Xm^4 Ym^2 → Xm^2 ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Ym^4 Xm^2 → Ym^2 ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Xm^2 Ym^3 → Ym ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Ym^2 Xm^3 → Xm ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Xm^2 Ym^4 → Ym^2 ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Ym^2 Xm^4 → Xm^2 ( (XH Yp + YH Xp - 3 Yp) / 2 )^2,
    Xm^3 Ym^3 → ( (XH Yp + YH Xp - 3 Yp) / 2 )^3,
    Ym^3 Xm^3 → ( (XH Yp + YH Xp - 3 Yp) / 2 )^3,


    Xp Ym → (2 XH YH + 3 YH + (ω^2 - ω) Δ) / 2,
    Ym Xp → (2 XH YH + 3 YH + (ω^2 - ω) Δ) / 2,
    Xm Yp → (2 XH YH + 3 YH + (ω - ω^2) Δ) / 2,
    Yp Xm → (2 XH YH + 3 YH + (ω - ω^2) Δ) / 2,
    Ym Xp → (2 XH YH + 3 YH + (ω^2 - ω) Δ) / 2,
    Xp Ym → (2 XH YH + 3 YH + (ω^2 - ω) Δ) / 2,
    Yp Xm → (2 XH YH + 3 YH + (ω - ω^2) Δ) / 2,
    Xm Yp → (2 XH YH + 3 YH + (ω - ω^2) Δ) / 2,

    Δ^2 → 1/4 (27 - 18 XH^2 + 8 XH^3 - XH^4 - 18 YH^2 - 24 XH YH^2 - 2 XH^2 YH^2 - YH^4),

    Δ Xp → 1/6 ((1 - ω) ω) (6 Xp YH + 4 XH Xp YH - 9 Yp + 6 XH Yp - XH^2 Yp + 3 YH^2 Yp),

    Δ Xm → 1/6 ((1 - ω) ω) (-6 Xm YH - 4 XH Xm YH + 9 Ym - 6 XH Ym + XH^2 Ym - 3 YH^2 Ym),

    Δ Yp → 1/6 ((1 - ω) ω) (9 Xp + 6 XH Xp - 3 XH^2 Xp + Xp YH^2 - 6 YH Yp - 4 XH YH Yp),

    Δ Ym → 1/6 ((1 - ω) ω) (-9 Xm - 6 XH Xm + 3 XH^2 Xm - Xm YH^2 + 6 YH Ym + 4 XH YH Ym)

  };

tweak1 = {
    X1 → - X2 - X3,
    Y1 → - Y2 - Y3
  };

tweak2 = {
    X2 → - X3 - X1,
    Y2 → - Y3 - Y1
  };
```

```
tweak3 = {
    X3 → - X1 - X2,
    Y3 → - Y1 - Y2
  };

XYΔReduce[e_] :=
 Factor[Expand[Factor[Expand[Factor[Expand[e //. XYΔSubs] //. XYΔSubs] //. XYΔSubs] //.
        XYΔSubs] //. XYΔSubs] //. XYΔSubs] //. XYΔSubs
```

$$\mu\nu\xi\text{Subs} = \left\{ x \to \frac{\mu + XH\,\xi}{1+\xi},\ y \to \frac{\nu + YH\,\xi}{1+\xi},\ z \to \sqrt{Z},\ Z \to \frac{\left(1 - x^2 - y^2\right)(1-\xi)}{-2\,\xi} \right\};$$

$$\text{un}\mu\nu\xi\text{Subs} = \left\{ \mu \to \frac{-2\,x + 2\,x^3 + 2\,x\,y^2 + 2\,x\,Z + XH - x^2\,XH - y^2\,XH}{-1 + x^2 + y^2 + 2\,Z},\ \nu \to \frac{-2\,y + 2\,x^2\,y + 2\,y^3 + 2\,y\,Z + YH - x^2\,YH - y^2\,YH}{-1 + x^2 + y^2 + 2\,Z},\ \xi \to \frac{-1 + x^2 + y^2}{-1 + x^2 + y^2 + 2\,Z} \right\};$$

```
(*
    Lemma 1 Check.
     using Weierstrass Subtittuions.
    t1 = Tan[ϕ1/2], x1 = Cos[ϕ1], y1 = Sin[ϕ1], etc.
    Since ϕ1+ϕ2+ϕ3 = 0, have t1 + t2 + t3 = t1 t2 t3.
*)
```

In[128]:=
```
tSubsSp
```

Out[128]=
$$\left\{ x1 \to \frac{1 - t1^2}{1 + t1^2},\ y1 \to \frac{2\,t1}{1 + t1^2},\ x2 \to \frac{1 - t2^2}{1 + t2^2},\ y2 \to \frac{2\,t2}{1 + t2^2},\ x3 \to \frac{1 - t3^2}{1 + t3^2},\ y3 \to \frac{2\,t3}{1 + t3^2},\ t1 \to \frac{t2 + t3}{-1 + t2\,t3} \right\}$$

In[129]:=
```
Factor[ {
   x1 - (x2 x3 - y2 y3),
   x2 - (x3 x1 - y3 y1),
   x3 - (x1 x2 - y1 y2),
   y1 + (x2 y3 + x3 y2),
   y2 + (x3 y1 + x1 y3),
   y3 + (x1 y2 + x2 y1),
   x1 x2 x3 - x1 y2 y3 - x2 y3 y1 - x3 y1 y2 - 1,
   y1 y2 y3 - y1 x2 x3 - y2 x3 x1 - y3 x1 x2
  } //. tSubsSp]
```

Out[129]= {0, 0, 0, 0, 0, 0, 0, 0}

```
(*
    Lemma 2 Check.
*)
```

In[130]:=
```
unXYΔSubs
```

Out[130]= {XH → x1 + x2 + x3, X1 → 2 x1 − x2 − x3, X2 → − x1 + 2 x2 − x3,
X3 → − x1 − x2 + 2 x3, YH → y1 + y2 + y3, Y1 → 2 y1 − y2 − y3, Y2 → − y1 + 2 y2 − y3,
Y3 → − y1 − y2 + 2 y3, Δ → − x2 y1 + x3 y1 + x1 y2 − x3 y2 − x1 y3 + x2 y3}

In[131]:=
```
Factor[Factor[{
     (x1^2 + x2^2 + x3^2) - (1 + ((XH - 1)^2 - YH^2) / 2),
     (y1^2 + y2^2 + y3^2) - (2 - ((XH - 1)^2 - YH^2) / 2),
     (x2 x3 + x3 x1 + x1 x2)  -  (((XH + 1)^2 + YH^2) / 4 - 1),
     (y2 y3 + y3 y1 + y1 y2)  -  (((XH - 1)^2 + YH^2) / 4 - 1),
     (x1 y2 + x1 y3 + x2 y3 + x2 y1 + x3 y1 + x3 y2)  + YH,
     (x1 y1 + x2 y2 + x3 y3)  -  (XH + 1) YH,
     x1 x2 x3  -  ((XH - 1)^2 - YH^2) / 4 ,
     y1 y2 y3  +  ((XH + 1) YH) / 2,
     (x1 + 1) (x2 + 1) (x3 + 1)  -  (XH + 1)^2 / 2
    } //. unXYΔSubs] //. tSubsSp]
```

Out[131]= {0, 0, 0, 0, 0, 0, 0, 0, 0}

```
(*   Lemma 3, Part (i) Check.   *)
```

In[132]:=
```
Factor[Factor[{XH - x1, YH - y1}. {x2 - x3, y2 - y3} //. unXYΔSubs] //. tSubsSp]
```

Out[132]= 0

```
(*   Lemma 3, Part (ii) Check.   *)
```

In[133]:= DSubs

Out[133]= $\{D1 \to (x2 - x3)^2 + (y2 - y3)^2,\ D2 \to (-x1 + x3)^2 + (-y1 + y3)^2,\ D3 \to (x1 - x2)^2 + (y1 - y2)^2\}$

In[134]:=
```
Factor[Factor[Factor[{
        D1 D2 D3 / 16,
        (1 - x2 x3 - y2 y3)
         (1 - x3 x1 - y3 y1) (1 - x1 x2 - y1 y2) / 2,  (* TYPO IN PAPER *)
        (2 D2 D3 + 2 D3 D1 + 2 D1 D2 - D1^2 - D2^2 - D3^2) / 16,
        (x1 y2 + x2 y3 + x3 y1 - x1 y3 - x2 y1 - x3 y2)^2 / 4,
        ((x1 - x2) (x2 - x3) (x3 - x1) / YH )^2 ,
        ((y1 - y2) (y2 - y3) (y3 - y1) / (XH + 1))^2,
        (27 + 8 XH (XH^2 - 3 YH^2) - (XH^2 + YH^2) (XH^2 + YH^2 + 18)) / 16
      } /. DSubs] //. unXYΔSubs] //. tSubsSp]
```

Out[134]=
$$\left\{\frac{4\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4},\right.$$
$$\frac{4\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4},$$
$$\frac{4\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4},$$
$$\frac{4\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4},$$
$$\frac{4\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4},$$
$$\frac{4\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4},$$
$$\left.\frac{4\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4}\right\}$$

In[135]:=
```
Factor[{%[[1]] / %[[2]], %[[2]] / %[[3]],
  %[[3]] / %[[4]], %[[4]] / %[[5]], %[[5]] / %[[6]], %[[6]] / %[[7]]}]
```

Out[135]= {1, 1, 1, 1, 1, 1}

```
(* Lemma 4 Check. *)
```

In[136]:=
```
list = {
  X1^2,
  X2 X3,
  Y1^2,
  Y2 Y3,
  X1 Y1,
  X2 Y3,
  X3 Y2,
  X1 Δ,
  Y1 Δ,
  Δ^2
 }
```

Out[136]= $\{X1^2, X2\,X3, Y1^2, Y2\,Y3, X1\,Y1, X2\,Y3, X3\,Y2, X1\,\Delta, Y1\,\Delta, \Delta^2\}$

In[137]:=
```
Factor[list] /. XYΔSubs
```

$$\left\{
\begin{array}{l}
\frac{1}{2}\left(9+3\,X1-6\,XH-X1\,XH+XH^2-3\,Y1\,YH-3\,YH^2\right),\\
\frac{1}{4}\left(-9+6\,XH-XH^2+3\,YH^2+2\,(3\,X1-X1\,XH-3\,Y1\,YH)\right),\\
\frac{1}{2}\left(9-3\,X1+6\,XH-3\,X1\,XH-3\,XH^2-Y1\,YH+YH^2\right),\\
\frac{1}{4}\left(-9-6\,XH+3\,XH^2-YH^2+2\,(-3\,X1-3\,X1\,XH-Y1\,YH)\right),\\
\frac{1}{2}(-3\,Y1+XH\,Y1+X1\,YH+2\,(3\,YH+2\,XH\,YH)),\\
\frac{1}{2}(-3\,Y1+XH\,Y1-3\,YH+X1\,YH-2\,XH\,YH+3\,\Delta),\\
\frac{1}{2}(-3\,Y1+XH\,Y1-3\,YH+X1\,YH-2\,XH\,YH-3\,\Delta),\\
\frac{1}{6}\left(2\,(-X2+X3)\,(3+2\,XH)\,YH+(Y2-Y3)\left(9-6\,XH+XH^2-3\,YH^2\right)\right),\\
\frac{1}{6}\left(2\,(3+2\,XH)\,(Y2-Y3)\,YH+(-X2+X3)\left(9+6\,XH-3\,XH^2+YH^2\right)\right),\\
\frac{1}{4}\left(27-18\,XH^2+8\,XH^3-XH^4-18\,YH^2-24\,XH\,YH^2-2\,XH^2\,YH^2-YH^4\right)
\end{array}
\right\}$$

In[138]:=
```
list1 = Factor[ list /. unXYΔSubs]
```

Out[138]=
$$\{(2\,x1 - x2 - x3)^2,\ (x1 + x2 - 2\,x3)\,(x1 - 2\,x2 + x3),\ (2\,y1 - y2 - y3)^2,$$
$$(y1 + y2 - 2\,y3)\,(y1 - 2\,y2 + y3),\ (2\,x1 - x2 - x3)\,(2\,y1 - y2 - y3),$$
$$(x1 - 2\,x2 + x3)\,(y1 + y2 - 2\,y3),\ (x1 + x2 - 2\,x3)\,(y1 - 2\,y2 + y3),$$
$$(2\,x1 - x2 - x3)\,(-x2\,y1 + x3\,y1 + x1\,y2 - x3\,y2 - x1\,y3 + x2\,y3),$$
$$-(2\,y1 - y2 - y3)\,(x2\,y1 - x3\,y1 - x1\,y2 + x3\,y2 + x1\,y3 - x2\,y3),$$
$$(x2\,y1 - x3\,y1 - x1\,y2 + x3\,y2 + x1\,y3 - x2\,y3)^2\}$$

In[139]:=
```
list2 = Factor[list1 //. tSubsSp]
```

Out[139]=
$$\left\{\frac{4\left(-t2^2 - 4\,t2\,t3 - t3^2 + 2\,t2^2\,t3^2\right)^2}{\left(1 + t2^2\right)^2\left(1 + t3^2\right)^2},\right.$$
$$\frac{4\left(t2^2 - 2\,t2\,t3 - 2\,t3^2 + t2^2\,t3^2\right)\left(-2\,t2^2 - 2\,t2\,t3 + t3^2 + t2^2\,t3^2\right)}{\left(1 + t2^2\right)^2\left(1 + t3^2\right)^2},$$
$$\frac{4\,(t2 + t3)^2\,(-3 + t2\,t3)^2}{\left(1 + t2^2\right)^2\left(1 + t3^2\right)^2}, -\frac{4\,t2\,t3\left(3 + t2^2 - 2\,t2\,t3\right)\left(-3 + 2\,t2\,t3 - t3^2\right)}{\left(1 + t2^2\right)^2\left(1 + t3^2\right)^2},$$
$$\frac{4\,(t2 + t3)\,(-3 + t2\,t3)\left(-t2^2 - 4\,t2\,t3 - t3^2 + 2\,t2^2\,t3^2\right)}{\left(1 + t2^2\right)^2\left(1 + t3^2\right)^2},$$
$$-\frac{4\,t3\left(3 + t2^2 - 2\,t2\,t3\right)\left(t2^2 - 2\,t2\,t3 - 2\,t3^2 + t2^2\,t3^2\right)}{\left(1 + t2^2\right)^2\left(1 + t3^2\right)^2},$$
$$\frac{4\,t2\left(-3 + 2\,t2\,t3 - t3^2\right)\left(-2\,t2^2 - 2\,t2\,t3 + t3^2 + t2^2\,t3^2\right)}{\left(1 + t2^2\right)^2\left(1 + t3^2\right)^2},$$
$$-\frac{8\,(t2 - t3)\left(-2\,t2 - t3 + t2^2\,t3\right)\left(-t2 - 2\,t3 + t2\,t3^2\right)\left(-t2^2 - 4\,t2\,t3 - t3^2 + 2\,t2^2\,t3^2\right)}{\left(1 + t2^2\right)^3\left(1 + t3^2\right)^3},$$
$$-\frac{8\,(t2 - t3)\,(t2 + t3)\,(-3 + t2\,t3)\left(-2\,t2 - t3 + t2^2\,t3\right)\left(-t2 - 2\,t3 + t2\,t3^2\right)}{\left(1 + t2^2\right)^3\left(1 + t3^2\right)^3},$$
$$\left.\frac{16\,(t2 - t3)^2\left(-2\,t2 - t3 + t2^2\,t3\right)^2\left(-t2 - 2\,t3 + t2\,t3^2\right)^2}{\left(1 + t2^2\right)^4\left(1 + t3^2\right)^4}\right\}$$

In[216]:=
```
list3 = Factor[XYΔReduce[list1] /. tweak3]
```

Out[216]=
$$\left\{\frac{1}{2}\left(9+3\,X1-6\,XH-X1\,XH+XH^2-3\,Y1\,YH-3\,YH^2\right),\right.$$
$$\frac{1}{4}\left(-9+6\,X1+6\,XH-2\,X1\,XH-XH^2-6\,Y1\,YH+3\,YH^2\right),$$
$$\frac{1}{2}\left(9-3\,X1+6\,XH-3\,X1\,XH-3\,XH^2-Y1\,YH+YH^2\right),$$
$$\frac{1}{4}\left(-9-6\,X1-6\,XH-6\,X1\,XH+3\,XH^2-2\,Y1\,YH-YH^2\right),$$
$$\frac{1}{2}\left(-3\,Y1+XH\,Y1+6\,YH+X1\,YH+4\,XH\,YH\right),\ \frac{1}{2}\left(-3\,Y1+XH\,Y1-3\,YH+X1\,YH-2\,XH\,YH+3\,\Delta\right),$$
$$\frac{1}{2}\left(-3\,Y1+XH\,Y1-3\,YH+X1\,YH-2\,XH\,YH-3\,\Delta\right),\ \frac{1}{6}\left(9\,Y1-6\,XH\,Y1+XH^2\,Y1+18\,Y2-\right.$$
$$\left.12\,XH\,Y2+2\,XH^2\,Y2-6\,X1\,YH-12\,X2\,YH-4\,X1\,XH\,YH-8\,X2\,XH\,YH-3\,Y1\,YH^2-6\,Y2\,YH^2\right),$$
$$\frac{1}{6}\left(-9\,X1-18\,X2-6\,X1\,XH-12\,X2\,XH+3\,X1\,XH^2+6\,X2\,XH^2+6\,Y1\,YH+\right.$$
$$\left.4\,XH\,Y1\,YH+12\,Y2\,YH+8\,XH\,Y2\,YH-X1\,YH^2-2\,X2\,YH^2\right),$$
$$\left.\frac{1}{4}\left(27-18\,XH^2+8\,XH^3-XH^4-18\,YH^2-24\,XH\,YH^2-2\,XH^2\,YH^2-YH^4\right)\right\}$$

In[217]:=
```
list4 = Factor[ list3 /. unXYΔSubs]
```

Out[217]=
$$\left\{\frac{1}{2}\left(9-x1^2-9\,x2+x1\,x2+2\,x2^2-9\,x3+x1\,x3+4\,x2\,x3+2\,x3^2-9\,y1^2-9\,y1\,y2-9\,y1\,y3\right),\right.$$
$$\frac{1}{4}\left(-9+18\,x1-5\,x1^2-4\,x1\,x2+x2^2-4\,x1\,x3+2\,x2\,x3+x3^2-9\,y1^2+9\,y2^2+18\,y2\,y3+9\,y3^2\right),$$
$$\frac{1}{2}\left(9-9\,x1^2+9\,x2-9\,x1\,x2+9\,x3-9\,x1\,x3-y1^2+y1\,y2+2\,y2^2+y1\,y3+4\,y2\,y3+2\,y3^2\right),$$
$$\frac{1}{4}\left(-9-18\,x1-9\,x1^2+9\,x2^2+18\,x2\,x3+9\,x3^2-5\,y1^2-4\,y1\,y2+y2^2-4\,y1\,y3+2\,y2\,y3+y3^2\right),\ \frac{1}{2}$$
$$(8\,x1\,y1+5\,x2\,y1+5\,x3\,y1+9\,y2+5\,x1\,y2+2\,x2\,y2+2\,x3\,y2+9\,y3+5\,x1\,y3+2\,x2\,y3+2\,x3\,y3),$$
$$\frac{1}{2}(-9\,y1+2\,x1\,y1-4\,x2\,y1+2\,x3\,y1+2\,x1\,y2-4\,x2\,y2-7\,x3\,y2-4\,x1\,y3-x2\,y3-4\,x3\,y3),$$
$$\frac{1}{2}(-9\,y1+2\,x1\,y1+2\,x2\,y1-4\,x3\,y1-4\,x1\,y2-4\,x2\,y2-x3\,y2+2\,x1\,y3-7\,x2\,y3-4\,x3\,y3),$$
$$\frac{1}{2}\left(-6\,x2\,y1-4\,x1\,x2\,y1-4\,x2^2\,y1+6\,x3\,y1+4\,x1\,x3\,y1+4\,x3^2\,y1+9\,y2-6\,x1\,y2+\right.$$
$$x1^2\,y2-12\,x2\,y2-2\,x1\,x2\,y2-3\,x2^2\,y2+6\,x1\,x3\,y2+2\,x2\,x3\,y2+5\,x3^2\,y2-$$
$$3\,y1^2\,y2-6\,y1\,y2^2-3\,y2^3-9\,y3+6\,x1\,y3-x1^2\,y3-6\,x1\,x2\,y3-5\,x2^2\,y3+12\,x3\,y3+$$
$$\left.2\,x1\,x3\,y3-2\,x2\,x3\,y3+3\,x3^2\,y3+3\,y1^2\,y3-3\,y2^2\,y3+6\,y1\,y3^2+3\,y2\,y3^2+3\,y3^3\right),$$
$$\frac{1}{2}\left(-9\,x2-6\,x1\,x2+3\,x1^2\,x2-6\,x2^2+6\,x1\,x2^2+3\,x2^3+9\,x3+6\,x1\,x3-3\,x1^2\,x3+\right.$$
$$3\,x2^2\,x3+6\,x3^2-6\,x1\,x3^2-3\,x2\,x3^2-3\,x3^3-x2\,y1^2+x3\,y1^2+6\,y1\,y2+4\,x1\,y1\,y2+$$
$$2\,x2\,y1\,y2+6\,x3\,y1\,y2+6\,y2^2+4\,x1\,y2^2+3\,x2\,y2^2+5\,x3\,y2^2-6\,y1\,y3-4\,x1\,y1\,y3-$$
$$\left.6\,x2\,y1\,y3-2\,x3\,y1\,y3-2\,x2\,y2\,y3+2\,x3\,y2\,y3-6\,y3^2-4\,x1\,y3^2-5\,x2\,y3^2-3\,x3\,y3^2\right),$$
$$\frac{1}{4}\left(27-18\,x1^2+8\,x1^3-x1^4-36\,x1\,x2+24\,x1^2\,x2-4\,x1^3\,x2-18\,x2^2+24\,x1\,x2^2-6\,x1^2\,x2^2+\right.$$
$$8\,x2^3-4\,x1\,x2^3-x2^4-36\,x1\,x3+24\,x1^2\,x3-4\,x1^3\,x3-36\,x2\,x3+48\,x1\,x2\,x3-12\,x1^2\,x2\,x3+$$
$$24\,x2^2\,x3-12\,x1\,x2^2\,x3-4\,x2^3\,x3-18\,x3^2+24\,x1\,x3^2-6\,x1^2\,x3^2+24\,x2\,x3^2-12\,x1\,x2\,x3^2-$$
$$6\,x2^2\,x3^2+8\,x3^3-4\,x1\,x3^3-4\,x2\,x3^3-x3^4-18\,y1^2-24\,x1\,y1^2-2\,x1^2\,y1^2-24\,x2\,y1^2-$$
$$4\,x1\,x2\,y1^2-2\,x2^2\,y1^2-24\,x3\,y1^2-4\,x1\,x3\,y1^2-4\,x2\,x3\,y1^2-2\,x3^2\,y1^2-y1^4-36\,y1\,y2-$$
$$48\,x1\,y1\,y2-4\,x1^2\,y1\,y2-48\,x2\,y1\,y2-8\,x1\,x2\,y1\,y2-4\,x2^2\,y1\,y2-48\,x3\,y1\,y2-$$
$$8\,x1\,x3\,y1\,y2-8\,x2\,x3\,y1\,y2-4\,x3^2\,y1\,y2-4\,y1^3\,y2-18\,y2^2-24\,x1\,y2^2-2\,x1^2\,y2^2-$$
$$24\,x2\,y2^2-4\,x1\,x2\,y2^2-2\,x2^2\,y2^2-24\,x3\,y2^2-4\,x1\,x3\,y2^2-4\,x2\,x3\,y2^2-2\,x3^2\,y2^2-$$
$$6\,y1^2\,y2^2-4\,y1\,y2^3-y2^4-36\,y1\,y3-48\,x1\,y1\,y3-4\,x1^2\,y1\,y3-48\,x2\,y1\,y3-8\,x1\,x2\,y1\,y3-$$
$$4\,x2^2\,y1\,y3-48\,x3\,y1\,y3-8\,x1\,x3\,y1\,y3-8\,x2\,x3\,y1\,y3-4\,x3^2\,y1\,y3-4\,y1^3\,y3-36\,y2\,y3-$$
$$48\,x1\,y2\,y3-4\,x1^2\,y2\,y3-48\,x2\,y2\,y3-8\,x1\,x2\,y2\,y3-4\,x2^2\,y2\,y3-48\,x3\,y2\,y3-$$
$$8\,x1\,x3\,y2\,y3-8\,x2\,x3\,y2\,y3-4\,x3^2\,y2\,y3-12\,y1^2\,y2\,y3-12\,y1\,y2^2\,y3-4\,y2^3\,y3-18\,y3^2-$$
$$24\,x1\,y3^2-2\,x1^2\,y3^2-24\,x2\,y3^2-4\,x1\,x2\,y3^2-2\,x2^2\,y3^2-24\,x3\,y3^2-4\,x1\,x3\,y3^2-$$
$$\left.\left.4\,x2\,x3\,y3^2-2\,x3^2\,y3^2-6\,y1^2\,y3^2-12\,y1\,y2\,y3^2-6\,y2^2\,y3^2-4\,y1\,y3^3-4\,y2\,y3^3-y3^4\right)\right\}$$

In[218]:=
```
list5 = Factor[list4 //. tSubsSp]
```

Out[218]=
$$\left\{\frac{4\left(-t2^2-4\,t2\,t3-t3^2+2\,t2^2\,t3^2\right)^2}{\left(1+t2^2\right)^2\left(1+t3^2\right)^2},\right.$$
$$\frac{4\left(t2^2-2\,t2\,t3-2\,t3^2+t2^2\,t3^2\right)\left(-2\,t2^2-2\,t2\,t3+t3^2+t2^2\,t3^2\right)}{\left(1+t2^2\right)^2\left(1+t3^2\right)^2},$$
$$\frac{4\,(t2+t3)^2\,(-3+t2\,t3)^2}{\left(1+t2^2\right)^2\left(1+t3^2\right)^2},-\frac{4\,t2\,t3\left(3+t2^2-2\,t2\,t3\right)\left(-3+2\,t2\,t3-t3^2\right)}{\left(1+t2^2\right)^2\left(1+t3^2\right)^2},$$
$$\frac{4\,(t2+t3)\,(-3+t2\,t3)\left(-t2^2-4\,t2\,t3-t3^2+2\,t2^2\,t3^2\right)}{\left(1+t2^2\right)^2\left(1+t3^2\right)^2},$$
$$-\frac{4\,t3\left(3+t2^2-2\,t2\,t3\right)\left(t2^2-2\,t2\,t3-2\,t3^2+t2^2\,t3^2\right)}{\left(1+t2^2\right)^2\left(1+t3^2\right)^2},$$
$$\frac{4\,t2\left(-3+2\,t2\,t3-t3^2\right)\left(-2\,t2^2-2\,t2\,t3+t3^2+t2^2\,t3^2\right)}{\left(1+t2^2\right)^2\left(1+t3^2\right)^2},$$
$$-\frac{8\,(t2-t3)\left(-2\,t2-t3+t2^2\,t3\right)\left(-t2-2\,t3+t2\,t3^2\right)\left(-t2^2-4\,t2\,t3-t3^2+2\,t2^2\,t3^2\right)}{\left(1+t2^2\right)^3\left(1+t3^2\right)^3},$$
$$-\frac{8\,(t2-t3)\,(t2+t3)\,(-3+t2\,t3)\left(-2\,t2-t3+t2^2\,t3\right)\left(-t2-2\,t3+t2\,t3^2\right)}{\left(1+t2^2\right)^3\left(1+t3^2\right)^3},$$
$$\left.\frac{16\,(t2-t3)^2\left(-2\,t2-t3+t2^2\,t3\right)^2\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\left(1+t3^2\right)^4}\right\}$$

In[219]:=
```
Factor[list2 - list5]
```

Out[219]=
```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
(* Lemma 5 Check. *)
```

In[144]:=
```
H = 1 - c1^2 - c2^2 - c3^2 + 2 c1 c2 c3   (*  H = η^2  *)
```

Out[144]=
$$1-c1^2-c2^2+2\,c1\,c2\,c3-c3^2$$

In[145]:=
```
Factor[Factor[Factor[Factor[Factor[ {
              D1 - ( (x2 - x3) ^ 2 + (y2 - y3) ^ 2 ),
              D1 - 2 (1 - x2 x3 - y2 y3),
              R1 - (1 - 2 (x1 x + y1 y) + x ^ 2 + y ^ 2 + z ^ 2),
              (R2 + R3 - D1) / 2 -
               ((x - x2) (x - x3) + (y - y2) (y - y3) + z ^ 2),     (* TYPO IN PAPER *)
              4 R1 R2 R3 H - D1 D2 D3 z ^ 2,
              4 R1 R2 R3 c1 ^ 2 - R1 (R2 + R3 - D1) ^ 2 ,
              8 R1 R2 R3 c1 c2 c3 - (R2 + R3 - D1) (R3 + R1 - D2) (R1 + R2 - D3) ,
              c1 ^ 2 / H - R1 (R2 + R3 - D1) ^ 2 / (D1 D2 D3 z ^ 2) ,
              c1 c2 c3 / H - (R2 + R3 - D1) (R3 + R1 - D2) (R1 + R2 - D3) / (2 D1 D2 D3 z ^ 2)
             } /. cSubs ] /. r2RSubs] /. DRSubs] /. Z → z ^ 2] /. tSubsSp]
```

Out[145]= {0, 0, 0, 0, 0, 0, 0, 0, 0}

(* Lemma 6 Check. *)

In[146]:=
```
Factor[Factor[Factor[Factor[ {
          x1 - (XH + X1) / 3,
          y1 - (YH + Y1) / 3,
          (1 - x1) y1 - ((3 - XH) Y1 - YH X1 - 2 XH YH) / 6,
          (1 + x1) x1 - ((3 + XH) X1 - YH Y1 + XH ^ 2 - YH ^ 2 + 3) / 6 ,
          D1 - ((9 - XH ^ 2 - YH ^ 2) + 2 (XH X1 + YH Y1)) / 3 ,
          R1 - (1 + x ^ 2 + y ^ 2 + z ^ 2 - 2 ((XH + X1) x + (YH + Y1) y) / 3 ) ,
          (R2 + R3 - D1) - (2 (x ^ 2 + y ^ 2 + z ^ 2) - 1 +
             (2 (X1 - 2 XH) x + 2 (Y1 - 2 YH) y + (XH ^ 2 + YH ^ 2 - 2 XH X1 - 2 YH Y1)) / 3)
         } /. DRSubs] /. Z → z ^ 2] /. unXYΔSubs ] //. tSubsSp]
```

Out[146]= {0, 0, 0, 0, 0, 0, 0}

(* Lemma 7 Check. *)

In[220]:=
```
Factor[ XYΔReduce[Factor[Factor[ {
           R1 R2 R3
         } /. DRSubs] /. Z → z ^ 2]] /. tweak3]
```

Out[220]=
$$\{1 - 2\,x^3 + x^6 - 2\,x\,XH + 2\,x^2\,XH + 2\,x^4\,XH - 2\,x^5\,XH + x^2\,XH^2 - 2\,x^3\,XH^2 + x^4\,XH^2 + 6\,x\,y^2 + 3\,x^4\,y^2 -$$
$$2\,XH\,y^2 - 4\,x^3\,XH\,y^2 + XH^2\,y^2 - 2\,x\,XH^2\,y^2 + 2\,x^2\,XH^2\,y^2 + 3\,x^2\,y^4 - 2\,XH\,y^4 - 2\,x\,XH\,y^4 +$$
$$XH^2\,y^4 + y^6 - 2\,y\,YH - 4\,x\,y\,YH - 4\,x^3\,y\,YH - 2\,x^4\,y\,YH + 4\,x^2\,XH\,y\,YH - 4\,x\,y^3\,YH - 4\,x^2\,y^3\,YH +$$
$$4\,XH\,y^3\,YH - 2\,y^5\,YH + x^2\,YH^2 + 2\,x^3\,YH^2 + x^4\,YH^2 + y^2\,YH^2 + 2\,x\,y^2\,YH^2 + 2\,x^2\,y^2\,YH^2 + y^4\,YH^2 +$$
$$3\,z^2 + 3\,x^2\,z^2 + 3\,x^4\,z^2 - 4\,x\,XH\,z^2 + 2\,x^2\,XH\,z^2 - 4\,x^3\,XH\,z^2 + x^2\,XH^2\,z^2 + 3\,y^2\,z^2 + 6\,x^2\,y^2\,z^2 -$$
$$2\,XH\,y^2\,z^2 - 4\,x\,XH\,y^2\,z^2 + XH^2\,y^2\,z^2 + 3\,y^4\,z^2 - 4\,y\,YH\,z^2 - 4\,x\,y\,YH\,z^2 - 4\,x^2\,y\,YH\,z^2 -$$
$$4\,y^3\,YH\,z^2 + x^2\,YH^2\,z^2 + y^2\,YH^2\,z^2 + 3\,z^4 + 3\,x^2\,z^4 - 2\,x\,XH\,z^4 + 3\,y^2\,z^4 - 2\,y\,YH\,z^4 + z^6\}$$

In[221]:=
```
Factor[CoefficientList[% , {z, y, x}]]
```

```
{
 {
  {
   {1, -2 XH, 2 XH + XH^2 + YH^2, -2 (1 + XH^2 - YH^2), 2 XH + XH^2 + YH^2, -2 XH, 1},
   {-2 YH, -4 YH, 4 XH YH, -4 YH, -2 YH, 0, 0},
   {-2 XH + XH^2 + YH^2, -2 (-3 + XH^2 - YH^2), 2 (XH^2 + YH^2), -4 XH, 3, 0, 0},
   {4 XH YH, -4 YH, -4 YH, 0, 0, 0, 0},
   {-2 XH + XH^2 + YH^2, -2 XH, 3, 0, 0, 0, 0},
   {-2 YH, 0, 0, 0, 0, 0, 0},
   {1, 0, 0, 0, 0, 0, 0}
  }, {
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
  }, {
   {3, -4 XH, 3 + 2 XH + XH^2 + YH^2, -4 XH, 3, 0, 0},
   {-4 YH, -4 YH, -4 YH, 0, 0, 0, 0},
   {3 - 2 XH + XH^2 + YH^2, -4 XH, 6, 0, 0, 0, 0},
   {-4 YH, 0, 0, 0, 0, 0, 0},
   {3, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
  }, {
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
  }, {
   {3, -2 XH, 3, 0, 0, 0, 0},
   {-2 YH, 0, 0, 0, 0, 0, 0},
   {3, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
  }, {
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
  }, {
   {1, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}}
 }
}
```

In[222]:=
```
Factor[ XYΔReduce[Factor[Factor[ {
          (R2 + R3 - D1) (R3 + R1 - D2) (R1 + R2 - D3)
        } /. DRSubs] /. Z → z^2]] /. tweak3]
```

Out[222]=
$$\{-1+9x^2+2x^3-18x^4+8x^6-10x\,XH-8x^2\,XH+30x^3\,XH+4x^4\,XH-16x^5\,XH+4\,XH^2+10x\,XH^2-18x^2\,XH^2-10x^3\,XH^2+14x^4\,XH^2-4\,XH^3+2x\,XH^3+8x^2\,XH^3-6x^3\,XH^3+XH^4-2x\,XH^4+x^2\,XH^4+9y^2-6x\,y^2-36x^2\,y^2+24x^4\,y^2+8\,XH\,y^2+30x\,XH\,y^2-32x^3\,XH\,y^2-10\,XH^2\,y^2-2x\,XH^2\,y^2+20x^2\,XH^2\,y^2-6x\,XH^3\,y^2+XH^4\,y^2-18y^4+24x^2\,y^4-4\,XH\,y^4-16x\,XH\,y^4+6\,XH^2\,y^4+8y^6-10\,y\,YH+16x\,y\,YH+30x^2\,y\,YH-8x^3\,y\,YH-16x^4\,y\,YH-20\,XH\,y\,YH-16x\,XH\,y\,YH+16x^2\,XH\,y\,YH+16x^3\,XH\,y\,YH+2\,XH^2\,y\,YH-8x\,XH^2\,y\,YH-6x^2\,XH^2\,y\,YH+4\,XH^3\,y\,YH+30y^3\,YH-8x\,y^3\,YH-32x^2\,y^3\,YH+16\,XH\,y^3\,YH+16x\,XH\,y^3\,YH-6\,XH^2\,y^3\,YH-16y^5\,YH+4\,YH^2-10x\,YH^2-10x^2\,YH^2+6x^3\,YH^2+6x^4\,YH^2+12\,XH\,YH^2+2x\,XH\,YH^2-8x^2\,XH\,YH^2-6x^3\,XH\,YH^2+2\,XH^2\,YH^2+2x^2\,XH^2\,YH^2-18y^2\,YH^2+14x\,y^2\,YH^2+20x^2\,y^2\,YH^2-16\,XH\,y^2\,YH^2-6x\,XH\,y^2\,YH^2+2\,XH^2\,y^2\,YH^2+14y^4\,YH^2+2\,y\,YH^3-8x\,y\,YH^3-6x^2\,y\,YH^3+4\,XH\,y\,YH^3-6y^3\,YH^3+YH^4+2x\,YH^4+x^2\,YH^4+y^2\,YH^4+6z^2-30x^2\,z^2+24x^4\,z^2+28x\,XH\,z^2+4x^2\,XH\,z^2-32x^3\,XH\,z^2-10\,XH^2\,z^2-8x\,XH^2\,z^2+18x^2\,XH^2\,z^2+4\,XH^3\,z^2-4x\,XH^3\,z^2-30y^2\,z^2+48x^2\,y^2\,z^2-4\,XH\,y^2\,z^2-32x\,XH\,y^2\,z^2+10\,XH^2\,y^2\,z^2+24y^4\,z^2+28\,y\,YH\,z^2-8x\,y\,YH\,z^2-32x^2\,y\,YH\,z^2+16\,XH\,y\,YH\,z^2+16x\,XH\,y\,YH\,z^2-4\,XH^2\,y\,YH\,z^2-32y^3\,YH\,z^2-10\,YH^2\,z^2+8x\,YH^2\,z^2+10x^2\,YH^2\,z^2-12\,XH\,YH^2\,z^2-4x\,XH\,YH^2\,z^2+18y^2\,YH^2\,z^2-4y\,YH^3\,z^2-12z^4+24x^2\,z^4-16x\,XH\,z^4+4\,XH^2\,z^4+24y^2\,z^4-16y\,YH\,z^4+4\,YH^2\,z^4+8z^6\}$$

In[223]:=
```
Factor[CoefficientList[% , {z, y, x}]]
```

Out[223]=
```
{
 {
  {
   {-1 + 4 XH^2 - 4 XH^3 + XH^4 + 4 YH^2 + 12 XH YH^2 + 2 XH^2 YH^2 + YH^4, -2 (-XH + XH^2 - YH^2)
      (-5 + XH^2 + YH^2), 9 - 8 XH - 18 XH^2 + 8 XH^3 + XH^4 - 10 YH^2 - 8 XH YH^2 + 2 XH^2 YH^2 + YH^4,
    -2 (-1 - 15 XH + 5 XH^2 + 3 XH^3 - 3 YH^2 + 3 XH YH^2), 2 (-9 + 2 XH + 7 XH^2 + 3 YH^2), -16 XH, 8},
   {2 (1 + 2 XH) YH (-5 + XH^2 + YH^2), -8 YH (-2 + 2 XH + XH^2 + YH^2),
    -2 YH (-15 - 8 XH + 3 XH^2 + 3 YH^2), 8 (-1 + 2 XH) YH, -16 YH, 0, 0},
   {9 + 8 XH - 10 XH^2 + XH^4 - 18 YH^2 - 16 XH YH^2 + 2 XH^2 YH^2 + YH^4,
    -2 (3 - 15 XH + XH^2 + 3 XH^3 - 7 YH^2 + 3 XH YH^2), 4 (-9 + 5 XH^2 + 5 YH^2), -32 XH, 24, 0, 0},
   {-2 YH (-15 - 8 XH + 3 XH^2 + 3 YH^2), 8 (-1 + 2 XH) YH, -32 YH, 0, 0, 0, 0},
   {2 (-9 - 2 XH + 3 XH^2 + 7 YH^2), -16 XH, 24, 0, 0, 0, 0},
   {-16 YH, 0, 0, 0, 0, 0, 0},
   {8, 0, 0, 0, 0, 0, 0}
  }, {
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
  }, {
   {2 (3 - 5 XH^2 + 2 XH^3 - 5 YH^2 - 6 XH YH^2),
    -4 (-7 XH + 2 XH^2 + XH^3 - 2 YH^2 + XH YH^2), 2 (-15 + 2 XH + 9 XH^2 + 5 YH^2), -32 XH, 24, 0, 0},
```

```
    {-4 YH (-7 - 4 XH + XH^2 + YH^2), 8 (-1 + 2 XH) YH, -32 YH, 0, 0, 0, 0},
    {2 (-15 - 2 XH + 5 XH^2 + 9 YH^2), -32 XH, 48, 0, 0, 0, 0},
    {-32 YH, 0, 0, 0, 0, 0, 0},
    {24, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
   }, {
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
   }, {
    {4 (-3 + XH^2 + YH^2), -16 XH, 24, 0, 0, 0, 0},
    {-16 YH, 0, 0, 0, 0, 0, 0},
    {24, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
   }, {
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}
   }, {
    {8, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}}
  }
 }
```

```
(*  Lemma 8 Check.  *)
```

In[238]:=
```
Factor[
 Factor[ Factor[ Factor[ (α + β c1 c2 c3 + γ1 c1^2 + γ2 c2^2 + γ3 c3^2) / H /. cSubs] /.
      r2RSubs] /. DRSubs] /. Z → z^2]
```

Out[238]=
$$\left(x^6\,\alpha - 2\,x^5\,x1\,\alpha + x^4\,x1^2\,\alpha - 2\,x^5\,x2\,\alpha + 4\,x^4\,x1\,x2\,\alpha - 2\,x^3\,x1^2\,x2\,\alpha + x^4\,x2^2\,\alpha - 2\,x^3\,x1\,x2^2\,\alpha + \text{··· 2177 ···} + x3^2\,z^4\,\gamma3 + 3\,y^2\,z^4\,\gamma3 - 2\,y\,y1\,z^4\,\gamma3 - 2\,y\,y2\,z^4\,\gamma3 + 2\,y1\,y2\,z^4\,\gamma3 - 2\,y\,y3\,z^4\,\gamma3 + y3^2\,z^4\,\gamma3 + z^6\,\gamma3\right) \Big/ \left((x2\,y1 - x3\,y1 - x1\,y2 + x3\,y2 + x1\,y3 - x2\,y3)^2\,z^2\right)$$

large output | show less | show more | show all | set size limit...

In[239]:=
```
{Numerator[%], Denominator[%]}
```

Out[239]=
```
{x^6 α - 2 x^5 x1 α + x^4 x1^2 α - 2 x^5 x2 α + 4 x^4 x1 x2 α - 2 x^3 x1^2 x2 α + x^4 x2^2 α -
  2 x^3 x1 x2^2 α + x^2 x1^2 x2^2 α - 2 x^5 x3 α + 4 x^4 x1 x3 α - 2 x^3 x1^2 x3 α + 4 x^4 x2 x3 α -
  8 x^3 x1 x2 x3 α + ··· 2159 ··· + 2 y1 y2 y3^2 z^2 γ3 + 3 x^2 z^4 γ3 - 2 x x1 z^4 γ3 - 2 x x2 z^4 γ3 +
  2 x1 x2 z^4 γ3 - 2 x x3 z^4 γ3 + x3^2 z^4 γ3 + 3 y^2 z^4 γ3 - 2 y y1 z^4 γ3 - 2 y y2 z^4 γ3 +
  2 y1 y2 z^4 γ3 - 2 y y3 z^4 γ3 + y3^2 z^4 γ3 + z^6 γ3, (··· 1 ···) (··· 1 ···)^2 (··· 1 ···)}
```

large output | show less | show more | show all | set size limit...

In[240]:=
```
Factor[CoefficientList[%, z]]
```

Out[240]=
```
{{··· 1 ···}, {0, 0, (x2 y1 - x3 y1 - x1 y2 + x3 y2 + x1 y3 - x2 y3)^2}}
```

large output | show less | show more | show all | set size limit...

In[241]:=
```
{Length[%[[1]]], Length[%[[2]]]}
```

Out[241]=
```
{7, 3}
```

In[242]:=
```
Factor[ %%[[1]][[7]] z^4 / %%[[2]][[3]] ]
```

Out[242]=
$$\frac{z^4\ (\alpha + \beta + \gamma 1 + \gamma 2 + \gamma 3)}{(x2\ y1 - x3\ y1 - x1\ y2 + x3\ y2 + x1\ y3 - x2\ y3)^2}$$

In[243]:=
```
Factor[ XYΔReduce[ (x2 y1 - x3 y1 - x1 y2 + x3 y2 + x1 y3 - x2 y3)^2 /. DSubs] /. tweak3]
```

Out[243]=
$$\frac{1}{4}\left(27 - 18\ XH^2 + 8\ XH^3 - XH^4 - 18\ YH^2 - 24\ XH\ YH^2 - 2\ XH^2\ YH^2 - YH^4\right)$$

In[244]:=
```
Factor[ XYΔReduce[ D1 D2 D3 / 4 /. DSubs] /. tweak3]
```

Out[244]=
$$\frac{1}{4}\left(27 - 18\ XH^2 + 8\ XH^3 - XH^4 - 18\ YH^2 - 24\ XH\ YH^2 - 2\ XH^2\ YH^2 - YH^4\right)$$

In[245]:=
```
Factor[%% - %]
```

Out[245]=
```
0
```

```
(*  Lemma 9 Check.  *)
```

In[246]:=
```
Factor[
 Factor[ Factor[ Factor[ (β (c1 c2 c3 - 1) + γ1 (c1^2 - 1) + γ2 (c2^2 - 1) + γ3 (c3^2 - 1)) /
          H /. cSubs] /. r2RSubs] /. DRSubs] /. Z → z^2]
```

Out[246]=
```
-((x^2 x1^2 y^2 β - x^2 x1 x2 y^2 β - x x1^2 x2 y^2 β + x^2 x2^2 y^2 β - x x1 x2^2 y^2 β + x1^2 x2^2 y^2 β -
    x^2 x1 x3 y^2 β - x x1^2 x3 y^2 β - x^2 x2 x3 y^2 β + 6 x x1 x2 x3 y^2 β - x1^2 x2 x3 y^2 β -
    x x2^2 x3 y^2 β - x1 x2^2 x3 y^2 β + x^2 x3^2 y^2 β - x x1 x3^2 y^2 β + x1^2 x3^2 y^2 β - x x2 x3^2 y^2 β -
    x1 x2 x3^2 y^2 β + x2^2 x3^2 y^2 β + x1^2 y^4 β - x1 x2 y^4 β + x2^2 y^4 β - x1 x3 y^4 β - x2 x3 y^4 β +
```

$$x3^2\,y^4\,\beta - 2\,x^3\,x1\,y\,y1\,\beta + x^3\,x2\,y\,y1\,\beta + 3\,x^2\,x1\,x2\,y\,y1\,\beta - x^2\,x2^2\,y\,y1\,\beta - x\,x1\,x2^2\,y\,y1\,\beta + x^3\,x3\,y\,y1\,\beta + 3\,x^2\,x1\,x3\,y\,y1\,\beta - 4\,x^2\,x2\,x3\,y\,y1\,\beta - 4\,x\,x1\,x2\,x3\,y\,y1\,\beta + 3\,x\,x2^2\,x3\,y\,y1\,\beta + x1\,x2^2\,x3\,y\,y1\,\beta - x^2\,x3^2\,y\,y1\,\beta - x\,x1\,x3^2\,y\,y1\,\beta + 3\,x\,x2\,x3^2\,y\,y1\,\beta + x1\,x2\,x3^2\,y\,y1\,\beta - 2\,x2^2\,x3^2\,y\,y1\,\beta - 2\,x\,x1\,y^3\,y1\,\beta + x\,x2\,y^3\,y1\,\beta + x1\,x2\,y^3\,y1\,\beta - 2\,x2^2\,y^3\,y1\,\beta + x\,x3\,y^3\,y1\,\beta + x1\,x3\,y^3\,y1\,\beta + 2\,x2\,x3\,y^3\,y1\,\beta - 2\,x3^2\,y^3\,y1\,\beta + x^4\,y1^2\,\beta - 2\,x^3\,x2\,y1^2\,\beta + x^2\,x2^2\,y1^2\,\beta - 2\,x^3\,x3\,y1^2\,\beta + 4\,x^2\,x2\,x3\,y1^2\,\beta - 2\,x\,x2^2\,x3\,y1^2\,\beta + x^2\,x3^2\,y1^2\,\beta + x2^2\,x3^2\,y1^2\,\beta + x^2\,y^2\,y1^2\,\beta - x\,x2\,y^2\,y1^2\,\beta + x2^2\,y^2\,y1^2\,\beta - x\,x3\,y^2\,y1^2\,\beta - x2\,x3\,y^2\,y1^2\,\beta + x3^2\,y^2\,y1^2\,\beta + x^3\,x1\,y\,y2\,\beta - x^2\,x1^2\,y\,y2\,\beta - 2\,x^3\,x2\,y\,y2\,\beta + 3\,x^2\,x1\,x2\,y\,y2\,\beta - x\,x1^2\,x2\,y\,y2\,\beta + x^3\,x3\,y\,y2\,\beta - 4\,x^2\,x1\,x3\,y\,y2\,\beta + 3\,x\,x1^2\,x3\,y\,y2\,\beta + 3\,x^2\,x2\,x3\,y\,y2\,\beta - 4\,x\,x1\,x2\,x3\,y\,y2\,\beta + x1^2\,x2\,x3\,y\,y2\,\beta - x^2\,x3^2\,y\,y2\,\beta + 3\,x\,x1\,x3^2\,y\,y2\,\beta - 2\,x1^2\,x3^2\,y\,y2\,\beta - x\,x2\,x3^2\,y\,y2\,\beta + x1\,x2\,x3^2\,y\,y2\,\beta + x\,x1\,y^3\,y2\,\beta - 2\,x1^2\,y^3\,y2\,\beta - 2\,x\,x2\,y^3\,y2\,\beta + x1\,x2\,y^3\,y2\,\beta + x\,x3\,y^3\,y2\,\beta + 2\,x1\,x3\,y^3\,y2\,\beta + x2\,x3\,y^3\,y2\,\beta - 2\,x3^2\,y^3\,y2\,\beta - x^4\,y1\,y2\,\beta + x^3\,x1\,y1\,y2\,\beta + x^3\,x2\,y1\,y2\,\beta - x^2\,x1\,x2\,y1\,y2\,\beta + 2\,x^3\,x3\,y1\,y2\,\beta - 2\,x^2\,x1\,x3\,y1\,y2\,\beta - 2\,x^2\,x2\,x3\,y1\,y2\,\beta + 2\,x\,x1\,x2\,x3\,y1\,y2\,\beta - x^2\,x3^2\,y1\,y2\,\beta + x\,x1\,x3^2\,y1\,y2\,\beta + x\,x2\,x3^2\,y1\,y2\,\beta - x1\,x2\,x3^2\,y1\,y2\,\beta - x^2\,y^2\,y1\,y2\,\beta + 3\,x\,x1\,y^2\,y1\,y2\,\beta + 3\,x\,x2\,y^2\,y1\,y2\,\beta - x1\,x2\,y^2\,y1\,y2\,\beta - 4\,x\,x3\,y^2\,y1\,y2\,\beta - 2\,x1\,x3\,y^2\,y1\,y2\,\beta - 2\,x2\,x3\,y^2\,y1\,y2\,\beta + 4\,x3^2\,y^2\,y1\,y2\,\beta - x^2\,y\,y1^2\,y2\,\beta - x\,x2\,y\,y1^2\,y2\,\beta + 3\,x\,x3\,y\,y1^2\,y2\,\beta + x2\,x3\,y\,y1^2\,y2\,\beta - 2\,x3^2\,y\,y1^2\,y2\,\beta + x^4\,y2^2\,\beta - 2\,x^3\,x1\,y2^2\,\beta + x^2\,x1^2\,y2^2\,\beta - 2\,x^3\,x3\,y2^2\,\beta + 4\,x^2\,x1\,x3\,y2^2\,\beta - 2\,x\,x1^2\,x3\,y2^2\,\beta + x^2\,x3^2\,y2^2\,\beta - 2\,x\,x1\,x3^2\,y2^2\,\beta + x1^2\,x3^2\,y2^2\,\beta + x^2\,y^2\,y2^2\,\beta - x\,x1\,y^2\,y2^2\,\beta + x1^2\,y^2\,y2^2\,\beta - x\,x3\,y^2\,y2^2\,\beta - x1\,x3\,y^2\,y2^2\,\beta + x3^2\,y^2\,y2^2\,\beta - x^2\,y\,y1\,y2^2\,\beta - x\,x1\,y\,y1\,y2^2\,\beta + 3\,x\,x3\,y\,y1\,y2^2\,\beta + x1\,x3\,y\,y1\,y2^2\,\beta - 2\,x3^2\,y\,y1\,y2^2\,\beta + x^2\,y1^2\,y2^2\,\beta - 2\,x\,x3\,y1^2\,y2^2\,\beta + x3^2\,y1^2\,y2^2\,\beta + x^3\,x1\,y\,y3\,\beta - x^2\,x1^2\,y\,y3\,\beta + x^3\,x2\,y\,y3\,\beta - 4\,x^2\,x1\,x2\,y\,y3\,\beta + 3\,x\,x1^2\,x2\,y\,y3\,\beta - x^2\,x2^2\,y\,y3\,\beta + 3\,x\,x1\,x2^2\,y\,y3\,\beta - 2\,x1^2\,x2^2\,y\,y3\,\beta - 2\,x^3\,x3\,y\,y3\,\beta + 3\,x^2\,x1\,x3\,y\,y3\,\beta - x\,x1^2\,x3\,y\,y3\,\beta + 3\,x^2\,x2\,x3\,y\,y3\,\beta - 4\,x\,x1\,x2\,x3\,y\,y3\,\beta + x1^2\,x2\,x3\,y\,y3\,\beta - x\,x2^2\,x3\,y\,y3\,\beta + x1\,x2^2\,x3\,y\,y3\,\beta + x\,x1\,y^3\,y3\,\beta - 2\,x1^2\,y^3\,y3\,\beta + x\,x2\,y^3\,y3\,\beta + 2\,x1\,x2\,y^3\,y3\,\beta - 2\,x2^2\,y^3\,y3\,\beta - 2\,x\,x3\,y^3\,y3\,\beta + x1\,x3\,y^3\,y3\,\beta + x2\,x3\,y^3\,y3\,\beta - x^4\,y1\,y3\,\beta + x^3\,x1\,y1\,y3\,\beta + 2\,x^3\,x2\,y1\,y3\,\beta - 2\,x^2\,x1\,x2\,y1\,y3\,\beta - x^2\,x2^2\,y1\,y3\,\beta + x\,x1\,x2^2\,y1\,y3\,\beta + x^3\,x3\,y1\,y3\,\beta - x^2\,x1\,x3\,y1\,y3\,\beta - 2\,x^2\,x2\,x3\,y1\,y3\,\beta + 2\,x\,x1\,x2\,x3\,y1\,y3\,\beta + x\,x2^2\,x3\,y1\,y3\,\beta - x1\,x2^2\,x3\,y1\,y3\,\beta - x^2\,y^2\,y1\,y3\,\beta + 3\,x\,x1\,y^2\,y1\,y3\,\beta - 4\,x\,x2\,y^2\,y1\,y3\,\beta - 2\,x1\,x2\,y^2\,y1\,y3\,\beta + 4\,x2^2\,y^2\,y1\,y3\,\beta + 3\,x\,x3\,y^2\,y1\,y3\,\beta - x1\,x3\,y^2\,y1\,y3\,\beta - 2\,x2\,x3\,y^2\,y1\,y3\,\beta - x^2\,y\,y1^2\,y3\,\beta + 3\,x\,x2\,y\,y1^2\,y3\,\beta - 2\,x2^2\,y\,y1^2\,y3\,\beta - x\,x3\,y\,y1^2\,y3\,\beta + x2\,x3\,y\,y1^2\,y3\,\beta - x^4\,y2\,y3\,\beta + 2\,x^3\,x1\,y2\,y3\,\beta - x^2\,x1^2\,y2\,y3\,\beta + x^3\,x2\,y2\,y3\,\beta - 2\,x^2\,x1\,x2\,y2\,y3\,\beta + x\,x1^2\,x2\,y2\,y3\,\beta + x^3\,x3\,y2\,y3\,\beta - 2\,x^2\,x1\,x3\,y2\,y3\,\beta + x\,x1^2\,x3\,y2\,y3\,\beta - x^2\,x2\,x3\,y2\,y3\,\beta + 2\,x\,x1\,x2\,x3\,y2\,y3\,\beta - x1^2\,x2\,x3\,y2\,y3\,\beta - x^2\,y^2\,y2\,y3\,\beta - 4\,x\,x1\,y^2\,y2\,y3\,\beta + 4\,x1^2\,y^2\,y2\,y3\,\beta + 3\,x\,x2\,y^2\,y2\,y3\,\beta - 2\,x1\,x2\,y^2\,y2\,y3\,\beta + 3\,x\,x3\,y^2\,y2\,y3\,\beta - 2\,x1\,x3\,y^2\,y2\,y3\,\beta - x2\,x3\,y^2\,y2\,y3\,\beta + 6\,x^2\,y\,y1\,y2\,y3\,\beta - 4\,x\,x1\,y\,y1\,y2\,y3\,\beta - 4\,x\,x2\,y\,y1\,y2\,y3\,\beta + 2\,x1\,x2\,y\,y1\,y2\,y3\,\beta - 4\,x\,x3\,y\,y1\,y2\,y3\,\beta + 2\,x1\,x3\,y\,y1\,y2\,y3\,\beta + 2\,x2\,x3\,y\,y1\,y2\,y3\,\beta - x^2\,y1^2\,y2\,y3\,\beta + x\,x2\,y1^2\,y2\,y3\,\beta + x\,x3\,y1^2\,y2\,y3\,\beta - x2\,x3\,y1^2\,y2\,y3\,\beta - x^2\,y\,y2^2\,y3\,\beta + 3\,x\,x1\,y\,y2^2\,y3\,\beta - 2\,x1^2\,y\,y2^2\,y3\,\beta - x\,x3\,y\,y2^2\,y3\,\beta + x1\,x3\,y\,y2^2\,y3\,\beta - x^2\,y1\,y2^2\,y3\,\beta + x\,x1\,y1\,y2^2\,y3\,\beta + x\,x3\,y1\,y2^2\,y3\,\beta - x1\,x3\,y1\,y2^2\,y3\,\beta + x^4\,y3^2\,\beta - 2\,x^3\,x1\,y3^2\,\beta + x^2\,x1^2\,y3^2\,\beta - 2\,x^3\,x2\,y3^2\,\beta + 4\,x^2\,x1\,x2\,y3^2\,\beta - 2\,x\,x1^2\,x2\,y3^2\,\beta + x^2\,x2^2\,y3^2\,\beta - 2\,x\,x1\,x2^2\,y3^2\,\beta + x1^2\,x2^2\,y3^2\,\beta + x^2\,y^2\,y3^2\,\beta - x\,x1\,y^2\,y3^2\,\beta + x1^2\,y^2\,y3^2\,\beta - x\,x2\,y^2\,y3^2\,\beta - x1\,x2\,y^2\,y3^2\,\beta + x2^2\,y^2\,y3^2\,\beta - x^2\,y\,y1\,y3^2\,\beta - x\,x1\,y\,y1\,y3^2\,\beta + 3\,x\,x2\,y\,y1\,y3^2\,\beta + x1\,x2\,y\,y1\,y3^2\,\beta - 2\,x2^2\,y\,y1\,y3^2\,\beta + x^2\,y1^2\,y3^2\,\beta - 2\,x\,x2\,y1^2\,y3^2\,\beta + x2^2\,y1^2\,y3^2\,\beta - x^2\,y\,y2\,y3^2\,\beta + 3\,x\,x1\,y\,y2\,y3^2\,\beta - 2\,x1^2\,y\,y2\,y3^2\,\beta - x\,x2\,y\,y2\,y3^2\,\beta +$$

$$x1\,x2\,y\,y2\,y3^2\,\beta - x^2\,y1\,y2\,y3^2\,\beta + x\,x1\,y1\,y2\,y3^2\,\beta + x\,x2\,y1\,y2\,y3^2\,\beta - x1\,x2\,y1\,y2\,y3^2\,\beta +$$
$$x^2\,y2^2\,y3^2\,\beta - 2\,x\,x1\,y2^2\,y3^2\,\beta + x1^2\,y2^2\,y3^2\,\beta + x^2\,x1^2\,z^2\,\beta - x^2\,x1\,x2\,z^2\,\beta - x\,x1^2\,x2\,z^2\,\beta +$$
$$x^2\,x2^2\,z^2\,\beta - x\,x1\,x2^2\,z^2\,\beta + x1^2\,x2^2\,z^2\,\beta - x^2\,x1\,x3\,z^2\,\beta - x\,x1^2\,x3\,z^2\,\beta - x^2\,x2\,x3\,z^2\,\beta +$$
$$6\,x\,x1\,x2\,x3\,z^2\,\beta - x1^2\,x2\,x3\,z^2\,\beta - x\,x2^2\,x3\,z^2\,\beta - x1\,x2^2\,x3\,z^2\,\beta + x^2\,x3^2\,z^2\,\beta -$$
$$x\,x1\,x3^2\,z^2\,\beta + x1^2\,x3^2\,z^2\,\beta - x\,x2\,x3^2\,z^2\,\beta - x1\,x2\,x3^2\,z^2\,\beta + x2^2\,x3^2\,z^2\,\beta + 2\,x1^2\,y^2\,z^2\,\beta -$$
$$2\,x1\,x2\,y^2\,z^2\,\beta + 2\,x2^2\,y^2\,z^2\,\beta - 2\,x1\,x3\,y^2\,z^2\,\beta - 2\,x2\,x3\,y^2\,z^2\,\beta + 2\,x3^2\,y^2\,z^2\,\beta -$$
$$2\,x\,x1\,y\,y1\,z^2\,\beta + x\,x2\,y\,y1\,z^2\,\beta + x1\,x2\,y\,y1\,z^2\,\beta - 2\,x2^2\,y\,y1\,z^2\,\beta + x\,x3\,y\,y1\,z^2\,\beta +$$
$$x1\,x3\,y\,y1\,z^2\,\beta + 2\,x2\,x3\,y\,y1\,z^2\,\beta - 2\,x3^2\,y\,y1\,z^2\,\beta + 2\,x^2\,y1^2\,z^2\,\beta - 2\,x\,x2\,y1^2\,z^2\,\beta +$$
$$x2^2\,y1^2\,z^2\,\beta - 2\,x\,x3\,y1^2\,z^2\,\beta + x3^2\,y1^2\,z^2\,\beta + y^2\,y1^2\,z^2\,\beta + x\,x1\,y\,y2\,z^2\,\beta - 2\,x1^2\,y\,y2\,z^2\,\beta -$$
$$2\,x\,x2\,y\,y2\,z^2\,\beta + x1\,x2\,y\,y2\,z^2\,\beta + x\,x3\,y\,y2\,z^2\,\beta + 2\,x1\,x3\,y\,y2\,z^2\,\beta + x2\,x3\,y\,y2\,z^2\,\beta -$$
$$2\,x3^2\,y\,y2\,z^2\,\beta - 2\,x^2\,y1\,y2\,z^2\,\beta + x\,x1\,y1\,y2\,z^2\,\beta + x\,x2\,y1\,y2\,z^2\,\beta + 2\,x\,x3\,y1\,y2\,z^2\,\beta -$$
$$x1\,x3\,y1\,y2\,z^2\,\beta - x2\,x3\,y1\,y2\,z^2\,\beta - y^2\,y1\,y2\,z^2\,\beta - y\,y1^2\,y2\,z^2\,\beta + 2\,x^2\,y2^2\,z^2\,\beta -$$
$$2\,x\,x1\,y2^2\,z^2\,\beta + x1^2\,y2^2\,z^2\,\beta - 2\,x\,x3\,y2^2\,z^2\,\beta + x3^2\,y2^2\,z^2\,\beta + y^2\,y2^2\,z^2\,\beta - y\,y1\,y2^2\,z^2\,\beta +$$
$$y1^2\,y2^2\,z^2\,\beta + x\,x1\,y\,y3\,z^2\,\beta - 2\,x1^2\,y\,y3\,z^2\,\beta + x\,x2\,y\,y3\,z^2\,\beta + 2\,x1\,x2\,y\,y3\,z^2\,\beta -$$
$$2\,x2^2\,y\,y3\,z^2\,\beta - 2\,x\,x3\,y\,y3\,z^2\,\beta + x1\,x3\,y\,y3\,z^2\,\beta + x2\,x3\,y\,y3\,z^2\,\beta - 2\,x^2\,y1\,y3\,z^2\,\beta +$$
$$x\,x1\,y1\,y3\,z^2\,\beta + 2\,x\,x2\,y1\,y3\,z^2\,\beta - x1\,x2\,y1\,y3\,z^2\,\beta + x\,x3\,y1\,y3\,z^2\,\beta - x2\,x3\,y1\,y3\,z^2\,\beta -$$
$$y^2\,y1\,y3\,z^2\,\beta - y\,y1^2\,y3\,z^2\,\beta - 2\,x^2\,y2\,y3\,z^2\,\beta + 2\,x\,x1\,y2\,y3\,z^2\,\beta + x\,x2\,y2\,y3\,z^2\,\beta -$$
$$x1\,x2\,y2\,y3\,z^2\,\beta + x\,x3\,y2\,y3\,z^2\,\beta - x1\,x3\,y2\,y3\,z^2\,\beta - y^2\,y2\,y3\,z^2\,\beta + 6\,y\,y1\,y2\,y3\,z^2\,\beta -$$
$$y1^2\,y2\,y3\,z^2\,\beta - y\,y2^2\,y3\,z^2\,\beta - y1\,y2^2\,y3\,z^2\,\beta + 2\,x^2\,y3^2\,z^2\,\beta - 2\,x\,x1\,y3^2\,z^2\,\beta + x1^2\,y3^2\,z^2\,\beta -$$
$$2\,x\,x2\,y3^2\,z^2\,\beta + x2^2\,y3^2\,z^2\,\beta + y^2\,y3^2\,z^2\,\beta - y\,y1\,y3^2\,z^2\,\beta + y1^2\,y3^2\,z^2\,\beta - y\,y2\,y3^2\,z^2\,\beta -$$
$$y1\,y2\,y3^2\,z^2\,\beta + y2^2\,y3^2\,z^2\,\beta + x1^2\,z^4\,\beta - x1\,x2\,z^4\,\beta + x2^2\,z^4\,\beta - x1\,x3\,z^4\,\beta - x2\,x3\,z^4\,\beta +$$
$$x3^2\,z^4\,\beta + y1^2\,z^4\,\beta - y1\,y2\,z^4\,\beta + y2^2\,z^4\,\beta - y1\,y3\,z^4\,\beta - y2\,y3\,z^4\,\beta + y3^2\,z^4\,\beta + x^2\,x2^2\,y^2\,\gamma1 -$$
$$2\,x\,x1\,x2^2\,y^2\,\gamma1 + x1^2\,x2^2\,y^2\,\gamma1 - 2\,x^2\,x2\,x3\,y^2\,\gamma1 + 4\,x\,x1\,x2\,x3\,y^2\,\gamma1 - 2\,x1^2\,x2\,x3\,y^2\,\gamma1 +$$
$$x^2\,x3^2\,y^2\,\gamma1 - 2\,x\,x1\,x3^2\,y^2\,\gamma1 + x1^2\,x3^2\,y^2\,\gamma1 + x2^2\,y^4\,\gamma1 - 2\,x2\,x3\,y^4\,\gamma1 + x3^2\,y^4\,\gamma1 -$$
$$2\,x2^2\,y^3\,y1\,\gamma1 + 4\,x2\,x3\,y^3\,y1\,\gamma1 - 2\,x3^2\,y^3\,y1\,\gamma1 + x2^2\,y^2\,y1^2\,\gamma1 - 2\,x2\,x3\,y^2\,y1^2\,\gamma1 +$$
$$x3^2\,y^2\,y1^2\,\gamma1 - 2\,x^3\,x2\,y\,y2\,\gamma1 + 4\,x^2\,x1\,x2\,y\,y2\,\gamma1 - 2\,x\,x1^2\,x2\,y\,y2\,\gamma1 + 2\,x^3\,x3\,y\,y2\,\gamma1 -$$
$$4\,x^2\,x1\,x3\,y\,y2\,\gamma1 + 2\,x\,x1^2\,x3\,y\,y2\,\gamma1 + 2\,x^2\,x2\,x3\,y\,y2\,\gamma1 - 4\,x\,x1\,x2\,x3\,y\,y2\,\gamma1 +$$
$$2\,x1^2\,x2\,x3\,y\,y2\,\gamma1 - 2\,x^2\,x3^2\,y\,y2\,\gamma1 + 4\,x\,x1\,x3^2\,y\,y2\,\gamma1 - 2\,x1^2\,x3^2\,y\,y2\,\gamma1 -$$
$$2\,x\,x2\,y^3\,y2\,\gamma1 + 2\,x\,x3\,y^3\,y2\,\gamma1 + 2\,x2\,x3\,y^3\,y2\,\gamma1 - 2\,x3^2\,y^3\,y2\,\gamma1 + 4\,x\,x2\,y^2\,y1\,y2\,\gamma1 -$$
$$4\,x\,x3\,y^2\,y1\,y2\,\gamma1 - 4\,x2\,x3\,y^2\,y1\,y2\,\gamma1 + 4\,x3^2\,y^2\,y1\,y2\,\gamma1 - 2\,x\,x2\,y\,y1^2\,y2\,\gamma1 +$$
$$2\,x\,x3\,y\,y1^2\,y2\,\gamma1 + 2\,x2\,x3\,y\,y1^2\,y2\,\gamma1 - 2\,x3^2\,y\,y1^2\,y2\,\gamma1 + x^4\,y2^2\,\gamma1 - 2\,x^3\,x1\,y2^2\,\gamma1 +$$
$$x^2\,x1^2\,y2^2\,\gamma1 - 2\,x^3\,x3\,y2^2\,\gamma1 + 4\,x^2\,x1\,x3\,y2^2\,\gamma1 - 2\,x\,x1^2\,x3\,y2^2\,\gamma1 + x^2\,x3^2\,y2^2\,\gamma1 -$$
$$2\,x\,x1\,x3^2\,y2^2\,\gamma1 + x1^2\,x3^2\,y2^2\,\gamma1 + x^2\,y^2\,y2^2\,\gamma1 - 2\,x\,x3\,y^2\,y2^2\,\gamma1 + x3^2\,y^2\,y2^2\,\gamma1 -$$
$$2\,x^2\,y\,y1\,y2^2\,\gamma1 + 4\,x\,x3\,y\,y1\,y2^2\,\gamma1 - 2\,x3^2\,y\,y1\,y2^2\,\gamma1 + x^2\,y1^2\,y2^2\,\gamma1 - 2\,x\,x3\,y1^2\,y2^2\,\gamma1 +$$
$$x3^2\,y1^2\,y2^2\,\gamma1 + 2\,x^3\,x2\,y\,y3\,\gamma1 - 4\,x^2\,x1\,x2\,y\,y3\,\gamma1 + 2\,x\,x1^2\,x2\,y\,y3\,\gamma1 - 2\,x^2\,x2^2\,y\,y3\,\gamma1 +$$
$$4\,x\,x1\,x2^2\,y\,y3\,\gamma1 - 2\,x1^2\,x2^2\,y\,y3\,\gamma1 - 2\,x^3\,x3\,y\,y3\,\gamma1 + 4\,x^2\,x1\,x3\,y\,y3\,\gamma1 -$$
$$2\,x\,x1^2\,x3\,y\,y3\,\gamma1 + 2\,x^2\,x2\,x3\,y\,y3\,\gamma1 - 4\,x\,x1\,x2\,x3\,y\,y3\,\gamma1 + 2\,x1^2\,x2\,x3\,y\,y3\,\gamma1 +$$
$$2\,x\,x2\,y^3\,y3\,\gamma1 - 2\,x2^2\,y^3\,y3\,\gamma1 - 2\,x\,x3\,y^3\,y3\,\gamma1 + 2\,x2\,x3\,y^3\,y3\,\gamma1 - 4\,x\,x2\,y^2\,y1\,y3\,\gamma1 +$$
$$4\,x2^2\,y^2\,y1\,y3\,\gamma1 + 4\,x\,x3\,y^2\,y1\,y3\,\gamma1 - 4\,x2\,x3\,y^2\,y1\,y3\,\gamma1 + 2\,x\,x2\,y\,y1^2\,y3\,\gamma1 -$$
$$2\,x2^2\,y\,y1^2\,y3\,\gamma1 - 2\,x\,x3\,y\,y1^2\,y3\,\gamma1 + 2\,x2\,x3\,y\,y1^2\,y3\,\gamma1 - 2\,x^4\,y2\,y3\,\gamma1 + 4\,x^3\,x1\,y2\,y3\,\gamma1 -$$
$$2\,x^2\,x1^2\,y2\,y3\,\gamma1 + 2\,x^3\,x2\,y2\,y3\,\gamma1 - 4\,x^2\,x1\,x2\,y2\,y3\,\gamma1 + 2\,x\,x1^2\,x2\,y2\,y3\,\gamma1 +$$
$$2\,x^3\,x3\,y2\,y3\,\gamma1 - 4\,x^2\,x1\,x3\,y2\,y3\,\gamma1 + 2\,x\,x1^2\,x3\,y2\,y3\,\gamma1 - 2\,x^2\,x2\,x3\,y2\,y3\,\gamma1 +$$
$$4\,x\,x1\,x2\,x3\,y2\,y3\,\gamma1 - 2\,x1^2\,x2\,x3\,y2\,y3\,\gamma1 - 2\,x^2\,y^2\,y2\,y3\,\gamma1 + 2\,x\,x2\,y^2\,y2\,y3\,\gamma1 +$$
$$2\,x\,x3\,y^2\,y2\,y3\,\gamma1 - 2\,x2\,x3\,y^2\,y2\,y3\,\gamma1 + 4\,x^2\,y\,y1\,y2\,y3\,\gamma1 - 4\,x\,x2\,y\,y1\,y2\,y3\,\gamma1 -$$

$$4\,x\,x3\,y\,y1\,y2\,y3\,\gamma1 + 4\,x2\,x3\,y\,y1\,y2\,y3\,\gamma1 - 2\,x^2\,y1^2\,y2\,y3\,\gamma1 + 2\,x\,x2\,y1^2\,y2\,y3\,\gamma1 +$$
$$2\,x\,x3\,y1^2\,y2\,y3\,\gamma1 - 2\,x2\,x3\,y1^2\,y2\,y3\,\gamma1 + x^4\,y3^2\,\gamma1 - 2\,x^3\,x1\,y3^2\,\gamma1 + x^2\,x1^2\,y3^2\,\gamma1 -$$
$$2\,x^3\,x2\,y3^2\,\gamma1 + 4\,x^2\,x1\,x2\,y3^2\,\gamma1 - 2\,x\,x1^2\,x2\,y3^2\,\gamma1 + x^2\,x2^2\,y3^2\,\gamma1 - 2\,x\,x1\,x2^2\,y3^2\,\gamma1 +$$
$$x1^2\,x2^2\,y3^2\,\gamma1 + x^2\,y^2\,y3^2\,\gamma1 - 2\,x\,x2\,y^2\,y3^2\,\gamma1 + x2^2\,y^2\,y3^2\,\gamma1 - 2\,x^2\,y\,y1\,y3^2\,\gamma1 +$$
$$4\,x\,x2\,y\,y1\,y3^2\,\gamma1 - 2\,x2^2\,y\,y1\,y3^2\,\gamma1 + x^2\,y1^2\,y3^2\,\gamma1 - 2\,x\,x2\,y1^2\,y3^2\,\gamma1 + x2^2\,y1^2\,y3^2\,\gamma1 +$$
$$x^2\,x2^2\,z^2\,\gamma1 - 2\,x\,x1\,x2^2\,z^2\,\gamma1 + x1^2\,x2^2\,z^2\,\gamma1 - 2\,x^2\,x2\,x3\,z^2\,\gamma1 + 4\,x\,x1\,x2\,x3\,z^2\,\gamma1 -$$
$$2\,x1^2\,x2\,x3\,z^2\,\gamma1 + x^2\,x3^2\,z^2\,\gamma1 - 2\,x\,x1\,x3^2\,z^2\,\gamma1 + x1^2\,x3^2\,z^2\,\gamma1 + 2\,x2^2\,y^2\,z^2\,\gamma1 -$$
$$4\,x2\,x3\,y^2\,z^2\,\gamma1 + 2\,x3^2\,y^2\,z^2\,\gamma1 - 2\,x2^2\,y\,y1\,z^2\,\gamma1 + 4\,x2\,x3\,y\,y1\,z^2\,\gamma1 - 2\,x3^2\,y\,y1\,z^2\,\gamma1 +$$
$$x2^2\,y1^2\,z^2\,\gamma1 - 2\,x2\,x3\,y1^2\,z^2\,\gamma1 + x3^2\,y1^2\,z^2\,\gamma1 - 2\,x\,x2\,y\,y2\,z^2\,\gamma1 + 2\,x\,x3\,y\,y2\,z^2\,\gamma1 +$$
$$2\,x2\,x3\,y\,y2\,z^2\,\gamma1 - 2\,x3^2\,y\,y2\,z^2\,\gamma1 + 2\,x^2\,y2^2\,z^2\,\gamma1 - 2\,x\,x1\,y2^2\,z^2\,\gamma1 + x1^2\,y2^2\,z^2\,\gamma1 -$$
$$2\,x\,x3\,y2^2\,z^2\,\gamma1 + x3^2\,y2^2\,z^2\,\gamma1 + y^2\,y2^2\,z^2\,\gamma1 - 2\,y\,y1\,y2^2\,z^2\,\gamma1 + y1^2\,y2^2\,z^2\,\gamma1 +$$
$$2\,x\,x2\,y\,y3\,z^2\,\gamma1 - 2\,x2^2\,y\,y3\,z^2\,\gamma1 - 2\,x\,x3\,y\,y3\,z^2\,\gamma1 + 2\,x2\,x3\,y\,y3\,z^2\,\gamma1 - 4\,x^2\,y2\,y3\,z^2\,\gamma1 +$$
$$4\,x\,x1\,y2\,y3\,z^2\,\gamma1 - 2\,x1^2\,y2\,y3\,z^2\,\gamma1 + 2\,x\,x2\,y2\,y3\,z^2\,\gamma1 + 2\,x\,x3\,y2\,y3\,z^2\,\gamma1 -$$
$$2\,x2\,x3\,y2\,y3\,z^2\,\gamma1 - 2\,y^2\,y2\,y3\,z^2\,\gamma1 + 4\,y\,y1\,y2\,y3\,z^2\,\gamma1 - 2\,y1^2\,y2\,y3\,z^2\,\gamma1 +$$
$$2\,x^2\,y3^2\,z^2\,\gamma1 - 2\,x\,x1\,y3^2\,z^2\,\gamma1 + x1^2\,y3^2\,z^2\,\gamma1 - 2\,x\,x2\,y3^2\,z^2\,\gamma1 + x2^2\,y3^2\,z^2\,\gamma1 +$$
$$y^2\,y3^2\,z^2\,\gamma1 - 2\,y\,y1\,y3^2\,z^2\,\gamma1 + y1^2\,y3^2\,z^2\,\gamma1 + x2^2\,z^4\,\gamma1 - 2\,x2\,x3\,z^4\,\gamma1 + x3^2\,z^4\,\gamma1 +$$
$$y2^2\,z^4\,\gamma1 - 2\,y2\,y3\,z^4\,\gamma1 + y3^2\,z^4\,\gamma1 + x^2\,x1^2\,y^2\,\gamma2 - 2\,x\,x1^2\,x2\,y^2\,\gamma2 + x1^2\,x2^2\,y^2\,\gamma2 -$$
$$2\,x^2\,x1\,x3\,y^2\,\gamma2 + 4\,x\,x1\,x2\,x3\,y^2\,\gamma2 - 2\,x1\,x2^2\,x3\,y^2\,\gamma2 + x^2\,x3^2\,y^2\,\gamma2 - 2\,x\,x2\,x3^2\,y^2\,\gamma2 +$$
$$x2^2\,x3^2\,y^2\,\gamma2 + x1^2\,y^4\,\gamma2 - 2\,x1\,x3\,y^4\,\gamma2 + x3^2\,y^4\,\gamma2 - 2\,x^3\,x1\,y\,y1\,\gamma2 + 4\,x^2\,x1\,x2\,y\,y1\,\gamma2 -$$
$$2\,x\,x1\,x2^2\,y\,y1\,\gamma2 + 2\,x^3\,x3\,y\,y1\,\gamma2 + 2\,x^2\,x1\,x3\,y\,y1\,\gamma2 - 4\,x^2\,x2\,x3\,y\,y1\,\gamma2 -$$
$$4\,x\,x1\,x2\,x3\,y\,y1\,\gamma2 + 2\,x\,x2^2\,x3\,y\,y1\,\gamma2 + 2\,x1\,x2^2\,x3\,y\,y1\,\gamma2 - 2\,x^2\,x3^2\,y\,y1\,\gamma2 +$$
$$4\,x\,x2\,x3^2\,y\,y1\,\gamma2 - 2\,x2^2\,x3^2\,y\,y1\,\gamma2 - 2\,x\,x1\,y^3\,y1\,\gamma2 + 2\,x\,x3\,y^3\,y1\,\gamma2 + 2\,x1\,x3\,y^3\,y1\,\gamma2 -$$
$$2\,x3^2\,y^3\,y1\,\gamma2 + x^4\,y1^2\,\gamma2 - 2\,x^3\,x2\,y1^2\,\gamma2 + x^2\,x2^2\,y1^2\,\gamma2 - 2\,x^3\,x3\,y1^2\,\gamma2 + 4\,x^2\,x2\,x3\,y1^2\,\gamma2 -$$
$$2\,x\,x2^2\,x3\,y1^2\,\gamma2 + x^2\,x3^2\,y1^2\,\gamma2 - 2\,x\,x2\,x3^2\,y1^2\,\gamma2 + x2^2\,x3^2\,y1^2\,\gamma2 + x^2\,y^2\,y1^2\,\gamma2 -$$
$$2\,x\,x3\,y^2\,y1^2\,\gamma2 + x3^2\,y^2\,y1^2\,\gamma2 - 2\,x1^2\,y^3\,y2\,\gamma2 + 4\,x1\,x3\,y^3\,y2\,\gamma2 - 2\,x3^2\,y^3\,y2\,\gamma2 +$$
$$4\,x\,x1\,y^2\,y1\,y2\,\gamma2 - 4\,x\,x3\,y^2\,y1\,y2\,\gamma2 - 4\,x1\,x3\,y^2\,y1\,y2\,\gamma2 + 4\,x3^2\,y^2\,y1\,y2\,\gamma2 -$$
$$2\,x^2\,y\,y1^2\,y2\,\gamma2 + 4\,x\,x3\,y\,y1^2\,y2\,\gamma2 - 2\,x3^2\,y\,y1^2\,y2\,\gamma2 + x1^2\,y^2\,y2^2\,\gamma2 - 2\,x1\,x3\,y^2\,y2^2\,\gamma2 +$$
$$x3^2\,y^2\,y2^2\,\gamma2 - 2\,x\,x1\,y\,y1\,y2^2\,\gamma2 + 2\,x\,x3\,y\,y1\,y2^2\,\gamma2 + 2\,x1\,x3\,y\,y1\,y2^2\,\gamma2 -$$
$$2\,x3^2\,y\,y1\,y2^2\,\gamma2 + x^2\,y1^2\,y2^2\,\gamma2 - 2\,x\,x3\,y1^2\,y2^2\,\gamma2 + x3^2\,y1^2\,y2^2\,\gamma2 + 2\,x^3\,x1\,y\,y3\,\gamma2 -$$
$$2\,x^2\,x1^2\,y\,y3\,\gamma2 - 4\,x^2\,x1\,x2\,y\,y3\,\gamma2 + 4\,x\,x1^2\,x2\,y\,y3\,\gamma2 + 2\,x\,x1\,x2^2\,y\,y3\,\gamma2 -$$
$$2\,x1^2\,x2^2\,y\,y3\,\gamma2 - 2\,x^3\,x3\,y\,y3\,\gamma2 + 2\,x^2\,x1\,x3\,y\,y3\,\gamma2 + 4\,x^2\,x2\,x3\,y\,y3\,\gamma2 -$$
$$4\,x\,x1\,x2\,x3\,y\,y3\,\gamma2 - 2\,x\,x2^2\,x3\,y\,y3\,\gamma2 + 2\,x1\,x2^2\,x3\,y\,y3\,\gamma2 + 2\,x\,x1\,y^3\,y3\,\gamma2 -$$
$$2\,x1^2\,y^3\,y3\,\gamma2 - 2\,x\,x3\,y^3\,y3\,\gamma2 + 2\,x1\,x3\,y^3\,y3\,\gamma2 - 2\,x^4\,y1\,y3\,\gamma2 + 2\,x^3\,x1\,y1\,y3\,\gamma2 +$$
$$4\,x^3\,x2\,y1\,y3\,\gamma2 - 4\,x^2\,x1\,x2\,y1\,y3\,\gamma2 - 2\,x^2\,x2^2\,y1\,y3\,\gamma2 + 2\,x\,x1\,x2^2\,y1\,y3\,\gamma2 +$$
$$2\,x^3\,x3\,y1\,y3\,\gamma2 - 2\,x^2\,x1\,x3\,y1\,y3\,\gamma2 - 4\,x^2\,x2\,x3\,y1\,y3\,\gamma2 + 4\,x\,x1\,x2\,x3\,y1\,y3\,\gamma2 +$$
$$2\,x\,x2^2\,x3\,y1\,y3\,\gamma2 - 2\,x1\,x2^2\,x3\,y1\,y3\,\gamma2 - 2\,x^2\,y^2\,y1\,y3\,\gamma2 + 2\,x\,x1\,y^2\,y1\,y3\,\gamma2 +$$
$$2\,x\,x3\,y^2\,y1\,y3\,\gamma2 - 2\,x1\,x3\,y^2\,y1\,y3\,\gamma2 - 4\,x\,x1\,y^2\,y2\,y3\,\gamma2 + 4\,x1^2\,y^2\,y2\,y3\,\gamma2 +$$
$$4\,x\,x3\,y^2\,y2\,y3\,\gamma2 - 4\,x1\,x3\,y^2\,y2\,y3\,\gamma2 + 4\,x^2\,y\,y1\,y2\,y3\,\gamma2 - 4\,x\,x1\,y\,y1\,y2\,y3\,\gamma2 -$$
$$4\,x\,x3\,y\,y1\,y2\,y3\,\gamma2 + 4\,x1\,x3\,y\,y1\,y2\,y3\,\gamma2 + 2\,x\,x1\,y\,y2^2\,y3\,\gamma2 - 2\,x1^2\,y\,y2^2\,y3\,\gamma2 -$$
$$2\,x\,x3\,y\,y2^2\,y3\,\gamma2 + 2\,x1\,x3\,y\,y2^2\,y3\,\gamma2 - 2\,x^2\,y1\,y2^2\,y3\,\gamma2 + 2\,x\,x1\,y1\,y2^2\,y3\,\gamma2 +$$
$$2\,x\,x3\,y1\,y2^2\,y3\,\gamma2 - 2\,x1\,x3\,y1\,y2^2\,y3\,\gamma2 + x^4\,y3^2\,\gamma2 - 2\,x^3\,x1\,y3^2\,\gamma2 + x^2\,x1^2\,y3^2\,\gamma2 -$$
$$2\,x^3\,x2\,y3^2\,\gamma2 + 4\,x^2\,x1\,x2\,y3^2\,\gamma2 - 2\,x\,x1^2\,x2\,y3^2\,\gamma2 + x^2\,x2^2\,y3^2\,\gamma2 - 2\,x\,x1\,x2^2\,y3^2\,\gamma2 +$$
$$x1^2\,x2^2\,y3^2\,\gamma2 + x^2\,y^2\,y3^2\,\gamma2 - 2\,x\,x1\,y^2\,y3^2\,\gamma2 + x1^2\,y^2\,y3^2\,\gamma2 - 2\,x^2\,y\,y2\,y3^2\,\gamma2 +$$
$$4\,x\,x1\,y\,y2\,y3^2\,\gamma2 - 2\,x1^2\,y\,y2\,y3^2\,\gamma2 + x^2\,y2^2\,y3^2\,\gamma2 - 2\,x\,x1\,y2^2\,y3^2\,\gamma2 + x1^2\,y2^2\,y3^2\,\gamma2 +$$

$$x^2\,x1^2\,z^2\,\gamma2 - 2\,x\,x1^2\,x2\,z^2\,\gamma2 + x1^2\,x2^2\,z^2\,\gamma2 - 2\,x^2\,x1\,x3\,z^2\,\gamma2 + 4\,x\,x1\,x2\,x3\,z^2\,\gamma2 -$$
$$2\,x1\,x2^2\,x3\,z^2\,\gamma2 + x^2\,x3^2\,z^2\,\gamma2 - 2\,x\,x2\,x3^2\,z^2\,\gamma2 + x2^2\,x3^2\,z^2\,\gamma2 + 2\,x1^2\,y^2\,z^2\,\gamma2 -$$
$$4\,x1\,x3\,y^2\,z^2\,\gamma2 + 2\,x3^2\,y^2\,z^2\,\gamma2 - 2\,x\,x1\,y\,y1\,z^2\,\gamma2 + 2\,x\,x3\,y\,y1\,z^2\,\gamma2 + 2\,x1\,x3\,y\,y1\,z^2\,\gamma2 -$$
$$2\,x3^2\,y\,y1\,z^2\,\gamma2 + 2\,x^2\,y1^2\,z^2\,\gamma2 - 2\,x\,x2\,y1^2\,z^2\,\gamma2 + x2^2\,y1^2\,z^2\,\gamma2 - 2\,x\,x3\,y1^2\,z^2\,\gamma2 +$$
$$x3^2\,y1^2\,z^2\,\gamma2 + y^2\,y1^2\,z^2\,\gamma2 - 2\,x1^2\,y\,y2\,z^2\,\gamma2 + 4\,x1\,x3\,y\,y2\,z^2\,\gamma2 - 2\,x3^2\,y\,y2\,z^2\,\gamma2 -$$
$$2\,y\,y1^2\,y2\,z^2\,\gamma2 + x1^2\,y2^2\,z^2\,\gamma2 - 2\,x1\,x3\,y2^2\,z^2\,\gamma2 + x3^2\,y2^2\,z^2\,\gamma2 + y1^2\,y2^2\,z^2\,\gamma2 +$$
$$2\,x\,x1\,y\,y3\,z^2\,\gamma2 - 2\,x1^2\,y\,y3\,z^2\,\gamma2 - 2\,x\,x3\,y\,y3\,z^2\,\gamma2 + 2\,x1\,x3\,y\,y3\,z^2\,\gamma2 - 4\,x^2\,y1\,y3\,z^2\,\gamma2 +$$
$$2\,x\,x1\,y1\,y3\,z^2\,\gamma2 + 4\,x\,x2\,y1\,y3\,z^2\,\gamma2 - 2\,x2^2\,y1\,y3\,z^2\,\gamma2 + 2\,x\,x3\,y1\,y3\,z^2\,\gamma2 -$$
$$2\,x1\,x3\,y1\,y3\,z^2\,\gamma2 - 2\,y^2\,y1\,y3\,z^2\,\gamma2 + 4\,y\,y1\,y2\,y3\,z^2\,\gamma2 - 2\,y1\,y2^2\,y3\,z^2\,\gamma2 +$$
$$2\,x^2\,y3^2\,z^2\,\gamma2 - 2\,x\,x1\,y3^2\,z^2\,\gamma2 + x1^2\,y3^2\,z^2\,\gamma2 - 2\,x\,x2\,y3^2\,z^2\,\gamma2 + x2^2\,y3^2\,z^2\,\gamma2 +$$
$$y^2\,y3^2\,z^2\,\gamma2 - 2\,y\,y2\,y3^2\,z^2\,\gamma2 + y2^2\,y3^2\,z^2\,\gamma2 + x1^2\,z^4\,\gamma2 - 2\,x1\,x3\,z^4\,\gamma2 + x3^2\,z^4\,\gamma2 +$$
$$y1^2\,z^4\,\gamma2 - 2\,y1\,y3\,z^4\,\gamma2 + y3^2\,z^4\,\gamma2 + x^2\,x1^2\,y^2\,\gamma3 - 2\,x^2\,x1\,x2\,y^2\,\gamma3 + x^2\,x2^2\,y^2\,\gamma3 -$$
$$2\,x\,x1^2\,x3\,y^2\,\gamma3 + 4\,x\,x1\,x2\,x3\,y^2\,\gamma3 - 2\,x\,x2^2\,x3\,y^2\,\gamma3 + x1^2\,x3^2\,y^2\,\gamma3 - 2\,x1\,x2\,x3^2\,y^2\,\gamma3 +$$
$$x2^2\,x3^2\,y^2\,\gamma3 + x1^2\,y^4\,\gamma3 - 2\,x1\,x2\,y^4\,\gamma3 + x2^2\,y^4\,\gamma3 - 2\,x^3\,x1\,y\,y1\,\gamma3 + 2\,x^3\,x2\,y\,y1\,\gamma3 +$$
$$2\,x^2\,x1\,x2\,y\,y1\,\gamma3 - 2\,x^2\,x2^2\,y\,y1\,\gamma3 + 4\,x^2\,x1\,x3\,y\,y1\,\gamma3 - 4\,x^2\,x2\,x3\,y\,y1\,\gamma3 -$$
$$4\,x\,x1\,x2\,x3\,y\,y1\,\gamma3 + 4\,x\,x2^2\,x3\,y\,y1\,\gamma3 - 2\,x\,x1\,x3^2\,y\,y1\,\gamma3 + 2\,x\,x2\,x3^2\,y\,y1\,\gamma3 +$$
$$2\,x1\,x2\,x3^2\,y\,y1\,\gamma3 - 2\,x2^2\,x3^2\,y\,y1\,\gamma3 - 2\,x\,x1\,y^3\,y1\,\gamma3 + 2\,x\,x2\,y^3\,y1\,\gamma3 + 2\,x1\,x2\,y^3\,y1\,\gamma3 -$$
$$2\,x2^2\,y^3\,y1\,\gamma3 + x^4\,y1^2\,\gamma3 - 2\,x^3\,x2\,y1^2\,\gamma3 + x^2\,x2^2\,y1^2\,\gamma3 - 2\,x^3\,x3\,y1^2\,\gamma3 + 4\,x^2\,x2\,x3\,y1^2\,\gamma3 -$$
$$2\,x\,x2^2\,x3\,y1^2\,\gamma3 + x^2\,x3^2\,y1^2\,\gamma3 - 2\,x\,x2\,x3^2\,y1^2\,\gamma3 + x2^2\,x3^2\,y1^2\,\gamma3 + x^2\,y^2\,y1^2\,\gamma3 -$$
$$2\,x\,x2\,y^2\,y1^2\,\gamma3 + x2^2\,y^2\,y1^2\,\gamma3 + 2\,x^3\,x1\,y\,y2\,\gamma3 - 2\,x^2\,x1^2\,y\,y2\,\gamma3 - 2\,x^3\,x2\,y\,y2\,\gamma3 +$$
$$2\,x^2\,x1\,x2\,y\,y2\,\gamma3 - 4\,x^2\,x1\,x3\,y\,y2\,\gamma3 + 4\,x\,x1^2\,x3\,y\,y2\,\gamma3 + 4\,x^2\,x2\,x3\,y\,y2\,\gamma3 -$$
$$4\,x\,x1\,x2\,x3\,y\,y2\,\gamma3 + 2\,x\,x1\,x3^2\,y\,y2\,\gamma3 - 2\,x1^2\,x3^2\,y\,y2\,\gamma3 - 2\,x\,x2\,x3^2\,y\,y2\,\gamma3 +$$
$$2\,x1\,x2\,x3^2\,y\,y2\,\gamma3 + 2\,x\,x1\,y^3\,y2\,\gamma3 - 2\,x1^2\,y^3\,y2\,\gamma3 - 2\,x\,x2\,y^3\,y2\,\gamma3 + 2\,x1\,x2\,y^3\,y2\,\gamma3 -$$
$$2\,x^4\,y1\,y2\,\gamma3 + 2\,x^3\,x1\,y1\,y2\,\gamma3 + 2\,x^3\,x2\,y1\,y2\,\gamma3 - 2\,x^2\,x1\,x2\,y1\,y2\,\gamma3 + 4\,x^3\,x3\,y1\,y2\,\gamma3 -$$
$$4\,x^2\,x1\,x3\,y1\,y2\,\gamma3 - 4\,x^2\,x2\,x3\,y1\,y2\,\gamma3 + 4\,x\,x1\,x2\,x3\,y1\,y2\,\gamma3 - 2\,x^2\,x3^2\,y1\,y2\,\gamma3 +$$
$$2\,x\,x1\,x3^2\,y1\,y2\,\gamma3 + 2\,x\,x2\,x3^2\,y1\,y2\,\gamma3 - 2\,x1\,x2\,x3^2\,y1\,y2\,\gamma3 - 2\,x^2\,y^2\,y1\,y2\,\gamma3 +$$
$$2\,x\,x1\,y^2\,y1\,y2\,\gamma3 + 2\,x\,x2\,y^2\,y1\,y2\,\gamma3 - 2\,x1\,x2\,y^2\,y1\,y2\,\gamma3 + x^4\,y2^2\,\gamma3 - 2\,x^3\,x1\,y2^2\,\gamma3 +$$
$$x^2\,x1^2\,y2^2\,\gamma3 - 2\,x^3\,x3\,y2^2\,\gamma3 + 4\,x^2\,x1\,x3\,y2^2\,\gamma3 - 2\,x\,x1^2\,x3\,y2^2\,\gamma3 + x^2\,x3^2\,y2^2\,\gamma3 -$$
$$2\,x\,x1\,x3^2\,y2^2\,\gamma3 + x1^2\,x3^2\,y2^2\,\gamma3 + x^2\,y^2\,y2^2\,\gamma3 - 2\,x\,x1\,y^2\,y2^2\,\gamma3 + x1^2\,y^2\,y2^2\,\gamma3 -$$
$$2\,x1^2\,y^3\,y3\,\gamma3 + 4\,x1\,x2\,y^3\,y3\,\gamma3 - 2\,x2^2\,y^3\,y3\,\gamma3 + 4\,x\,x1\,y^2\,y1\,y3\,\gamma3 - 4\,x\,x2\,y^2\,y1\,y3\,\gamma3 -$$
$$4\,x1\,x2\,y^2\,y1\,y3\,\gamma3 + 4\,x2^2\,y^2\,y1\,y3\,\gamma3 - 2\,x^2\,y\,y1^2\,y3\,\gamma3 + 4\,x\,x2\,y\,y1^2\,y3\,\gamma3 -$$
$$2\,x2^2\,y\,y1^2\,y3\,\gamma3 - 4\,x\,x1\,y^2\,y2\,y3\,\gamma3 + 4\,x1^2\,y^2\,y2\,y3\,\gamma3 + 4\,x\,x2\,y^2\,y2\,y3\,\gamma3 -$$
$$4\,x1\,x2\,y^2\,y2\,y3\,\gamma3 + 4\,x^2\,y\,y1\,y2\,y3\,\gamma3 - 4\,x\,x1\,y\,y1\,y2\,y3\,\gamma3 - 4\,x\,x2\,y\,y1\,y2\,y3\,\gamma3 +$$
$$4\,x1\,x2\,y\,y1\,y2\,y3\,\gamma3 - 2\,x^2\,y\,y2^2\,y3\,\gamma3 + 4\,x\,x1\,y\,y2^2\,y3\,\gamma3 - 2\,x1^2\,y\,y2^2\,y3\,\gamma3 +$$
$$x1^2\,y^2\,y3^2\,\gamma3 - 2\,x1\,x2\,y^2\,y3^2\,\gamma3 + x2^2\,y^2\,y3^2\,\gamma3 - 2\,x\,x1\,y\,y1\,y3^2\,\gamma3 + 2\,x\,x2\,y\,y1\,y3^2\,\gamma3 +$$
$$2\,x1\,x2\,y\,y1\,y3^2\,\gamma3 - 2\,x2^2\,y\,y1\,y3^2\,\gamma3 + x^2\,y1^2\,y3^2\,\gamma3 - 2\,x\,x2\,y1^2\,y3^2\,\gamma3 + x2^2\,y1^2\,y3^2\,\gamma3 +$$
$$2\,x\,x1\,y\,y2\,y3^2\,\gamma3 - 2\,x1^2\,y\,y2\,y3^2\,\gamma3 - 2\,x\,x2\,y\,y2\,y3^2\,\gamma3 + 2\,x1\,x2\,y\,y2\,y3^2\,\gamma3 -$$
$$2\,x^2\,y1\,y2\,y3^2\,\gamma3 + 2\,x\,x1\,y1\,y2\,y3^2\,\gamma3 + 2\,x\,x2\,y1\,y2\,y3^2\,\gamma3 - 2\,x1\,x2\,y1\,y2\,y3^2\,\gamma3 +$$
$$x^2\,y2^2\,y3^2\,\gamma3 - 2\,x\,x1\,y2^2\,y3^2\,\gamma3 + x1^2\,y2^2\,y3^2\,\gamma3 + x^2\,x1^2\,z^2\,\gamma3 - 2\,x^2\,x1\,x2\,z^2\,\gamma3 +$$
$$x^2\,x2^2\,z^2\,\gamma3 - 2\,x\,x1^2\,x3\,z^2\,\gamma3 + 4\,x\,x1\,x2\,x3\,z^2\,\gamma3 - 2\,x\,x2^2\,x3\,z^2\,\gamma3 + x1^2\,x3^2\,z^2\,\gamma3 -$$
$$2\,x1\,x2\,x3^2\,z^2\,\gamma3 + x2^2\,x3^2\,z^2\,\gamma3 + 2\,x1^2\,y^2\,z^2\,\gamma3 - 4\,x1\,x2\,y^2\,z^2\,\gamma3 + 2\,x2^2\,y^2\,z^2\,\gamma3 -$$
$$2\,x\,x1\,y\,y1\,z^2\,\gamma3 + 2\,x\,x2\,y\,y1\,z^2\,\gamma3 + 2\,x1\,x2\,y\,y1\,z^2\,\gamma3 - 2\,x2^2\,y\,y1\,z^2\,\gamma3 + 2\,x^2\,y1^2\,z^2\,\gamma3 -$$
$$2\,x\,x2\,y1^2\,z^2\,\gamma3 + x2^2\,y1^2\,z^2\,\gamma3 - 2\,x\,x3\,y1^2\,z^2\,\gamma3 + x3^2\,y1^2\,z^2\,\gamma3 + y^2\,y1^2\,z^2\,\gamma3 +$$
$$2\,x\,x1\,y\,y2\,z^2\,\gamma3 - 2\,x1^2\,y\,y2\,z^2\,\gamma3 - 2\,x\,x2\,y\,y2\,z^2\,\gamma3 + 2\,x1\,x2\,y\,y2\,z^2\,\gamma3 - 4\,x^2\,y1\,y2\,z^2\,\gamma3 +$$

$$2\,x\,x1\,y1\,y2\,z^2\,\gamma3 + 2\,x\,x2\,y1\,y2\,z^2\,\gamma3 - 2\,x1\,x2\,y1\,y2\,z^2\,\gamma3 + 4\,x\,x3\,y1\,y2\,z^2\,\gamma3 - 2\,x3^2\,y1\,y2\,z^2\,\gamma3 - 2\,y^2\,y1\,y2\,z^2\,\gamma3 + 2\,x^2\,y2^2\,z^2\,\gamma3 - 2\,x\,x1\,y2^2\,z^2\,\gamma3 + x1^2\,y2^2\,z^2\,\gamma3 - 2\,x\,x3\,y2^2\,z^2\,\gamma3 + x3^2\,y2^2\,z^2\,\gamma3 + y^2\,y2^2\,z^2\,\gamma3 - 2\,x1^2\,y\,y3\,z^2\,\gamma3 + 4\,x1\,x2\,y\,y3\,z^2\,\gamma3 - 2\,x2^2\,y\,y3\,z^2\,\gamma3 - 2\,y\,y1^2\,y3\,z^2\,\gamma3 + 4\,y\,y1\,y2\,y3\,z^2\,\gamma3 - 2\,y\,y2^2\,y3\,z^2\,\gamma3 + x1^2\,y3^2\,z^2\,\gamma3 - 2\,x1\,x2\,y3^2\,z^2\,\gamma3 + x2^2\,y3^2\,z^2\,\gamma3 + y1^2\,y3^2\,z^2\,\gamma3 - 2\,y1\,y2\,y3^2\,z^2\,\gamma3 + y2^2\,y3^2\,z^2\,\gamma3 + x1^2\,z^4\,\gamma3 - 2\,x1\,x2\,z^4\,\gamma3 + x2^2\,z^4\,\gamma3 + y1^2\,z^4\,\gamma3 - 2\,y1\,y2\,z^4\,\gamma3 + y2^2\,z^4\,\gamma3\Big) \Big/ \Big((x2\,y1 - x3\,y1 - x1\,y2 + x3\,y2 + x1\,y3 - x2\,y3)^2\,z^2\Big)\Big)$$

In[247]:=
```
{Numerator[%], Denominator[%]}
```

Out[247]=

$$\Big\{-x^2\,x1^2\,y^2\,\beta + x^2\,x1\,x2\,y^2\,\beta + x\,x1^2\,x2\,y^2\,\beta - x^2\,x2^2\,y^2\,\beta + x\,x1\,x2^2\,y^2\,\beta - x1^2\,x2^2\,y^2\,\beta + x^2\,x1\,x3\,y^2\,\beta + x\,x1^2\,x3\,y^2\,\beta + x^2\,x2\,x3\,y^2\,\beta - 6\,x\,x1\,x2\,x3\,y^2\,\beta + x1^2\,x2\,x3\,y^2\,\beta + x\,x2^2\,x3\,y^2\,\beta + x1\,x2^2\,x3\,y^2\,\beta - x^2\,x3^2\,y^2\,\beta + x\,x1\,x3^2\,y^2\,\beta - x1^2\,x3^2\,y^2\,\beta + x\,x2\,x3^2\,y^2\,\beta + x1\,x2\,x3^2\,y^2\,\beta - x2^2\,x3^2\,y^2\,\beta - x1^2\,y^4\,\beta + x1\,x2\,y^4\,\beta - x2^2\,y^4\,\beta + x1\,x3\,y^4\,\beta + x2\,x3\,y^4\,\beta - x3^2\,y^4\,\beta + 2\,x^3\,x1\,y\,y1\,\beta - x^3\,x2\,y\,y1\,\beta - 3\,x^2\,x1\,x2\,y\,y1\,\beta + x^2\,x2^2\,y\,y1\,\beta + x\,x1\,x2^2\,y\,y1\,\beta - x^3\,x3\,y\,y1\,\beta - 3\,x^2\,x1\,x3\,y\,y1\,\beta + 4\,x^2\,x2\,x3\,y\,y1\,\beta + 4\,x\,x1\,x2\,x3\,y\,y1\,\beta - 3\,x\,x2^2\,x3\,y\,y1\,\beta - x1\,x2^2\,x3\,y\,y1\,\beta + x^2\,x3^2\,y\,y1\,\beta + x\,x1\,x3^2\,y\,y1\,\beta - 3\,x\,x2\,x3^2\,y\,y1\,\beta - x1\,x2\,x3^2\,y\,y1\,\beta + 2\,x2^2\,x3^2\,y\,y1\,\beta + 2\,x\,x1\,y^3\,y1\,\beta - x\,x2\,y^3\,y1\,\beta - x1\,x2\,y^3\,y1\,\beta + 2\,x2^2\,y^3\,y1\,\beta - x\,x3\,y^3\,y1\,\beta - x1\,x3\,y^3\,y1\,\beta - 2\,x2\,x3\,y^3\,y1\,\beta + 2\,x3^2\,y^3\,y1\,\beta - x^4\,y1^2\,\beta + 2\,x^3\,x2\,y1^2\,\beta - x^2\,x2^2\,y1^2\,\beta + 2\,x^3\,x3\,y1^2\,\beta - 4\,x^2\,x2\,x3\,y1^2\,\beta + 2\,x\,x2^2\,x3\,y1^2\,\beta - x^2\,x3^2\,y1^2\,\beta + 2\,x\,x2\,x3^2\,y1^2\,\beta - x2^2\,x3^2\,y1^2\,\beta - x^2\,y^2\,y1^2\,\beta + x\,x2\,y^2\,y1^2\,\beta - x2^2\,y^2\,y1^2\,\beta + x\,x3\,y^2\,y1^2\,\beta + x2\,x3\,y^2\,y1^2\,\beta - x3^2\,y^2\,y1^2\,\beta - x^3\,x1\,y\,y2\,\beta + x^2\,x1^2\,y\,y2\,\beta + 2\,x^3\,x2\,y\,y2\,\beta - 3\,x^2\,x1\,x2\,y\,y2\,\beta + x\,x1^2\,x2\,y\,y2\,\beta - x^3\,x3\,y\,y2\,\beta + 4\,x^2\,x1\,x3\,y\,y2\,\beta - 3\,x\,x1^2\,x3\,y\,y2\,\beta - 3\,x^2\,x2\,x3\,y\,y2\,\beta + 4\,x\,x1\,x2\,x3\,y\,y2\,\beta - x1^2\,x2\,x3\,y\,y2\,\beta + x^2\,x3^2\,y\,y2\,\beta - 3\,x\,x1\,x3^2\,y\,y2\,\beta + 2\,x1^2\,x3^2\,y\,y2\,\beta + x\,x2\,x3^2\,y\,y2\,\beta - x1\,x2\,x3^2\,y\,y2\,\beta - x\,x1\,y^3\,y2\,\beta + 2\,x1^2\,y^3\,y2\,\beta + 2\,x\,x2\,y^3\,y2\,\beta - x1\,x2\,y^3\,y2\,\beta - x\,x3\,y^3\,y2\,\beta - 2\,x1\,x3\,y^3\,y2\,\beta - x2\,x3\,y^3\,y2\,\beta + 2\,x3^2\,y^3\,y2\,\beta + x^4\,y1\,y2\,\beta - x^3\,x1\,y1\,y2\,\beta - x^3\,x2\,y1\,y2\,\beta + x^2\,x1\,x2\,y1\,y2\,\beta - 2\,x^3\,x3\,y1\,y2\,\beta + 2\,x^2\,x1\,x3\,y1\,y2\,\beta + 2\,x^2\,x2\,x3\,y1\,y2\,\beta - 2\,x\,x1\,x2\,x3\,y1\,y2\,\beta + x^2\,x3^2\,y1\,y2\,\beta - x\,x1\,x3^2\,y1\,y2\,\beta - x\,x2\,x3^2\,y1\,y2\,\beta + x1\,x2\,x3^2\,y1\,y2\,\beta + x^2\,y^2\,y1\,y2\,\beta - 3\,x\,x1\,y^2\,y1\,y2\,\beta - 3\,x\,x2\,y^2\,y1\,y2\,\beta + x1\,x2\,y^2\,y1\,y2\,\beta + 4\,x\,x3\,y^2\,y1\,y2\,\beta + 2\,x1\,x3\,y^2\,y1\,y2\,\beta + 2\,x2\,x3\,y^2\,y1\,y2\,\beta - 4\,x3^2\,y^2\,y1\,y2\,\beta + x^2\,y\,y1^2\,y2\,\beta + x\,x2\,y\,y1^2\,y2\,\beta - 3\,x\,x3\,y\,y1^2\,y2\,\beta - x2\,x3\,y\,y1^2\,y2\,\beta + 2\,x3^2\,y\,y1^2\,y2\,\beta - x^4\,y2^2\,\beta + 2\,x^3\,x1\,y2^2\,\beta - x^2\,x1^2\,y2^2\,\beta + 2\,x^3\,x3\,y2^2\,\beta - 4\,x^2\,x1\,x3\,y2^2\,\beta + 2\,x\,x1^2\,x3\,y2^2\,\beta - x^2\,x3^2\,y2^2\,\beta + 2\,x\,x1\,x3^2\,y2^2\,\beta - x1^2\,x3^2\,y2^2\,\beta - x^2\,y^2\,y2^2\,\beta + x\,x1\,y^2\,y2^2\,\beta - x1^2\,y^2\,y2^2\,\beta + x\,x3\,y^2\,y2^2\,\beta + x1\,x3\,y^2\,y2^2\,\beta - x3^2\,y^2\,y2^2\,\beta + x^2\,y\,y1\,y2^2\,\beta + x\,x1\,y\,y1\,y2^2\,\beta - 3\,x\,x3\,y\,y1\,y2^2\,\beta - x1\,x3\,y\,y1\,y2^2\,\beta + 2\,x3^2\,y\,y1\,y2^2\,\beta - x^2\,y1^2\,y2^2\,\beta + 2\,x\,x3\,y1^2\,y2^2\,\beta - x3^2\,y1^2\,y2^2\,\beta - x^3\,x1\,y\,y3\,\beta + x^2\,x1^2\,y\,y3\,\beta - x^3\,x2\,y\,y3\,\beta + 4\,x^2\,x1\,x2\,y\,y3\,\beta - 3\,x\,x1^2\,x2\,y\,y3\,\beta + x^2\,x2^2\,y\,y3\,\beta - 3\,x\,x1\,x2^2\,y\,y3\,\beta + 2\,x1^2\,x2^2\,y\,y3\,\beta + 2\,x^3\,x3\,y\,y3\,\beta - 3\,x^2\,x1\,x3\,y\,y3\,\beta + x\,x1^2\,x3\,y\,y3\,\beta - 3\,x^2\,x2\,x3\,y\,y3\,\beta + 4\,x\,x1\,x2\,x3\,y\,y3\,\beta - x1^2\,x2\,x3\,y\,y3\,\beta + x\,x2^2\,x3\,y\,y3\,\beta - x1\,x2^2\,x3\,y\,y3\,\beta - x\,x1\,y^3\,y3\,\beta + 2\,x1^2\,y^3\,y3\,\beta - x\,x2\,y^3\,y3\,\beta - 2\,x1\,x2\,y^3\,y3\,\beta + 2\,x2^2\,y^3\,y3\,\beta + 2\,x\,x3\,y^3\,y3\,\beta - x1\,x3\,y^3\,y3\,\beta - x2\,x3\,y^3\,y3\,\beta + x^4\,y1\,y3\,\beta - x^3\,x1\,y1\,y3\,\beta - 2\,x^3\,x2\,y1\,y3\,\beta + 2\,x^2\,x1\,x2\,y1\,y3\,\beta + x^2\,x2^2\,y1\,y3\,\beta - x\,x1\,x2^2\,y1\,y3\,\beta - x^3\,x3\,y1\,y3\,\beta + x^2\,x1\,x3\,y1\,y3\,\beta + 2\,x^2\,x2\,x3\,y1\,y3\,\beta - 2\,x\,x1\,x2\,x3\,y1\,y3\,\beta - x\,x2^2\,x3\,y1\,y3\,\beta + x1\,x2^2\,x3\,y1\,y3\,\beta + x^2\,y^2\,y1\,y3\,\beta - 3\,x\,x1\,y^2\,y1\,y3\,\beta + 4\,x\,x2\,y^2\,y1\,y3\,\beta + 2\,x1\,x2\,y^2\,y1\,y3\,\beta - 4\,x2^2\,y^2\,y1\,y3\,\beta - 3\,x\,x3\,y^2\,y1\,y3\,\beta + x1\,x3\,y^2\,y1\,y3\,\beta + 2\,x2\,x3\,y^2\,y1\,y3\,\beta + x^2\,y\,y1^2\,y3\,\beta - 3\,x\,x2\,y\,y1^2\,y3\,\beta + 2\,x2^2\,y\,y1^2\,y3\,\beta + x\,x3\,y\,y1^2\,y3\,\beta - x2\,x3\,y\,y1^2\,y3\,\beta + x^4\,y2\,y3\,\beta - 2\,x^3\,x1\,y2\,y3\,\beta + x^2\,x1^2\,y2\,y3\,\beta - x^3\,x2\,y2\,y3\,\beta + 2\,x^2\,x1\,x2\,y2\,y3\,\beta -$$

$$x\,x1^2\,x2\,y2\,y3\,\beta - x^3\,x3\,y2\,y3\,\beta + 2\,x^2\,x1\,x3\,y2\,y3\,\beta - x\,x1^2\,x3\,y2\,y3\,\beta + x^2\,x2\,x3\,y2\,y3\,\beta -$$
$$2\,x\,x1\,x2\,x3\,y2\,y3\,\beta + x1^2\,x2\,x3\,y2\,y3\,\beta + x^2\,y^2\,y2\,y3\,\beta + 4\,x\,x1\,y^2\,y2\,y3\,\beta - 4\,x1^2\,y^2\,y2\,y3\,\beta -$$
$$3\,x\,x2\,y^2\,y2\,y3\,\beta + 2\,x1\,x2\,y^2\,y2\,y3\,\beta - 3\,x\,x3\,y^2\,y2\,y3\,\beta + 2\,x1\,x3\,y^2\,y2\,y3\,\beta + x2\,x3\,y^2\,y2\,y3\,\beta -$$
$$6\,x^2\,y\,y1\,y2\,y3\,\beta + 4\,x\,x1\,y\,y1\,y2\,y3\,\beta + 4\,x\,x2\,y\,y1\,y2\,y3\,\beta - 2\,x1\,x2\,y\,y1\,y2\,y3\,\beta +$$
$$4\,x\,x3\,y\,y1\,y2\,y3\,\beta - 2\,x1\,x3\,y\,y1\,y2\,y3\,\beta - 2\,x2\,x3\,y\,y1\,y2\,y3\,\beta + x^2\,y1^2\,y2\,y3\,\beta - x\,x2\,y1^2\,y2\,y3\,\beta -$$
$$x\,x3\,y1^2\,y2\,y3\,\beta + x2\,x3\,y1^2\,y2\,y3\,\beta + x^2\,y\,y2^2\,y3\,\beta - 3\,x\,x1\,y\,y2^2\,y3\,\beta + 2\,x1^2\,y\,y2^2\,y3\,\beta +$$
$$x\,x3\,y\,y2^2\,y3\,\beta - x1\,x3\,y\,y2^2\,y3\,\beta + x^2\,y1\,y2^2\,y3\,\beta - x\,x1\,y1\,y2^2\,y3\,\beta - x\,x3\,y1\,y2^2\,y3\,\beta +$$
$$x1\,x3\,y1\,y2^2\,y3\,\beta - x^4\,y3^2\,\beta + 2\,x^3\,x1\,y3^2\,\beta - x^2\,x1^2\,y3^2\,\beta + 2\,x^3\,x2\,y3^2\,\beta - 4\,x^2\,x1\,x2\,y3^2\,\beta +$$
$$2\,x\,x1^2\,x2\,y3^2\,\beta - x^2\,x2^2\,y3^2\,\beta + 2\,x\,x1\,x2^2\,y3^2\,\beta - x1^2\,x2^2\,y3^2\,\beta - x^2\,y^2\,y3^2\,\beta + x\,x1\,y^2\,y3^2\,\beta -$$
$$x1^2\,y^2\,y3^2\,\beta + x\,x2\,y^2\,y3^2\,\beta + x1\,x2\,y^2\,y3^2\,\beta - x2^2\,y^2\,y3^2\,\beta + x^2\,y\,y1\,y3^2\,\beta + x\,x1\,y\,y1\,y3^2\,\beta -$$
$$3\,x\,x2\,y\,y1\,y3^2\,\beta - x1\,x2\,y\,y1\,y3^2\,\beta + 2\,x2^2\,y\,y1\,y3^2\,\beta - x^2\,y1^2\,y3^2\,\beta + 2\,x\,x2\,y1^2\,y3^2\,\beta -$$
$$x2^2\,y1^2\,y3^2\,\beta + x^2\,y\,y2\,y3^2\,\beta - 3\,x\,x1\,y\,y2\,y3^2\,\beta + 2\,x1^2\,y\,y2\,y3^2\,\beta + x\,x2\,y\,y2\,y3^2\,\beta -$$
$$x1\,x2\,y\,y2\,y3^2\,\beta + x^2\,y1\,y2\,y3^2\,\beta - x\,x1\,y1\,y2\,y3^2\,\beta - x\,x2\,y1\,y2\,y3^2\,\beta + x1\,x2\,y1\,y2\,y3^2\,\beta -$$
$$x^2\,y2^2\,y3^2\,\beta + 2\,x\,x1\,y2^2\,y3^2\,\beta - x1^2\,y2^2\,y3^2\,\beta - x^2\,x1^2\,z^2\,\beta + x^2\,x1\,x2\,z^2\,\beta + x\,x1^2\,x2\,z^2\,\beta -$$
$$x^2\,x2^2\,z^2\,\beta + x\,x1\,x2^2\,z^2\,\beta - x1^2\,x2^2\,z^2\,\beta + x^2\,x1\,x3\,z^2\,\beta + x\,x1^2\,x3\,z^2\,\beta + x^2\,x2\,x3\,z^2\,\beta -$$
$$6\,x\,x1\,x2\,x3\,z^2\,\beta + x1^2\,x2\,x3\,z^2\,\beta + x\,x2^2\,x3\,z^2\,\beta + x1\,x2^2\,x3\,z^2\,\beta - x^2\,x3^2\,z^2\,\beta + x\,x1\,x3^2\,z^2\,\beta -$$
$$x1^2\,x3^2\,z^2\,\beta + x\,x2\,x3^2\,z^2\,\beta + x1\,x2\,x3^2\,z^2\,\beta - x2^2\,x3^2\,z^2\,\beta - 2\,x1^2\,y^2\,z^2\,\beta + 2\,x1\,x2\,y^2\,z^2\,\beta -$$
$$2\,x2^2\,y^2\,z^2\,\beta + 2\,x1\,x3\,y^2\,z^2\,\beta + 2\,x2\,x3\,y^2\,z^2\,\beta - 2\,x3^2\,y^2\,z^2\,\beta + 2\,x\,x1\,y\,y1\,z^2\,\beta - x\,x2\,y\,y1\,z^2\,\beta -$$
$$x1\,x2\,y\,y1\,z^2\,\beta + 2\,x2^2\,y\,y1\,z^2\,\beta - x\,x3\,y\,y1\,z^2\,\beta - x1\,x3\,y\,y1\,z^2\,\beta - 2\,x2\,x3\,y\,y1\,z^2\,\beta +$$
$$2\,x3^2\,y\,y1\,z^2\,\beta - 2\,x^2\,y1^2\,z^2\,\beta + 2\,x\,x2\,y1^2\,z^2\,\beta - x2^2\,y1^2\,z^2\,\beta + 2\,x\,x3\,y1^2\,z^2\,\beta - x3^2\,y1^2\,z^2\,\beta -$$
$$y^2\,y1^2\,z^2\,\beta - x\,x1\,y\,y2\,z^2\,\beta + 2\,x1^2\,y\,y2\,z^2\,\beta + 2\,x\,x2\,y\,y2\,z^2\,\beta - x1\,x2\,y\,y2\,z^2\,\beta - x\,x3\,y\,y2\,z^2\,\beta -$$
$$2\,x1\,x3\,y\,y2\,z^2\,\beta - x2\,x3\,y\,y2\,z^2\,\beta + 2\,x3^2\,y\,y2\,z^2\,\beta + 2\,x^2\,y1\,y2\,z^2\,\beta - x\,x1\,y1\,y2\,z^2\,\beta -$$
$$x\,x2\,y1\,y2\,z^2\,\beta - 2\,x\,x3\,y1\,y2\,z^2\,\beta + x1\,x3\,y1\,y2\,z^2\,\beta + x2\,x3\,y1\,y2\,z^2\,\beta + y^2\,y1\,y2\,z^2\,\beta +$$
$$y\,y1^2\,y2\,z^2\,\beta - 2\,x^2\,y2^2\,z^2\,\beta + 2\,x\,x1\,y2^2\,z^2\,\beta - x1^2\,y2^2\,z^2\,\beta + 2\,x\,x3\,y2^2\,z^2\,\beta - x3^2\,y2^2\,z^2\,\beta -$$
$$y^2\,y2^2\,z^2\,\beta + y\,y1\,y2^2\,z^2\,\beta - y1^2\,y2^2\,z^2\,\beta - x\,x1\,y\,y3\,z^2\,\beta + 2\,x1^2\,y\,y3\,z^2\,\beta - x\,x2\,y\,y3\,z^2\,\beta -$$
$$2\,x1\,x2\,y\,y3\,z^2\,\beta + 2\,x2^2\,y\,y3\,z^2\,\beta + 2\,x\,x3\,y\,y3\,z^2\,\beta - x1\,x3\,y\,y3\,z^2\,\beta - x2\,x3\,y\,y3\,z^2\,\beta +$$
$$2\,x^2\,y1\,y3\,z^2\,\beta - x\,x1\,y1\,y3\,z^2\,\beta - 2\,x\,x2\,y1\,y3\,z^2\,\beta + x1\,x2\,y1\,y3\,z^2\,\beta - x\,x3\,y1\,y3\,z^2\,\beta +$$
$$x2\,x3\,y1\,y3\,z^2\,\beta + y^2\,y1\,y3\,z^2\,\beta + y\,y1^2\,y3\,z^2\,\beta + 2\,x^2\,y2\,y3\,z^2\,\beta - 2\,x\,x1\,y2\,y3\,z^2\,\beta -$$
$$x\,x2\,y2\,y3\,z^2\,\beta + x1\,x2\,y2\,y3\,z^2\,\beta - x\,x3\,y2\,y3\,z^2\,\beta + x1\,x3\,y2\,y3\,z^2\,\beta + y^2\,y2\,y3\,z^2\,\beta -$$
$$6\,y\,y1\,y2\,y3\,z^2\,\beta + y1^2\,y2\,y3\,z^2\,\beta + y\,y2^2\,y3\,z^2\,\beta + y1\,y2^2\,y3\,z^2\,\beta - 2\,x^2\,y3^2\,z^2\,\beta + 2\,x\,x1\,y3^2\,z^2\,\beta -$$
$$x1^2\,y3^2\,z^2\,\beta + 2\,x\,x2\,y3^2\,z^2\,\beta - x2^2\,y3^2\,z^2\,\beta - y^2\,y3^2\,z^2\,\beta + y\,y1\,y3^2\,z^2\,\beta - y1^2\,y3^2\,z^2\,\beta +$$
$$y\,y2\,y3^2\,z^2\,\beta + y1\,y2\,y3^2\,z^2\,\beta - y2^2\,y3^2\,z^2\,\beta - x1^2\,z^4\,\beta + x1\,x2\,z^4\,\beta - x2^2\,z^4\,\beta + x1\,x3\,z^4\,\beta +$$
$$x2\,x3\,z^4\,\beta - x3^2\,z^4\,\beta - y1^2\,z^4\,\beta + y1\,y2\,z^4\,\beta - y2^2\,z^4\,\beta + y1\,y3\,z^4\,\beta + y2\,y3\,z^4\,\beta - y3^2\,z^4\,\beta -$$
$$x^2\,x2^2\,y^2\,\gamma 1 + 2\,x\,x1\,x2^2\,y^2\,\gamma 1 - x1^2\,x2^2\,y^2\,\gamma 1 + 2\,x^2\,x2\,x3\,y^2\,\gamma 1 - 4\,x\,x1\,x2\,x3\,y^2\,\gamma 1 +$$
$$2\,x1^2\,x2\,x3\,y^2\,\gamma 1 - x^2\,x3^2\,y^2\,\gamma 1 + 2\,x\,x1\,x3^2\,y^2\,\gamma 1 - x1^2\,x3^2\,y^2\,\gamma 1 - x2^2\,y^4\,\gamma 1 + 2\,x2\,x3\,y^4\,\gamma 1 -$$
$$x3^2\,y^4\,\gamma 1 + 2\,x2^2\,y^3\,y1\,\gamma 1 - 4\,x2\,x3\,y^3\,y1\,\gamma 1 + 2\,x3^2\,y^3\,y1\,\gamma 1 - x2^2\,y^2\,y1^2\,\gamma 1 + 2\,x2\,x3\,y^2\,y1^2\,\gamma 1 -$$
$$x3^2\,y^2\,y1^2\,\gamma 1 + 2\,x^3\,x2\,y\,y2\,\gamma 1 - 4\,x^2\,x1\,x2\,y\,y2\,\gamma 1 + 2\,x\,x1^2\,x2\,y\,y2\,\gamma 1 - 2\,x^3\,x3\,y\,y2\,\gamma 1 +$$
$$4\,x^2\,x1\,x3\,y\,y2\,\gamma 1 - 2\,x\,x1^2\,x3\,y\,y2\,\gamma 1 - 2\,x^2\,x2\,x3\,y\,y2\,\gamma 1 + 4\,x\,x1\,x2\,x3\,y\,y2\,\gamma 1 -$$
$$2\,x1^2\,x2\,x3\,y\,y2\,\gamma 1 + 2\,x^2\,x3^2\,y\,y2\,\gamma 1 - 4\,x\,x1\,x3^2\,y\,y2\,\gamma 1 + 2\,x1^2\,x3^2\,y\,y2\,\gamma 1 + 2\,x\,x2\,y^3\,y2\,\gamma 1 -$$
$$2\,x\,x3\,y^3\,y2\,\gamma 1 - 2\,x2\,x3\,y^3\,y2\,\gamma 1 + 2\,x3^2\,y^3\,y2\,\gamma 1 - 4\,x\,x2\,y^2\,y1\,y2\,\gamma 1 + 4\,x\,x3\,y^2\,y1\,y2\,\gamma 1 +$$
$$4\,x2\,x3\,y^2\,y1\,y2\,\gamma 1 - 4\,x3^2\,y^2\,y1\,y2\,\gamma 1 + 2\,x\,x2\,y\,y1^2\,y2\,\gamma 1 - 2\,x\,x3\,y\,y1^2\,y2\,\gamma 1 -$$
$$2\,x2\,x3\,y\,y1^2\,y2\,\gamma 1 + 2\,x3^2\,y\,y1^2\,y2\,\gamma 1 - x^4\,y2^2\,\gamma 1 + 2\,x^3\,x1\,y2^2\,\gamma 1 - x^2\,x1^2\,y2^2\,\gamma 1 +$$
$$2\,x^3\,x3\,y2^2\,\gamma 1 - 4\,x^2\,x1\,x3\,y2^2\,\gamma 1 + 2\,x\,x1^2\,x3\,y2^2\,\gamma 1 - x^2\,x3^2\,y2^2\,\gamma 1 + 2\,x\,x1\,x3^2\,y2^2\,\gamma 1 -$$
$$x1^2\,x3^2\,y2^2\,\gamma 1 - x^2\,y^2\,y2^2\,\gamma 1 + 2\,x\,x3\,y^2\,y2^2\,\gamma 1 - x3^2\,y^2\,y2^2\,\gamma 1 + 2\,x^2\,y\,y1\,y2^2\,\gamma 1 -$$

$$
\begin{aligned}
&4\,x\,x3\,y\,y1\,y2^2\,\gamma1 + 2\,x3^2\,y\,y1\,y2^2\,\gamma1 - x^2\,y1^2\,y2^2\,\gamma1 + 2\,x\,x3\,y1^2\,y2^2\,\gamma1 - x3^2\,y1^2\,y2^2\,\gamma1 - \\
&2\,x^3\,x2\,y\,y3\,\gamma1 + 4\,x^2\,x1\,x2\,y\,y3\,\gamma1 - 2\,x\,x1^2\,x2\,y\,y3\,\gamma1 + 2\,x^2\,x2^2\,y\,y3\,\gamma1 - 4\,x\,x1\,x2^2\,y\,y3\,\gamma1 + \\
&2\,x1^2\,x2^2\,y\,y3\,\gamma1 + 2\,x^3\,x3\,y\,y3\,\gamma1 - 4\,x^2\,x1\,x3\,y\,y3\,\gamma1 + 2\,x\,x1^2\,x3\,y\,y3\,\gamma1 - 2\,x^2\,x2\,x3\,y\,y3\,\gamma1 + \\
&4\,x\,x1\,x2\,x3\,y\,y3\,\gamma1 - 2\,x1^2\,x2\,x3\,y\,y3\,\gamma1 - 2\,x\,x2\,y^3\,y3\,\gamma1 + 2\,x2^2\,y^3\,y3\,\gamma1 + 2\,x\,x3\,y^3\,y3\,\gamma1 - \\
&2\,x2\,x3\,y^3\,y3\,\gamma1 + 4\,x\,x2\,y^2\,y1\,y3\,\gamma1 - 4\,x2^2\,y^2\,y1\,y3\,\gamma1 - 4\,x\,x3\,y^2\,y1\,y3\,\gamma1 + 4\,x2\,x3\,y^2\,y1\,y3\,\gamma1 - \\
&2\,x\,x2\,y\,y1^2\,y3\,\gamma1 + 2\,x2^2\,y\,y1^2\,y3\,\gamma1 + 2\,x\,x3\,y\,y1^2\,y3\,\gamma1 - 2\,x2\,x3\,y\,y1^2\,y3\,\gamma1 + 2\,x^4\,y2\,y3\,\gamma1 - \\
&4\,x^3\,x1\,y2\,y3\,\gamma1 + 2\,x^2\,x1^2\,y2\,y3\,\gamma1 - 2\,x^3\,x2\,y2\,y3\,\gamma1 + 4\,x^2\,x1\,x2\,y2\,y3\,\gamma1 - 2\,x\,x1^2\,x2\,y2\,y3\,\gamma1 - \\
&2\,x^3\,x3\,y2\,y3\,\gamma1 + 4\,x^2\,x1\,x3\,y2\,y3\,\gamma1 - 2\,x\,x1^2\,x3\,y2\,y3\,\gamma1 + 2\,x^2\,x2\,x3\,y2\,y3\,\gamma1 - \\
&4\,x\,x1\,x2\,x3\,y2\,y3\,\gamma1 + 2\,x1^2\,x2\,x3\,y2\,y3\,\gamma1 + 2\,x^2\,y^2\,y2\,y3\,\gamma1 - 2\,x\,x2\,y^2\,y2\,y3\,\gamma1 - \\
&2\,x\,x3\,y^2\,y2\,y3\,\gamma1 + 2\,x2\,x3\,y^2\,y2\,y3\,\gamma1 - 4\,x^2\,y\,y1\,y2\,y3\,\gamma1 + 4\,x\,x2\,y\,y1\,y2\,y3\,\gamma1 + \\
&4\,x\,x3\,y\,y1\,y2\,y3\,\gamma1 - 4\,x2\,x3\,y\,y1\,y2\,y3\,\gamma1 + 2\,x^2\,y1^2\,y2\,y3\,\gamma1 - 2\,x\,x2\,y1^2\,y2\,y3\,\gamma1 - \\
&2\,x\,x3\,y1^2\,y2\,y3\,\gamma1 + 2\,x2\,x3\,y1^2\,y2\,y3\,\gamma1 - x^4\,y3^2\,\gamma1 + 2\,x^3\,x1\,y3^2\,\gamma1 - x^2\,x1^2\,y3^2\,\gamma1 + \\
&2\,x^3\,x2\,y3^2\,\gamma1 - 4\,x^2\,x1\,x2\,y3^2\,\gamma1 + 2\,x\,x1^2\,x2\,y3^2\,\gamma1 - x^2\,x2^2\,y3^2\,\gamma1 + 2\,x\,x1\,x2^2\,y3^2\,\gamma1 - \\
&x1^2\,x2^2\,y3^2\,\gamma1 - x^2\,y^2\,y3^2\,\gamma1 + 2\,x\,x2\,y^2\,y3^2\,\gamma1 - x2^2\,y^2\,y3^2\,\gamma1 + 2\,x^2\,y\,y1\,y3^2\,\gamma1 - \\
&4\,x\,x2\,y\,y1\,y3^2\,\gamma1 + 2\,x2^2\,y\,y1\,y3^2\,\gamma1 - x^2\,y1^2\,y3^2\,\gamma1 + 2\,x\,x2\,y1^2\,y3^2\,\gamma1 - x2^2\,y1^2\,y3^2\,\gamma1 - \\
&x^2\,x2^2\,z^2\,\gamma1 + 2\,x\,x1\,x2^2\,z^2\,\gamma1 - x1^2\,x2^2\,z^2\,\gamma1 + 2\,x^2\,x2\,x3\,z^2\,\gamma1 - 4\,x\,x1\,x2\,x3\,z^2\,\gamma1 + \\
&2\,x1^2\,x2\,x3\,z^2\,\gamma1 - x^2\,x3^2\,z^2\,\gamma1 + 2\,x\,x1\,x3^2\,z^2\,\gamma1 - x1^2\,x3^2\,z^2\,\gamma1 - 2\,x2^2\,y^2\,z^2\,\gamma1 + \\
&4\,x2\,x3\,y^2\,z^2\,\gamma1 - 2\,x3^2\,y^2\,z^2\,\gamma1 + 2\,x2^2\,y\,y1\,z^2\,\gamma1 - 4\,x2\,x3\,y\,y1\,z^2\,\gamma1 + 2\,x3^2\,y\,y1\,z^2\,\gamma1 - \\
&x2^2\,y1^2\,z^2\,\gamma1 + 2\,x2\,x3\,y1^2\,z^2\,\gamma1 - x3^2\,y1^2\,z^2\,\gamma1 + 2\,x\,x2\,y\,y2\,z^2\,\gamma1 - 2\,x\,x3\,y\,y2\,z^2\,\gamma1 - \\
&2\,x2\,x3\,y\,y2\,z^2\,\gamma1 + 2\,x3^2\,y\,y2\,z^2\,\gamma1 - 2\,x^2\,y2^2\,z^2\,\gamma1 + 2\,x\,x1\,y2^2\,z^2\,\gamma1 - x1^2\,y2^2\,z^2\,\gamma1 + \\
&2\,x\,x3\,y2^2\,z^2\,\gamma1 - x3^2\,y2^2\,z^2\,\gamma1 - y^2\,y2^2\,z^2\,\gamma1 + 2\,y\,y1\,y2^2\,z^2\,\gamma1 - y1^2\,y2^2\,z^2\,\gamma1 - 2\,x\,x2\,y\,y3\,z^2\,\gamma1 + \\
&2\,x2^2\,y\,y3\,z^2\,\gamma1 + 2\,x\,x3\,y\,y3\,z^2\,\gamma1 - 2\,x2\,x3\,y\,y3\,z^2\,\gamma1 + 4\,x^2\,y2\,y3\,z^2\,\gamma1 - 4\,x\,x1\,y2\,y3\,z^2\,\gamma1 + \\
&2\,x1^2\,y2\,y3\,z^2\,\gamma1 - 2\,x\,x2\,y2\,y3\,z^2\,\gamma1 - 2\,x\,x3\,y2\,y3\,z^2\,\gamma1 + 2\,x2\,x3\,y2\,y3\,z^2\,\gamma1 + \\
&2\,y^2\,y2\,y3\,z^2\,\gamma1 - 4\,y\,y1\,y2\,y3\,z^2\,\gamma1 + 2\,y1^2\,y2\,y3\,z^2\,\gamma1 - 2\,x^2\,y3^2\,z^2\,\gamma1 + 2\,x\,x1\,y3^2\,z^2\,\gamma1 - \\
&x1^2\,y3^2\,z^2\,\gamma1 + 2\,x\,x2\,y3^2\,z^2\,\gamma1 - x2^2\,y3^2\,z^2\,\gamma1 - y^2\,y3^2\,z^2\,\gamma1 + 2\,y\,y1\,y3^2\,z^2\,\gamma1 - y1^2\,y3^2\,z^2\,\gamma1 - \\
&x2^2\,z^4\,\gamma1 + 2\,x2\,x3\,z^4\,\gamma1 - x3^2\,z^4\,\gamma1 - y2^2\,z^4\,\gamma1 + 2\,y2\,y3\,z^4\,\gamma1 - y3^2\,z^4\,\gamma1 - x^2\,x1^2\,y^2\,\gamma2 + \\
&2\,x\,x1^2\,x2\,y^2\,\gamma2 - x1^2\,x2^2\,y^2\,\gamma2 + 2\,x^2\,x1\,x3\,y^2\,\gamma2 - 4\,x\,x1\,x2\,x3\,y^2\,\gamma2 + 2\,x1\,x2^2\,x3\,y^2\,\gamma2 - \\
&x^2\,x3^2\,y^2\,\gamma2 + 2\,x\,x2\,x3^2\,y^2\,\gamma2 - x2^2\,x3^2\,y^2\,\gamma2 - x1^2\,y^4\,\gamma2 + 2\,x1\,x3\,y^4\,\gamma2 - x3^2\,y^4\,\gamma2 + \\
&2\,x^3\,x1\,y\,y1\,\gamma2 - 4\,x^2\,x1\,x2\,y\,y1\,\gamma2 + 2\,x\,x1\,x2^2\,y\,y1\,\gamma2 - 2\,x^3\,x3\,y\,y1\,\gamma2 - 2\,x^2\,x1\,x3\,y\,y1\,\gamma2 + \\
&4\,x^2\,x2\,x3\,y\,y1\,\gamma2 + 4\,x\,x1\,x2\,x3\,y\,y1\,\gamma2 - 2\,x\,x2^2\,x3\,y\,y1\,\gamma2 - 2\,x1\,x2^2\,x3\,y\,y1\,\gamma2 + \\
&2\,x^2\,x3^2\,y\,y1\,\gamma2 - 4\,x\,x2\,x3^2\,y\,y1\,\gamma2 + 2\,x2^2\,x3^2\,y\,y1\,\gamma2 + 2\,x\,x1\,y^3\,y1\,\gamma2 - 2\,x\,x3\,y^3\,y1\,\gamma2 - \\
&2\,x1\,x3\,y^3\,y1\,\gamma2 + 2\,x3^2\,y^3\,y1\,\gamma2 - x^4\,y1^2\,\gamma2 + 2\,x^3\,x2\,y1^2\,\gamma2 - x^2\,x2^2\,y1^2\,\gamma2 + 2\,x^3\,x3\,y1^2\,\gamma2 - \\
&4\,x^2\,x2\,x3\,y1^2\,\gamma2 + 2\,x\,x2^2\,x3\,y1^2\,\gamma2 - x^2\,x3^2\,y1^2\,\gamma2 + 2\,x\,x2\,x3^2\,y1^2\,\gamma2 - x2^2\,x3^2\,y1^2\,\gamma2 - \\
&x^2\,y^2\,y1^2\,\gamma2 + 2\,x\,x3\,y^2\,y1^2\,\gamma2 - x3^2\,y^2\,y1^2\,\gamma2 + 2\,x1^2\,y^3\,y2\,\gamma2 - 4\,x1\,x3\,y^3\,y2\,\gamma2 + \\
&2\,x3^2\,y^3\,y2\,\gamma2 - 4\,x\,x1\,y^2\,y1\,y2\,\gamma2 + 4\,x\,x3\,y^2\,y1\,y2\,\gamma2 + 4\,x1\,x3\,y^2\,y1\,y2\,\gamma2 - 4\,x3^2\,y^2\,y1\,y2\,\gamma2 + \\
&2\,x^2\,y\,y1^2\,y2\,\gamma2 - 4\,x\,x3\,y\,y1^2\,y2\,\gamma2 + 2\,x3^2\,y\,y1^2\,y2\,\gamma2 - x1^2\,y^2\,y2^2\,\gamma2 + 2\,x1\,x3\,y^2\,y2^2\,\gamma2 - \\
&x3^2\,y^2\,y2^2\,\gamma2 + 2\,x\,x1\,y\,y1\,y2^2\,\gamma2 - 2\,x\,x3\,y\,y1\,y2^2\,\gamma2 - 2\,x1\,x3\,y\,y1\,y2^2\,\gamma2 + 2\,x3^2\,y\,y1\,y2^2\,\gamma2 - \\
&x^2\,y1^2\,y2^2\,\gamma2 + 2\,x\,x3\,y1^2\,y2^2\,\gamma2 - x3^2\,y1^2\,y2^2\,\gamma2 - 2\,x^3\,x1\,y\,y3\,\gamma2 + 2\,x^2\,x1^2\,y\,y3\,\gamma2 + \\
&4\,x^2\,x1\,x2\,y\,y3\,\gamma2 - 4\,x\,x1^2\,x2\,y\,y3\,\gamma2 - 2\,x\,x1\,x2^2\,y\,y3\,\gamma2 + 2\,x1^2\,x2^2\,y\,y3\,\gamma2 + 2\,x^3\,x3\,y\,y3\,\gamma2 - \\
&2\,x^2\,x1\,x3\,y\,y3\,\gamma2 - 4\,x^2\,x2\,x3\,y\,y3\,\gamma2 + 4\,x\,x1\,x2\,x3\,y\,y3\,\gamma2 + 2\,x\,x2^2\,x3\,y\,y3\,\gamma2 - \\
&2\,x1\,x2^2\,x3\,y\,y3\,\gamma2 - 2\,x\,x1\,y^3\,y3\,\gamma2 + 2\,x1^2\,y^3\,y3\,\gamma2 + 2\,x\,x3\,y^3\,y3\,\gamma2 - 2\,x1\,x3\,y^3\,y3\,\gamma2 + \\
&2\,x^4\,y1\,y3\,\gamma2 - 2\,x^3\,x1\,y1\,y3\,\gamma2 - 4\,x^3\,x2\,y1\,y3\,\gamma2 + 4\,x^2\,x1\,x2\,y1\,y3\,\gamma2 + 2\,x^2\,x2^2\,y1\,y3\,\gamma2 - \\
&2\,x\,x1\,x2^2\,y1\,y3\,\gamma2 - 2\,x^3\,x3\,y1\,y3\,\gamma2 + 2\,x^2\,x1\,x3\,y1\,y3\,\gamma2 + 4\,x^2\,x2\,x3\,y1\,y3\,\gamma2 - \\
&4\,x\,x1\,x2\,x3\,y1\,y3\,\gamma2 - 2\,x\,x2^2\,x3\,y1\,y3\,\gamma2 + 2\,x1\,x2^2\,x3\,y1\,y3\,\gamma2 + 2\,x^2\,y^2\,y1\,y3\,\gamma2 -
\end{aligned}
$$

$$2\,x\,x1\,y^2\,y1\,y3\,\gamma2 - 2\,x\,x3\,y^2\,y1\,y3\,\gamma2 + 2\,x1\,x3\,y^2\,y1\,y3\,\gamma2 + 4\,x\,x1\,y^2\,y2\,y3\,\gamma2 -$$
$$4\,x1^2\,y^2\,y2\,y3\,\gamma2 - 4\,x\,x3\,y^2\,y2\,y3\,\gamma2 + 4\,x1\,x3\,y^2\,y2\,y3\,\gamma2 - 4\,x^2\,y\,y1\,y2\,y3\,\gamma2 +$$
$$4\,x\,x1\,y\,y1\,y2\,y3\,\gamma2 + 4\,x\,x3\,y\,y1\,y2\,y3\,\gamma2 - 4\,x1\,x3\,y\,y1\,y2\,y3\,\gamma2 - 2\,x\,x1\,y\,y2^2\,y3\,\gamma2 +$$
$$2\,x1^2\,y\,y2^2\,y3\,\gamma2 + 2\,x\,x3\,y\,y2^2\,y3\,\gamma2 - 2\,x1\,x3\,y\,y2^2\,y3\,\gamma2 + 2\,x^2\,y1\,y2^2\,y3\,\gamma2 -$$
$$2\,x\,x1\,y1\,y2^2\,y3\,\gamma2 - 2\,x\,x3\,y1\,y2^2\,y3\,\gamma2 + 2\,x1\,x3\,y1\,y2^2\,y3\,\gamma2 - x^4\,y3^2\,\gamma2 + 2\,x^3\,x1\,y3^2\,\gamma2 -$$
$$x^2\,x1^2\,y3^2\,\gamma2 + 2\,x^3\,x2\,y3^2\,\gamma2 - 4\,x^2\,x1\,x2\,y3^2\,\gamma2 + 2\,x\,x1^2\,x2\,y3^2\,\gamma2 - x^2\,x2^2\,y3^2\,\gamma2 +$$
$$2\,x\,x1\,x2^2\,y3^2\,\gamma2 - x1^2\,x2^2\,y3^2\,\gamma2 - x^2\,y^2\,y3^2\,\gamma2 + 2\,x\,x1\,y^2\,y3^2\,\gamma2 - x1^2\,y^2\,y3^2\,\gamma2 +$$
$$2\,x^2\,y\,y2\,y3^2\,\gamma2 - 4\,x\,x1\,y\,y2\,y3^2\,\gamma2 + 2\,x1^2\,y\,y2\,y3^2\,\gamma2 - x^2\,y2^2\,y3^2\,\gamma2 + 2\,x\,x1\,y2^2\,y3^2\,\gamma2 -$$
$$x1^2\,y2^2\,y3^2\,\gamma2 - x^2\,x1^2\,z^2\,\gamma2 + 2\,x\,x1^2\,x2\,z^2\,\gamma2 - x1^2\,x2^2\,z^2\,\gamma2 + 2\,x^2\,x1\,x3\,z^2\,\gamma2 -$$
$$4\,x\,x1\,x2\,x3\,z^2\,\gamma2 + 2\,x1\,x2^2\,x3\,z^2\,\gamma2 - x^2\,x3^2\,z^2\,\gamma2 + 2\,x\,x2\,x3^2\,z^2\,\gamma2 - x2^2\,x3^2\,z^2\,\gamma2 -$$
$$2\,x1^2\,y^2\,z^2\,\gamma2 + 4\,x1\,x3\,y^2\,z^2\,\gamma2 - 2\,x3^2\,y^2\,z^2\,\gamma2 + 2\,x\,x1\,y\,y1\,z^2\,\gamma2 - 2\,x\,x3\,y\,y1\,z^2\,\gamma2 -$$
$$2\,x1\,x3\,y\,y1\,z^2\,\gamma2 + 2\,x3^2\,y\,y1\,z^2\,\gamma2 - 2\,x^2\,y1^2\,z^2\,\gamma2 + 2\,x\,x2\,y1^2\,z^2\,\gamma2 - x2^2\,y1^2\,z^2\,\gamma2 +$$
$$2\,x\,x3\,y1^2\,z^2\,\gamma2 - x3^2\,y1^2\,z^2\,\gamma2 - y^2\,y1^2\,z^2\,\gamma2 + 2\,x1^2\,y\,y2\,z^2\,\gamma2 - 4\,x1\,x3\,y\,y2\,z^2\,\gamma2 +$$
$$2\,x3^2\,y\,y2\,z^2\,\gamma2 + 2\,y\,y1^2\,y2\,z^2\,\gamma2 - x1^2\,y2^2\,z^2\,\gamma2 + 2\,x1\,x3\,y2^2\,z^2\,\gamma2 - x3^2\,y2^2\,z^2\,\gamma2 -$$
$$y1^2\,y2^2\,z^2\,\gamma2 - 2\,x\,x1\,y\,y3\,z^2\,\gamma2 + 2\,x1^2\,y\,y3\,z^2\,\gamma2 + 2\,x\,x3\,y\,y3\,z^2\,\gamma2 - 2\,x1\,x3\,y\,y3\,z^2\,\gamma2 +$$
$$4\,x^2\,y1\,y3\,z^2\,\gamma2 - 2\,x\,x1\,y1\,y3\,z^2\,\gamma2 - 4\,x\,x2\,y1\,y3\,z^2\,\gamma2 + 2\,x2^2\,y1\,y3\,z^2\,\gamma2 -$$
$$2\,x\,x3\,y1\,y3\,z^2\,\gamma2 + 2\,x1\,x3\,y1\,y3\,z^2\,\gamma2 + 2\,y^2\,y1\,y3\,z^2\,\gamma2 - 4\,y\,y1\,y2\,y3\,z^2\,\gamma2 +$$
$$2\,y1\,y2^2\,y3\,z^2\,\gamma2 - 2\,x^2\,y3^2\,z^2\,\gamma2 + 2\,x\,x1\,y3^2\,z^2\,\gamma2 - x1^2\,y3^2\,z^2\,\gamma2 + 2\,x\,x2\,y3^2\,z^2\,\gamma2 -$$
$$x2^2\,y3^2\,z^2\,\gamma2 - y^2\,y3^2\,z^2\,\gamma2 + 2\,y\,y2\,y3^2\,z^2\,\gamma2 - y2^2\,y3^2\,z^2\,\gamma2 - x1^2\,z^4\,\gamma2 + 2\,x1\,x3\,z^4\,\gamma2 -$$
$$x3^2\,z^4\,\gamma2 - y1^2\,z^4\,\gamma2 + 2\,y1\,y3\,z^4\,\gamma2 - y3^2\,z^4\,\gamma2 - x^2\,x1^2\,y^2\,\gamma3 + 2\,x^2\,x1\,x2\,y^2\,\gamma3 -$$
$$x^2\,x2^2\,y^2\,\gamma3 + 2\,x\,x1^2\,x3\,y^2\,\gamma3 - 4\,x\,x1\,x2\,x3\,y^2\,\gamma3 + 2\,x\,x2^2\,x3\,y^2\,\gamma3 - x1^2\,x3^2\,y^2\,\gamma3 +$$
$$2\,x1\,x2\,x3^2\,y^2\,\gamma3 - x2^2\,x3^2\,y^2\,\gamma3 - x1^2\,y^4\,\gamma3 + 2\,x1\,x2\,y^4\,\gamma3 - x2^2\,y^4\,\gamma3 + 2\,x^3\,x1\,y\,y1\,\gamma3 -$$
$$2\,x^3\,x2\,y\,y1\,\gamma3 - 2\,x^2\,x1\,x2\,y\,y1\,\gamma3 + 2\,x^2\,x2^2\,y\,y1\,\gamma3 - 4\,x^2\,x1\,x3\,y\,y1\,\gamma3 + 4\,x^2\,x2\,x3\,y\,y1\,\gamma3 +$$
$$4\,x\,x1\,x2\,x3\,y\,y1\,\gamma3 - 4\,x\,x2^2\,x3\,y\,y1\,\gamma3 + 2\,x\,x1\,x3^2\,y\,y1\,\gamma3 - 2\,x\,x2\,x3^2\,y\,y1\,\gamma3 -$$
$$2\,x1\,x2\,x3^2\,y\,y1\,\gamma3 + 2\,x2^2\,x3^2\,y\,y1\,\gamma3 + 2\,x\,x1\,y^3\,y1\,\gamma3 - 2\,x\,x2\,y^3\,y1\,\gamma3 - 2\,x1\,x2\,y^3\,y1\,\gamma3 +$$
$$2\,x2^2\,y^3\,y1\,\gamma3 - x^4\,y1^2\,\gamma3 + 2\,x^3\,x2\,y1^2\,\gamma3 - x^2\,x2^2\,y1^2\,\gamma3 + 2\,x^3\,x3\,y1^2\,\gamma3 - 4\,x^2\,x2\,x3\,y1^2\,\gamma3 +$$
$$2\,x\,x2^2\,x3\,y1^2\,\gamma3 - x^2\,x3^2\,y1^2\,\gamma3 + 2\,x\,x2\,x3^2\,y1^2\,\gamma3 - x2^2\,x3^2\,y1^2\,\gamma3 - x^2\,y^2\,y1^2\,\gamma3 +$$
$$2\,x\,x2\,y^2\,y1^2\,\gamma3 - x2^2\,y^2\,y1^2\,\gamma3 - 2\,x^3\,x1\,y\,y2\,\gamma3 + 2\,x^2\,x1^2\,y\,y2\,\gamma3 + 2\,x^3\,x2\,y\,y2\,\gamma3 -$$
$$2\,x^2\,x1\,x2\,y\,y2\,\gamma3 + 4\,x^2\,x1\,x3\,y\,y2\,\gamma3 - 4\,x\,x1^2\,x3\,y\,y2\,\gamma3 - 4\,x^2\,x2\,x3\,y\,y2\,\gamma3 +$$
$$4\,x\,x1\,x2\,x3\,y\,y2\,\gamma3 - 2\,x\,x1\,x3^2\,y\,y2\,\gamma3 + 2\,x1^2\,x3^2\,y\,y2\,\gamma3 + 2\,x\,x2\,x3^2\,y\,y2\,\gamma3 -$$
$$2\,x1\,x2\,x3^2\,y\,y2\,\gamma3 - 2\,x\,x1\,y^3\,y2\,\gamma3 + 2\,x1^2\,y^3\,y2\,\gamma3 + 2\,x\,x2\,y^3\,y2\,\gamma3 - 2\,x1\,x2\,y^3\,y2\,\gamma3 +$$
$$2\,x^4\,y1\,y2\,\gamma3 - 2\,x^3\,x1\,y1\,y2\,\gamma3 - 2\,x^3\,x2\,y1\,y2\,\gamma3 + 2\,x^2\,x1\,x2\,y1\,y2\,\gamma3 - 4\,x^3\,x3\,y1\,y2\,\gamma3 +$$
$$4\,x^2\,x1\,x3\,y1\,y2\,\gamma3 + 4\,x^2\,x2\,x3\,y1\,y2\,\gamma3 - 4\,x\,x1\,x2\,x3\,y1\,y2\,\gamma3 + 2\,x^2\,x3^2\,y1\,y2\,\gamma3 -$$
$$2\,x\,x1\,x3^2\,y1\,y2\,\gamma3 - 2\,x\,x2\,x3^2\,y1\,y2\,\gamma3 + 2\,x1\,x2\,x3^2\,y1\,y2\,\gamma3 + 2\,x^2\,y^2\,y1\,y2\,\gamma3 -$$
$$2\,x\,x1\,y^2\,y1\,y2\,\gamma3 - 2\,x\,x2\,y^2\,y1\,y2\,\gamma3 + 2\,x1\,x2\,y^2\,y1\,y2\,\gamma3 - x^4\,y2^2\,\gamma3 + 2\,x^3\,x1\,y2^2\,\gamma3 -$$
$$x^2\,x1^2\,y2^2\,\gamma3 + 2\,x^3\,x3\,y2^2\,\gamma3 - 4\,x^2\,x1\,x3\,y2^2\,\gamma3 + 2\,x\,x1^2\,x3\,y2^2\,\gamma3 - x^2\,x3^2\,y2^2\,\gamma3 +$$
$$2\,x\,x1\,x3^2\,y2^2\,\gamma3 - x1^2\,x3^2\,y2^2\,\gamma3 - x^2\,y^2\,y2^2\,\gamma3 + 2\,x\,x1\,y^2\,y2^2\,\gamma3 - x1^2\,y^2\,y2^2\,\gamma3 +$$
$$2\,x1^2\,y^3\,y3\,\gamma3 - 4\,x1\,x2\,y^3\,y3\,\gamma3 + 2\,x2^2\,y^3\,y3\,\gamma3 - 4\,x\,x1\,y^2\,y1\,y3\,\gamma3 + 4\,x\,x2\,y^2\,y1\,y3\,\gamma3 +$$
$$4\,x1\,x2\,y^2\,y1\,y3\,\gamma3 - 4\,x2^2\,y^2\,y1\,y3\,\gamma3 + 2\,x^2\,y\,y1^2\,y3\,\gamma3 - 4\,x\,x2\,y\,y1^2\,y3\,\gamma3 + 2\,x2^2\,y\,y1^2\,y3\,\gamma3 +$$
$$4\,x\,x1\,y^2\,y2\,y3\,\gamma3 - 4\,x1^2\,y^2\,y2\,y3\,\gamma3 - 4\,x\,x2\,y^2\,y2\,y3\,\gamma3 + 4\,x1\,x2\,y^2\,y2\,y3\,\gamma3 -$$
$$4\,x^2\,y\,y1\,y2\,y3\,\gamma3 + 4\,x\,x1\,y\,y1\,y2\,y3\,\gamma3 + 4\,x\,x2\,y\,y1\,y2\,y3\,\gamma3 - 4\,x1\,x2\,y\,y1\,y2\,y3\,\gamma3 +$$
$$2\,x^2\,y\,y2^2\,y3\,\gamma3 - 4\,x\,x1\,y\,y2^2\,y3\,\gamma3 + 2\,x1^2\,y\,y2^2\,y3\,\gamma3 - x1^2\,y^2\,y3^2\,\gamma3 + 2\,x1\,x2\,y^2\,y3^2\,\gamma3 -$$
$$x2^2\,y^2\,y3^2\,\gamma3 + 2\,x\,x1\,y\,y1\,y3^2\,\gamma3 - 2\,x\,x2\,y\,y1\,y3^2\,\gamma3 - 2\,x1\,x2\,y\,y1\,y3^2\,\gamma3 + 2\,x2^2\,y\,y1\,y3^2\,\gamma3 -$$
$$x^2\,y1^2\,y3^2\,\gamma3 + 2\,x\,x2\,y1^2\,y3^2\,\gamma3 - x2^2\,y1^2\,y3^2\,\gamma3 - 2\,x\,x1\,y\,y2\,y3^2\,\gamma3 + 2\,x1^2\,y\,y2\,y3^2\,\gamma3 +$$

$$2\,x\,x2\,y\,y2\,y3^2\,\gamma3 - 2\,x1\,x2\,y\,y2\,y3^2\,\gamma3 + 2\,x^2\,y1\,y2\,y3^2\,\gamma3 - 2\,x\,x1\,y1\,y2\,y3^2\,\gamma3 -$$
$$2\,x\,x2\,y1\,y2\,y3^2\,\gamma3 + 2\,x1\,x2\,y1\,y2\,y3^2\,\gamma3 - x^2\,y2^2\,y3^2\,\gamma3 + 2\,x\,x1\,y2^2\,y3^2\,\gamma3 - x1^2\,y2^2\,y3^2\,\gamma3 -$$
$$x^2\,x1^2\,z^2\,\gamma3 + 2\,x^2\,x1\,x2\,z^2\,\gamma3 - x^2\,x2^2\,z^2\,\gamma3 + 2\,x\,x1^2\,x3\,z^2\,\gamma3 - 4\,x\,x1\,x2\,x3\,z^2\,\gamma3 +$$
$$2\,x\,x2^2\,x3\,z^2\,\gamma3 - x1^2\,x3^2\,z^2\,\gamma3 + 2\,x1\,x2\,x3^2\,z^2\,\gamma3 - x2^2\,x3^2\,z^2\,\gamma3 - 2\,x1^2\,y^2\,z^2\,\gamma3 +$$
$$4\,x1\,x2\,y^2\,z^2\,\gamma3 - 2\,x2^2\,y^2\,z^2\,\gamma3 + 2\,x\,x1\,y\,y1\,z^2\,\gamma3 - 2\,x\,x2\,y\,y1\,z^2\,\gamma3 - 2\,x1\,x2\,y\,y1\,z^2\,\gamma3 +$$
$$2\,x2^2\,y\,y1\,z^2\,\gamma3 - 2\,x^2\,y1^2\,z^2\,\gamma3 + 2\,x\,x2\,y1^2\,z^2\,\gamma3 - x2^2\,y1^2\,z^2\,\gamma3 + 2\,x\,x3\,y1^2\,z^2\,\gamma3 -$$
$$x3^2\,y1^2\,z^2\,\gamma3 - y^2\,y1^2\,z^2\,\gamma3 - 2\,x\,x1\,y\,y2\,z^2\,\gamma3 + 2\,x1^2\,y\,y2\,z^2\,\gamma3 + 2\,x\,x2\,y\,y2\,z^2\,\gamma3 -$$
$$2\,x1\,x2\,y\,y2\,z^2\,\gamma3 + 4\,x^2\,y1\,y2\,z^2\,\gamma3 - 2\,x\,x1\,y1\,y2\,z^2\,\gamma3 - 2\,x\,x2\,y1\,y2\,z^2\,\gamma3 + 2\,x1\,x2\,y1\,y2\,z^2\,\gamma3 -$$
$$4\,x\,x3\,y1\,y2\,z^2\,\gamma3 + 2\,x3^2\,y1\,y2\,z^2\,\gamma3 + 2\,y^2\,y1\,y2\,z^2\,\gamma3 - 2\,x^2\,y2^2\,z^2\,\gamma3 + 2\,x\,x1\,y2^2\,z^2\,\gamma3 -$$
$$x1^2\,y2^2\,z^2\,\gamma3 + 2\,x\,x3\,y2^2\,z^2\,\gamma3 - x3^2\,y2^2\,z^2\,\gamma3 - y^2\,y2^2\,z^2\,\gamma3 + 2\,x1^2\,y\,y3\,z^2\,\gamma3 -$$
$$4\,x1\,x2\,y\,y3\,z^2\,\gamma3 + 2\,x2^2\,y\,y3\,z^2\,\gamma3 + 2\,y\,y1^2\,y3\,z^2\,\gamma3 - 4\,y\,y1\,y2\,y3\,z^2\,\gamma3 + 2\,y\,y2^2\,y3\,z^2\,\gamma3 -$$
$$x1^2\,y3^2\,z^2\,\gamma3 + 2\,x1\,x2\,y3^2\,z^2\,\gamma3 - x2^2\,y3^2\,z^2\,\gamma3 - y1^2\,y3^2\,z^2\,\gamma3 + 2\,y1\,y2\,y3^2\,z^2\,\gamma3 -$$
$$y2^2\,y3^2\,z^2\,\gamma3 - x1^2\,z^4\,\gamma3 + 2\,x1\,x2\,z^4\,\gamma3 - x2^2\,z^4\,\gamma3 - y1^2\,z^4\,\gamma3 + 2\,y1\,y2\,z^4\,\gamma3 - y2^2\,z^4\,\gamma3,$$
$$(x2\,y1 - x3\,y1 - x1\,y2 + x3\,y2 + x1\,y3 - x2\,y3)^2\,z^2\}$$

In[248]:= **Factor[CoefficientList[%, z]]**

Out[248]= $\{\{-x^2\,x1^2\,y^2\,\beta + x^2\,x1\,x2\,y^2\,\beta + x\,x1^2\,x2\,y^2\,\beta - x^2\,x2^2\,y^2\,\beta + x\,x1\,x2^2\,y^2\,\beta - x1^2\,x2^2\,y^2\,\beta +$
$x^2\,x1\,x3\,y^2\,\beta + x\,x1^2\,x3\,y^2\,\beta + x^2\,x2\,x3\,y^2\,\beta - 6\,x\,x1\,x2\,x3\,y^2\,\beta + x1^2\,x2\,x3\,y^2\,\beta +$
$x\,x2^2\,x3\,y^2\,\beta + x1\,x2^2\,x3\,y^2\,\beta - x^2\,x3^2\,y^2\,\beta + x\,x1\,x3^2\,y^2\,\beta - x1^2\,x3^2\,y^2\,\beta + x\,x2\,x3^2\,y^2\,\beta +$
$x1\,x2\,x3^2\,y^2\,\beta - x2^2\,x3^2\,y^2\,\beta - x1^2\,y^4\,\beta + x1\,x2\,y^4\,\beta - x2^2\,y^4\,\beta + x1\,x3\,y^4\,\beta + x2\,x3\,y^4\,\beta -$
$x3^2\,y^4\,\beta + 2\,x^3\,x1\,y\,y1\,\beta - x^3\,x2\,y\,y1\,\beta - 3\,x^2\,x1\,x2\,y\,y1\,\beta + x^2\,x2^2\,y\,y1\,\beta + x\,x1\,x2^2\,y\,y1\,\beta -$
$x^3\,x3\,y\,y1\,\beta - 3\,x^2\,x1\,x3\,y\,y1\,\beta + 4\,x^2\,x2\,x3\,y\,y1\,\beta + 4\,x\,x1\,x2\,x3\,y\,y1\,\beta - 3\,x\,x2^2\,x3\,y\,y1\,\beta -$
$x1\,x2^2\,x3\,y\,y1\,\beta + x^2\,x3^2\,y\,y1\,\beta + x\,x1\,x3^2\,y\,y1\,\beta - 3\,x\,x2\,x3^2\,y\,y1\,\beta - x1\,x2\,x3^2\,y\,y1\,\beta +$
$2\,x2^2\,x3^2\,y\,y1\,\beta + 2\,x\,x1\,y^3\,y1\,\beta - x\,x2\,y^3\,y1\,\beta - x1\,x2\,y^3\,y1\,\beta + 2\,x2^2\,y^3\,y1\,\beta - x\,x3\,y^3\,y1\,\beta -$
$x1\,x3\,y^3\,y1\,\beta - 2\,x2\,x3\,y^3\,y1\,\beta + 2\,x3^2\,y^3\,y1\,\beta - x^4\,y1^2\,\beta + 2\,x^3\,x2\,y1^2\,\beta - x^2\,x2^2\,y1^2\,\beta +$
$2\,x^3\,x3\,y1^2\,\beta - 4\,x^2\,x2\,x3\,y1^2\,\beta + 2\,x\,x2^2\,x3\,y1^2\,\beta - x^2\,x3^2\,y1^2\,\beta + 2\,x\,x2\,x3^2\,y1^2\,\beta -$
$x2^2\,x3^2\,y1^2\,\beta - x^2\,y^2\,y1^2\,\beta + x\,x2\,y^2\,y1^2\,\beta - x2^2\,y^2\,y1^2\,\beta + x\,x3\,y^2\,y1^2\,\beta + x2\,x3\,y^2\,y1^2\,\beta -$
$x3^2\,y^2\,y1^2\,\beta - x^3\,x1\,y\,y2\,\beta + x^2\,x1^2\,y\,y2\,\beta + 2\,x^3\,x2\,y\,y2\,\beta - 3\,x^2\,x1\,x2\,y\,y2\,\beta + x\,x1^2\,x2\,y\,y2\,\beta -$
$x^3\,x3\,y\,y2\,\beta + 4\,x^2\,x1\,x3\,y\,y2\,\beta - 3\,x\,x1^2\,x3\,y\,y2\,\beta - 3\,x^2\,x2\,x3\,y\,y2\,\beta + 4\,x\,x1\,x2\,x3\,y\,y2\,\beta -$
$x1^2\,x2\,x3\,y\,y2\,\beta + x^2\,x3^2\,y\,y2\,\beta - 3\,x\,x1\,x3^2\,y\,y2\,\beta + 2\,x1^2\,x3^2\,y\,y2\,\beta + x\,x2\,x3^2\,y\,y2\,\beta -$
$x1\,x2\,x3^2\,y\,y2\,\beta - x\,x1\,y^3\,y2\,\beta + 2\,x1^2\,y^3\,y2\,\beta + 2\,x\,x2\,y^3\,y2\,\beta - x1\,x2\,y^3\,y2\,\beta - x\,x3\,y^3\,y2\,\beta -$
$2\,x1\,x3\,y^3\,y2\,\beta - x2\,x3\,y^3\,y2\,\beta + 2\,x3^2\,y^3\,y2\,\beta + x^4\,y1\,y2\,\beta - x^3\,x1\,y1\,y2\,\beta - x^3\,x2\,y1\,y2\,\beta +$
$x^2\,x1\,x2\,y1\,y2\,\beta - 2\,x^3\,x3\,y1\,y2\,\beta + 2\,x^2\,x1\,x3\,y1\,y2\,\beta + 2\,x^2\,x2\,x3\,y1\,y2\,\beta - 2\,x\,x1\,x2\,x3\,y1\,y2\,\beta +$
$x^2\,x3^2\,y1\,y2\,\beta - x\,x1\,x3^2\,y1\,y2\,\beta - x\,x2\,x3^2\,y1\,y2\,\beta + x1\,x2\,x3^2\,y1\,y2\,\beta + x^2\,y^2\,y1\,y2\,\beta -$
$3\,x\,x1\,y^2\,y1\,y2\,\beta - 3\,x\,x2\,y^2\,y1\,y2\,\beta + x1\,x2\,y^2\,y1\,y2\,\beta + 4\,x\,x3\,y^2\,y1\,y2\,\beta + 2\,x1\,x3\,y^2\,y1\,y2\,\beta +$
$2\,x2\,x3\,y^2\,y1\,y2\,\beta - 4\,x3^2\,y^2\,y1\,y2\,\beta + x^2\,y\,y1^2\,y2\,\beta + x\,x2\,y\,y1^2\,y2\,\beta - 3\,x\,x3\,y\,y1^2\,y2\,\beta -$
$x2\,x3\,y\,y1^2\,y2\,\beta + 2\,x3^2\,y\,y1^2\,y2\,\beta - x^4\,y2^2\,\beta + 2\,x^3\,x1\,y2^2\,\beta - x^2\,x1^2\,y2^2\,\beta + 2\,x^3\,x3\,y2^2\,\beta -$
$4\,x^2\,x1\,x3\,y2^2\,\beta + 2\,x\,x1^2\,x3\,y2^2\,\beta - x^2\,x3^2\,y2^2\,\beta + 2\,x\,x1\,x3^2\,y2^2\,\beta - x1^2\,x3^2\,y2^2\,\beta -$
$x^2\,y^2\,y2^2\,\beta + x\,x1\,y^2\,y2^2\,\beta - x1^2\,y^2\,y2^2\,\beta + x\,x3\,y^2\,y2^2\,\beta + x1\,x3\,y^2\,y2^2\,\beta - x3^2\,y^2\,y2^2\,\beta +$
$x^2\,y\,y1\,y2^2\,\beta + x\,x1\,y\,y1\,y2^2\,\beta - 3\,x\,x3\,y\,y1\,y2^2\,\beta - x1\,x3\,y\,y1\,y2^2\,\beta + 2\,x3^2\,y\,y1\,y2^2\,\beta -$
$x^2\,y1^2\,y2^2\,\beta + 2\,x\,x3\,y1^2\,y2^2\,\beta - x3^2\,y1^2\,y2^2\,\beta - x^3\,x1\,y\,y3\,\beta + x^2\,x1^2\,y\,y3\,\beta - x^3\,x2\,y\,y3\,\beta +$
$4\,x^2\,x1\,x2\,y\,y3\,\beta - 3\,x\,x1^2\,x2\,y\,y3\,\beta + x^2\,x2^2\,y\,y3\,\beta - 3\,x\,x1\,x2^2\,y\,y3\,\beta + 2\,x1^2\,x2^2\,y\,y3\,\beta +$
$2\,x^3\,x3\,y\,y3\,\beta - 3\,x^2\,x1\,x3\,y\,y3\,\beta + x\,x1^2\,x3\,y\,y3\,\beta - 3\,x^2\,x2\,x3\,y\,y3\,\beta + 4\,x\,x1\,x2\,x3\,y\,y3\,\beta -$

$$x1^2\,x2\,x3\,y\,y3\,\beta + x\,x2^2\,x3\,y\,y3\,\beta - x1\,x2^2\,x3\,y\,y3\,\beta - x\,x1\,y^3\,y3\,\beta + 2\,x1^2\,y^3\,y3\,\beta - x\,x2\,y^3\,y3\,\beta -$$
$$2\,x1\,x2\,y^3\,y3\,\beta + 2\,x2^2\,y^3\,y3\,\beta + 2\,x\,x3\,y^3\,y3\,\beta - x1\,x3\,y^3\,y3\,\beta - x2\,x3\,y^3\,y3\,\beta + x^4\,y1\,y3\,\beta -$$
$$x^3\,x1\,y1\,y3\,\beta - 2\,x^3\,x2\,y1\,y3\,\beta + 2\,x^2\,x1\,x2\,y1\,y3\,\beta + x^2\,x2^2\,y1\,y3\,\beta - x\,x1\,x2^2\,y1\,y3\,\beta -$$
$$x^3\,x3\,y1\,y3\,\beta + x^2\,x1\,x3\,y1\,y3\,\beta + 2\,x^2\,x2\,x3\,y1\,y3\,\beta - 2\,x\,x1\,x2\,x3\,y1\,y3\,\beta - x\,x2^2\,x3\,y1\,y3\,\beta +$$
$$x1\,x2^2\,x3\,y1\,y3\,\beta + x^2\,y^2\,y1\,y3\,\beta - 3\,x\,x1\,y^2\,y1\,y3\,\beta + 4\,x\,x2\,y^2\,y1\,y3\,\beta + 2\,x1\,x2\,y^2\,y1\,y3\,\beta -$$
$$4\,x2^2\,y^2\,y1\,y3\,\beta - 3\,x\,x3\,y^2\,y1\,y3\,\beta + x1\,x3\,y^2\,y1\,y3\,\beta + 2\,x2\,x3\,y^2\,y1\,y3\,\beta + x^2\,y\,y1^2\,y3\,\beta -$$
$$3\,x\,x2\,y\,y1^2\,y3\,\beta + 2\,x2^2\,y\,y1^2\,y3\,\beta + x\,x3\,y\,y1^2\,y3\,\beta - x2\,x3\,y\,y1^2\,y3\,\beta + x^4\,y2\,y3\,\beta -$$
$$2\,x^3\,x1\,y2\,y3\,\beta + x^2\,x1^2\,y2\,y3\,\beta - x^3\,x2\,y2\,y3\,\beta + 2\,x^2\,x1\,x2\,y2\,y3\,\beta - x\,x1^2\,x2\,y2\,y3\,\beta -$$
$$x^3\,x3\,y2\,y3\,\beta + 2\,x^2\,x1\,x3\,y2\,y3\,\beta - x\,x1^2\,x3\,y2\,y3\,\beta + x^2\,x2\,x3\,y2\,y3\,\beta - 2\,x\,x1\,x2\,x3\,y2\,y3\,\beta +$$
$$x1^2\,x2\,x3\,y2\,y3\,\beta + x^2\,y^2\,y2\,y3\,\beta + 4\,x\,x1\,y^2\,y2\,y3\,\beta - 4\,x1^2\,y^2\,y2\,y3\,\beta - 3\,x\,x2\,y^2\,y2\,y3\,\beta +$$
$$2\,x1\,x2\,y^2\,y2\,y3\,\beta - 3\,x\,x3\,y^2\,y2\,y3\,\beta + 2\,x1\,x3\,y^2\,y2\,y3\,\beta + x2\,x3\,y^2\,y2\,y3\,\beta - 6\,x^2\,y\,y1\,y2\,y3\,\beta +$$
$$4\,x\,x1\,y\,y1\,y2\,y3\,\beta + 4\,x\,x2\,y\,y1\,y2\,y3\,\beta - 2\,x1\,x2\,y\,y1\,y2\,y3\,\beta + 4\,x\,x3\,y\,y1\,y2\,y3\,\beta -$$
$$2\,x1\,x3\,y\,y1\,y2\,y3\,\beta - 2\,x2\,x3\,y\,y1\,y2\,y3\,\beta + x^2\,y1^2\,y2\,y3\,\beta - x\,x2\,y1^2\,y2\,y3\,\beta -$$
$$x\,x3\,y1^2\,y2\,y3\,\beta + x2\,x3\,y1^2\,y2\,y3\,\beta + x^2\,y\,y2^2\,y3\,\beta - 3\,x\,x1\,y\,y2^2\,y3\,\beta + 2\,x1^2\,y\,y2^2\,y3\,\beta +$$
$$x\,x3\,y\,y2^2\,y3\,\beta - x1\,x3\,y\,y2^2\,y3\,\beta + x^2\,y1\,y2^2\,y3\,\beta - x\,x1\,y1\,y2^2\,y3\,\beta - x\,x3\,y1\,y2^2\,y3\,\beta +$$
$$x1\,x3\,y1\,y2^2\,y3\,\beta - x^4\,y3^2\,\beta + 2\,x^3\,x1\,y3^2\,\beta - x^2\,x1^2\,y3^2\,\beta + 2\,x^3\,x2\,y3^2\,\beta - 4\,x^2\,x1\,x2\,y3^2\,\beta +$$
$$2\,x\,x1^2\,x2\,y3^2\,\beta - x^2\,x2^2\,y3^2\,\beta + 2\,x\,x1\,x2^2\,y3^2\,\beta - x1^2\,x2^2\,y3^2\,\beta - x^2\,y^2\,y3^2\,\beta + x\,x1\,y^2\,y3^2\,\beta -$$
$$x1^2\,y^2\,y3^2\,\beta + x\,x2\,y^2\,y3^2\,\beta + x1\,x2\,y^2\,y3^2\,\beta - x2^2\,y^2\,y3^2\,\beta + x^2\,y\,y1\,y3^2\,\beta + x\,x1\,y\,y1\,y3^2\,\beta -$$
$$3\,x\,x2\,y\,y1\,y3^2\,\beta - x1\,x2\,y\,y1\,y3^2\,\beta + 2\,x2^2\,y\,y1\,y3^2\,\beta - x^2\,y1^2\,y3^2\,\beta + 2\,x\,x2\,y1^2\,y3^2\,\beta -$$
$$x2^2\,y1^2\,y3^2\,\beta + x^2\,y\,y2\,y3^2\,\beta - 3\,x\,x1\,y\,y2\,y3^2\,\beta + 2\,x1^2\,y\,y2\,y3^2\,\beta + x\,x2\,y\,y2\,y3^2\,\beta -$$
$$x1\,x2\,y\,y2\,y3^2\,\beta + x^2\,y1\,y2\,y3^2\,\beta - x\,x1\,y1\,y2\,y3^2\,\beta - x\,x2\,y1\,y2\,y3^2\,\beta + x1\,x2\,y1\,y2\,y3^2\,\beta -$$
$$x^2\,y2^2\,y3^2\,\beta + 2\,x\,x1\,y2^2\,y3^2\,\beta - x1^2\,y2^2\,y3^2\,\beta - x^2\,x2^2\,y^2\,\gamma1 + 2\,x\,x1\,x2^2\,y^2\,\gamma1 - x1^2\,x2^2\,y^2\,\gamma1 +$$
$$2\,x^2\,x2\,x3\,y^2\,\gamma1 - 4\,x\,x1\,x2\,x3\,y^2\,\gamma1 + 2\,x1^2\,x2\,x3\,y^2\,\gamma1 - x^2\,x3^2\,y^2\,\gamma1 + 2\,x\,x1\,x3^2\,y^2\,\gamma1 -$$
$$x1^2\,x3^2\,y^2\,\gamma1 - x2^2\,y^4\,\gamma1 + 2\,x2\,x3\,y^4\,\gamma1 - x3^2\,y^4\,\gamma1 + 2\,x2^2\,y^3\,y1\,\gamma1 - 4\,x2\,x3\,y^3\,y1\,\gamma1 +$$
$$2\,x3^2\,y^3\,y1\,\gamma1 - x2^2\,y^2\,y1^2\,\gamma1 + 2\,x2\,x3\,y^2\,y1^2\,\gamma1 - x3^2\,y^2\,y1^2\,\gamma1 + 2\,x^3\,x2\,y\,y2\,\gamma1 -$$
$$4\,x^2\,x1\,x2\,y\,y2\,\gamma1 + 2\,x\,x1^2\,x2\,y\,y2\,\gamma1 - 2\,x^3\,x3\,y\,y2\,\gamma1 + 4\,x^2\,x1\,x3\,y\,y2\,\gamma1 - 2\,x\,x1^2\,x3\,y\,y2\,\gamma1 -$$
$$2\,x^2\,x2\,x3\,y\,y2\,\gamma1 + 4\,x\,x1\,x2\,x3\,y\,y2\,\gamma1 - 2\,x1^2\,x2\,x3\,y\,y2\,\gamma1 + 2\,x^2\,x3^2\,y\,y2\,\gamma1 -$$
$$4\,x\,x1\,x3^2\,y\,y2\,\gamma1 + 2\,x1^2\,x3^2\,y\,y2\,\gamma1 + 2\,x\,x2\,y^3\,y2\,\gamma1 - 2\,x\,x3\,y^3\,y2\,\gamma1 - 2\,x2\,x3\,y^3\,y2\,\gamma1 +$$
$$2\,x3^2\,y^3\,y2\,\gamma1 - 4\,x\,x2\,y^2\,y1\,y2\,\gamma1 + 4\,x\,x3\,y^2\,y1\,y2\,\gamma1 + 4\,x2\,x3\,y^2\,y1\,y2\,\gamma1 - 4\,x3^2\,y^2\,y1\,y2\,\gamma1 +$$
$$2\,x\,x2\,y\,y1^2\,y2\,\gamma1 - 2\,x\,x3\,y\,y1^2\,y2\,\gamma1 - 2\,x2\,x3\,y\,y1^2\,y2\,\gamma1 + 2\,x3^2\,y\,y1^2\,y2\,\gamma1 - x^4\,y2^2\,\gamma1 +$$
$$2\,x^3\,x1\,y2^2\,\gamma1 - x^2\,x1^2\,y2^2\,\gamma1 + 2\,x^3\,x3\,y2^2\,\gamma1 - 4\,x^2\,x1\,x3\,y2^2\,\gamma1 + 2\,x\,x1^2\,x3\,y2^2\,\gamma1 -$$
$$x^2\,x3^2\,y2^2\,\gamma1 + 2\,x\,x1\,x3^2\,y2^2\,\gamma1 - x1^2\,x3^2\,y2^2\,\gamma1 - x^2\,y^2\,y2^2\,\gamma1 + 2\,x\,x3\,y^2\,y2^2\,\gamma1 -$$
$$x3^2\,y^2\,y2^2\,\gamma1 + 2\,x^2\,y\,y1\,y2^2\,\gamma1 - 4\,x\,x3\,y\,y1\,y2^2\,\gamma1 + 2\,x3^2\,y\,y1\,y2^2\,\gamma1 - x^2\,y1^2\,y2^2\,\gamma1 +$$
$$2\,x\,x3\,y1^2\,y2^2\,\gamma1 - x3^2\,y1^2\,y2^2\,\gamma1 - 2\,x^3\,x2\,y\,y3\,\gamma1 + 4\,x^2\,x1\,x2\,y\,y3\,\gamma1 - 2\,x\,x1^2\,x2\,y\,y3\,\gamma1 +$$
$$2\,x^2\,x2^2\,y\,y3\,\gamma1 - 4\,x\,x1\,x2^2\,y\,y3\,\gamma1 + 2\,x1^2\,x2^2\,y\,y3\,\gamma1 + 2\,x^3\,x3\,y\,y3\,\gamma1 - 4\,x^2\,x1\,x3\,y\,y3\,\gamma1 +$$
$$2\,x\,x1^2\,x3\,y\,y3\,\gamma1 - 2\,x^2\,x2\,x3\,y\,y3\,\gamma1 + 4\,x\,x1\,x2\,x3\,y\,y3\,\gamma1 - 2\,x1^2\,x2\,x3\,y\,y3\,\gamma1 -$$
$$2\,x\,x2\,y^3\,y3\,\gamma1 + 2\,x2^2\,y^3\,y3\,\gamma1 + 2\,x\,x3\,y^3\,y3\,\gamma1 - 2\,x2\,x3\,y^3\,y3\,\gamma1 + 4\,x\,x2\,y^2\,y1\,y3\,\gamma1 -$$
$$4\,x2^2\,y^2\,y1\,y3\,\gamma1 - 4\,x\,x3\,y^2\,y1\,y3\,\gamma1 + 4\,x2\,x3\,y^2\,y1\,y3\,\gamma1 - 2\,x\,x2\,y\,y1^2\,y3\,\gamma1 +$$
$$2\,x2^2\,y\,y1^2\,y3\,\gamma1 + 2\,x\,x3\,y\,y1^2\,y3\,\gamma1 - 2\,x2\,x3\,y\,y1^2\,y3\,\gamma1 + 2\,x^4\,y2\,y3\,\gamma1 - 4\,x^3\,x1\,y2\,y3\,\gamma1 +$$
$$2\,x^2\,x1^2\,y2\,y3\,\gamma1 - 2\,x^3\,x2\,y2\,y3\,\gamma1 + 4\,x^2\,x1\,x2\,y2\,y3\,\gamma1 - 2\,x\,x1^2\,x2\,y2\,y3\,\gamma1 -$$
$$2\,x^3\,x3\,y2\,y3\,\gamma1 + 4\,x^2\,x1\,x3\,y2\,y3\,\gamma1 - 2\,x\,x1^2\,x3\,y2\,y3\,\gamma1 + 2\,x^2\,x2\,x3\,y2\,y3\,\gamma1 -$$
$$4\,x\,x1\,x2\,x3\,y2\,y3\,\gamma1 + 2\,x1^2\,x2\,x3\,y2\,y3\,\gamma1 + 2\,x^2\,y^2\,y2\,y3\,\gamma1 - 2\,x\,x2\,y^2\,y2\,y3\,\gamma1 -$$
$$2\,x\,x3\,y^2\,y2\,y3\,\gamma1 + 2\,x2\,x3\,y^2\,y2\,y3\,\gamma1 - 4\,x^2\,y\,y1\,y2\,y3\,\gamma1 + 4\,x\,x2\,y\,y1\,y2\,y3\,\gamma1 +$$
$$4\,x\,x3\,y\,y1\,y2\,y3\,\gamma1 - 4\,x2\,x3\,y\,y1\,y2\,y3\,\gamma1 + 2\,x^2\,y1^2\,y2\,y3\,\gamma1 - 2\,x\,x2\,y1^2\,y2\,y3\,\gamma1 -$$

$$
\begin{aligned}
&2\,x\,x3\,y1^2\,y2\,y3\,\gamma1 + 2\,x2\,x3\,y1^2\,y2\,y3\,\gamma1 - x^4\,y3^2\,\gamma1 + 2\,x^3\,x1\,y3^2\,\gamma1 - x^2\,x1^2\,y3^2\,\gamma1 + \\
&2\,x^3\,x2\,y3^2\,\gamma1 - 4\,x^2\,x1\,x2\,y3^2\,\gamma1 + 2\,x\,x1^2\,x2\,y3^2\,\gamma1 - x^2\,x2^2\,y3^2\,\gamma1 + 2\,x\,x1\,x2^2\,y3^2\,\gamma1 - \\
&x1^2\,x2^2\,y3^2\,\gamma1 - x^2\,y^2\,y3^2\,\gamma1 + 2\,x\,x2\,y^2\,y3^2\,\gamma1 - x2^2\,y^2\,y3^2\,\gamma1 + 2\,x^2\,y\,y1\,y3^2\,\gamma1 - \\
&4\,x\,x2\,y\,y1\,y3^2\,\gamma1 + 2\,x2^2\,y\,y1\,y3^2\,\gamma1 - x^2\,y1^2\,y3^2\,\gamma1 + 2\,x\,x2\,y1^2\,y3^2\,\gamma1 - x2^2\,y1^2\,y3^2\,\gamma1 - \\
&x^2\,x1^2\,y^2\,\gamma2 + 2\,x\,x1^2\,x2\,y^2\,\gamma2 - x1^2\,x2^2\,y^2\,\gamma2 + 2\,x^2\,x1\,x3\,y^2\,\gamma2 - 4\,x\,x1\,x2\,x3\,y^2\,\gamma2 + \\
&2\,x1\,x2^2\,x3\,y^2\,\gamma2 - x^2\,x3^2\,y^2\,\gamma2 + 2\,x\,x2\,x3^2\,y^2\,\gamma2 - x2^2\,x3^2\,y^2\,\gamma2 - x1^2\,y^4\,\gamma2 + 2\,x1\,x3\,y^4\,\gamma2 - \\
&x3^2\,y^4\,\gamma2 + 2\,x^3\,x1\,y\,y1\,\gamma2 - 4\,x^2\,x1\,x2\,y\,y1\,\gamma2 + 2\,x\,x1\,x2^2\,y\,y1\,\gamma2 - 2\,x^3\,x3\,y\,y1\,\gamma2 - \\
&2\,x^2\,x1\,x3\,y\,y1\,\gamma2 + 4\,x^2\,x2\,x3\,y\,y1\,\gamma2 + 4\,x\,x1\,x2\,x3\,y\,y1\,\gamma2 - 2\,x\,x2^2\,x3\,y\,y1\,\gamma2 - \\
&2\,x1\,x2^2\,x3\,y\,y1\,\gamma2 + 2\,x^2\,x3^2\,y\,y1\,\gamma2 - 4\,x\,x2\,x3^2\,y\,y1\,\gamma2 + 2\,x2^2\,x3^2\,y\,y1\,\gamma2 + 2\,x\,x1\,y^3\,y1\,\gamma2 - \\
&2\,x\,x3\,y^3\,y1\,\gamma2 - 2\,x1\,x3\,y^3\,y1\,\gamma2 + 2\,x3^2\,y^3\,y1\,\gamma2 - x^4\,y1^2\,\gamma2 + 2\,x^3\,x2\,y1^2\,\gamma2 - \\
&x^2\,x2^2\,y1^2\,\gamma2 + 2\,x^3\,x3\,y1^2\,\gamma2 - 4\,x^2\,x2\,x3\,y1^2\,\gamma2 + 2\,x\,x2^2\,x3\,y1^2\,\gamma2 - x^2\,x3^2\,y1^2\,\gamma2 + \\
&2\,x\,x2\,x3^2\,y1^2\,\gamma2 - x2^2\,x3^2\,y1^2\,\gamma2 - x^2\,y^2\,y1^2\,\gamma2 + 2\,x\,x3\,y^2\,y1^2\,\gamma2 - x3^2\,y^2\,y1^2\,\gamma2 + \\
&2\,x1^2\,y^3\,y2\,\gamma2 - 4\,x1\,x3\,y^3\,y2\,\gamma2 + 2\,x3^2\,y^3\,y2\,\gamma2 - 4\,x\,x1\,y^2\,y1\,y2\,\gamma2 + 4\,x\,x3\,y^2\,y1\,y2\,\gamma2 + \\
&4\,x1\,x3\,y^2\,y1\,y2\,\gamma2 - 4\,x3^2\,y^2\,y1\,y2\,\gamma2 + 2\,x^2\,y\,y1^2\,y2\,\gamma2 - 4\,x\,x3\,y\,y1^2\,y2\,\gamma2 + \\
&2\,x3^2\,y\,y1^2\,y2\,\gamma2 - x1^2\,y^2\,y2^2\,\gamma2 + 2\,x1\,x3\,y^2\,y2^2\,\gamma2 - x3^2\,y^2\,y2^2\,\gamma2 + 2\,x\,x1\,y\,y1\,y2^2\,\gamma2 - \\
&2\,x\,x3\,y\,y1\,y2^2\,\gamma2 - 2\,x1\,x3\,y\,y1\,y2^2\,\gamma2 + 2\,x3^2\,y\,y1\,y2^2\,\gamma2 - x^2\,y1^2\,y2^2\,\gamma2 + 2\,x\,x3\,y1^2\,y2^2\,\gamma2 - \\
&x3^2\,y1^2\,y2^2\,\gamma2 - 2\,x^3\,x1\,y\,y3\,\gamma2 + 2\,x^2\,x1^2\,y\,y3\,\gamma2 + 4\,x^2\,x1\,x2\,y\,y3\,\gamma2 - 4\,x\,x1^2\,x2\,y\,y3\,\gamma2 - \\
&2\,x\,x1\,x2^2\,y\,y3\,\gamma2 + 2\,x1^2\,x2^2\,y\,y3\,\gamma2 + 2\,x^3\,x3\,y\,y3\,\gamma2 - 2\,x^2\,x1\,x3\,y\,y3\,\gamma2 - 4\,x^2\,x2\,x3\,y\,y3\,\gamma2 + \\
&4\,x\,x1\,x2\,x3\,y\,y3\,\gamma2 + 2\,x\,x2^2\,x3\,y\,y3\,\gamma2 - 2\,x1\,x2^2\,x3\,y\,y3\,\gamma2 - 2\,x\,x1\,y^3\,y3\,\gamma2 + 2\,x1^2\,y^3\,y3\,\gamma2 + \\
&2\,x\,x3\,y^3\,y3\,\gamma2 - 2\,x1\,x3\,y^3\,y3\,\gamma2 + 2\,x^4\,y1\,y3\,\gamma2 - 2\,x^3\,x1\,y1\,y3\,\gamma2 - 4\,x^3\,x2\,y1\,y3\,\gamma2 + \\
&4\,x^2\,x1\,x2\,y1\,y3\,\gamma2 + 2\,x^2\,x2^2\,y1\,y3\,\gamma2 - 2\,x\,x1\,x2^2\,y1\,y3\,\gamma2 - 2\,x^3\,x3\,y1\,y3\,\gamma2 + \\
&2\,x^2\,x1\,x3\,y1\,y3\,\gamma2 + 4\,x^2\,x2\,x3\,y1\,y3\,\gamma2 - 4\,x\,x1\,x2\,x3\,y1\,y3\,\gamma2 - 2\,x\,x2^2\,x3\,y1\,y3\,\gamma2 + \\
&2\,x1\,x2^2\,x3\,y1\,y3\,\gamma2 + 2\,x^2\,y^2\,y1\,y3\,\gamma2 - 2\,x\,x1\,y^2\,y1\,y3\,\gamma2 - 2\,x\,x3\,y^2\,y1\,y3\,\gamma2 + \\
&2\,x1\,x3\,y^2\,y1\,y3\,\gamma2 + 4\,x\,x1\,y^2\,y2\,y3\,\gamma2 - 4\,x1^2\,y^2\,y2\,y3\,\gamma2 - 4\,x\,x3\,y^2\,y2\,y3\,\gamma2 + \\
&4\,x1\,x3\,y^2\,y2\,y3\,\gamma2 - 4\,x^2\,y\,y1\,y2\,y3\,\gamma2 + 4\,x\,x1\,y\,y1\,y2\,y3\,\gamma2 + 4\,x\,x3\,y\,y1\,y2\,y3\,\gamma2 - \\
&4\,x1\,x3\,y\,y1\,y2\,y3\,\gamma2 - 2\,x\,x1\,y\,y2^2\,y3\,\gamma2 + 2\,x1^2\,y\,y2^2\,y3\,\gamma2 + 2\,x\,x3\,y\,y2^2\,y3\,\gamma2 - \\
&2\,x1\,x3\,y\,y2^2\,y3\,\gamma2 + 2\,x^2\,y1\,y2^2\,y3\,\gamma2 - 2\,x\,x1\,y1\,y2^2\,y3\,\gamma2 - 2\,x\,x3\,y1\,y2^2\,y3\,\gamma2 + \\
&2\,x1\,x3\,y1\,y2^2\,y3\,\gamma2 - x^4\,y3^2\,\gamma2 + 2\,x^3\,x1\,y3^2\,\gamma2 - x^2\,x1^2\,y3^2\,\gamma2 + 2\,x^3\,x2\,y3^2\,\gamma2 - \\
&4\,x^2\,x1\,x2\,y3^2\,\gamma2 + 2\,x\,x1^2\,x2\,y3^2\,\gamma2 - x^2\,x2^2\,y3^2\,\gamma2 + 2\,x\,x1\,x2^2\,y3^2\,\gamma2 - x1^2\,x2^2\,y3^2\,\gamma2 - \\
&x^2\,y^2\,y3^2\,\gamma2 + 2\,x\,x1\,y^2\,y3^2\,\gamma2 - x1^2\,y^2\,y3^2\,\gamma2 + 2\,x^2\,y\,y2\,y3^2\,\gamma2 - 4\,x\,x1\,y\,y2\,y3^2\,\gamma2 + \\
&2\,x1^2\,y\,y2\,y3^2\,\gamma2 - x^2\,y2^2\,y3^2\,\gamma2 + 2\,x\,x1\,y2^2\,y3^2\,\gamma2 - x1^2\,y2^2\,y3^2\,\gamma2 - x^2\,x1^2\,y^2\,\gamma3 + \\
&2\,x^2\,x1\,x2\,y^2\,\gamma3 - x^2\,x2^2\,y^2\,\gamma3 + 2\,x\,x1^2\,x3\,y^2\,\gamma3 - 4\,x\,x1\,x2\,x3\,y^2\,\gamma3 + 2\,x\,x2^2\,x3\,y^2\,\gamma3 - \\
&x1^2\,x3^2\,y^2\,\gamma3 + 2\,x1\,x2\,x3^2\,y^2\,\gamma3 - x2^2\,x3^2\,y^2\,\gamma3 - x1^2\,y^4\,\gamma3 + 2\,x1\,x2\,y^4\,\gamma3 - x2^2\,y^4\,\gamma3 + \\
&2\,x^3\,x1\,y\,y1\,\gamma3 - 2\,x^3\,x2\,y\,y1\,\gamma3 - 2\,x^2\,x1\,x2\,y\,y1\,\gamma3 + 2\,x^2\,x2^2\,y\,y1\,\gamma3 - 4\,x^2\,x1\,x3\,y\,y1\,\gamma3 + \\
&4\,x^2\,x2\,x3\,y\,y1\,\gamma3 + 4\,x\,x1\,x2\,x3\,y\,y1\,\gamma3 - 4\,x\,x2^2\,x3\,y\,y1\,\gamma3 + 2\,x\,x1\,x3^2\,y\,y1\,\gamma3 - \\
&2\,x\,x2\,x3^2\,y\,y1\,\gamma3 - 2\,x1\,x2\,x3^2\,y\,y1\,\gamma3 + 2\,x2^2\,x3^2\,y\,y1\,\gamma3 + 2\,x\,x1\,y^3\,y1\,\gamma3 - 2\,x\,x2\,y^3\,y1\,\gamma3 - \\
&2\,x1\,x2\,y^3\,y1\,\gamma3 + 2\,x2^2\,y^3\,y1\,\gamma3 - x^4\,y1^2\,\gamma3 + 2\,x^3\,x2\,y1^2\,\gamma3 - x^2\,x2^2\,y1^2\,\gamma3 + 2\,x^3\,x3\,y1^2\,\gamma3 - \\
&4\,x^2\,x2\,x3\,y1^2\,\gamma3 + 2\,x\,x2^2\,x3\,y1^2\,\gamma3 - x^2\,x3^2\,y1^2\,\gamma3 + 2\,x\,x2\,x3^2\,y1^2\,\gamma3 - x2^2\,x3^2\,y1^2\,\gamma3 - \\
&x^2\,y^2\,y1^2\,\gamma3 + 2\,x\,x2\,y^2\,y1^2\,\gamma3 - x2^2\,y^2\,y1^2\,\gamma3 - 2\,x^3\,x1\,y\,y2\,\gamma3 + 2\,x^2\,x1^2\,y\,y2\,\gamma3 + \\
&2\,x^3\,x2\,y\,y2\,\gamma3 - 2\,x^2\,x1\,x2\,y\,y2\,\gamma3 + 4\,x^2\,x1\,x3\,y\,y2\,\gamma3 - 4\,x\,x1^2\,x3\,y\,y2\,\gamma3 - 4\,x^2\,x2\,x3\,y\,y2\,\gamma3 + \\
&4\,x\,x1\,x2\,x3\,y\,y2\,\gamma3 - 2\,x\,x1\,x3^2\,y\,y2\,\gamma3 + 2\,x1^2\,x3^2\,y\,y2\,\gamma3 + 2\,x\,x2\,x3^2\,y\,y2\,\gamma3 - \\
&2\,x1\,x2\,x3^2\,y\,y2\,\gamma3 - 2\,x\,x1\,y^3\,y2\,\gamma3 + 2\,x1^2\,y^3\,y2\,\gamma3 + 2\,x\,x2\,y^3\,y2\,\gamma3 - 2\,x1\,x2\,y^3\,y2\,\gamma3 + \\
&2\,x^4\,y1\,y2\,\gamma3 - 2\,x^3\,x1\,y1\,y2\,\gamma3 - 2\,x^3\,x2\,y1\,y2\,\gamma3 + 2\,x^2\,x1\,x2\,y1\,y2\,\gamma3 - 4\,x^3\,x3\,y1\,y2\,\gamma3 + \\
&4\,x^2\,x1\,x3\,y1\,y2\,\gamma3 + 4\,x^2\,x2\,x3\,y1\,y2\,\gamma3 - 4\,x\,x1\,x2\,x3\,y1\,y2\,\gamma3 + 2\,x^2\,x3^2\,y1\,y2\,\gamma3 -
\end{aligned}
$$

$$
\begin{aligned}
&2\,x\,x1\,x3^2\,y1\,y2\,\gamma3 - 2\,x\,x2\,x3^2\,y1\,y2\,\gamma3 + 2\,x1\,x2\,x3^2\,y1\,y2\,\gamma3 + 2\,x^2\,y^2\,y1\,y2\,\gamma3 - \\
&2\,x\,x1\,y^2\,y1\,y2\,\gamma3 - 2\,x\,x2\,y^2\,y1\,y2\,\gamma3 + 2\,x1\,x2\,y^2\,y1\,y2\,\gamma3 - x^4\,y2^2\,\gamma3 + 2\,x^3\,x1\,y2^2\,\gamma3 - \\
&x^2\,x1^2\,y2^2\,\gamma3 + 2\,x^3\,x3\,y2^2\,\gamma3 - 4\,x^2\,x1\,x3\,y2^2\,\gamma3 + 2\,x\,x1^2\,x3\,y2^2\,\gamma3 - x^2\,x3^2\,y2^2\,\gamma3 + \\
&2\,x\,x1\,x3^2\,y2^2\,\gamma3 - x1^2\,x3^2\,y2^2\,\gamma3 - x^2\,y^2\,y2^2\,\gamma3 + 2\,x\,x1\,y^2\,y2^2\,\gamma3 - x1^2\,y^2\,y2^2\,\gamma3 + \\
&2\,x1^2\,y^3\,y3\,\gamma3 - 4\,x1\,x2\,y^3\,y3\,\gamma3 + 2\,x2^2\,y^3\,y3\,\gamma3 - 4\,x\,x1\,y^2\,y1\,y3\,\gamma3 + 4\,x\,x2\,y^2\,y1\,y3\,\gamma3 + \\
&4\,x1\,x2\,y^2\,y1\,y3\,\gamma3 - 4\,x2^2\,y^2\,y1\,y3\,\gamma3 + 2\,x^2\,y\,y1^2\,y3\,\gamma3 - 4\,x\,x2\,y\,y1^2\,y3\,\gamma3 + 2\,x2^2\,y\,y1^2\,y3\,\gamma3 + \\
&4\,x\,x1\,y^2\,y2\,y3\,\gamma3 - 4\,x1^2\,y^2\,y2\,y3\,\gamma3 - 4\,x\,x2\,y^2\,y2\,y3\,\gamma3 + 4\,x1\,x2\,y^2\,y2\,y3\,\gamma3 - \\
&4\,x^2\,y\,y1\,y2\,y3\,\gamma3 + 4\,x\,x1\,y\,y1\,y2\,y3\,\gamma3 + 4\,x\,x2\,y\,y1\,y2\,y3\,\gamma3 - 4\,x1\,x2\,y\,y1\,y2\,y3\,\gamma3 + \\
&2\,x^2\,y\,y2^2\,y3\,\gamma3 - 4\,x\,x1\,y\,y2^2\,y3\,\gamma3 + 2\,x1^2\,y\,y2^2\,y3\,\gamma3 - x1^2\,y^2\,y3^2\,\gamma3 + 2\,x1\,x2\,y^2\,y3^2\,\gamma3 - \\
&x2^2\,y^2\,y3^2\,\gamma3 + 2\,x\,x1\,y\,y1\,y3^2\,\gamma3 - 2\,x\,x2\,y\,y1\,y3^2\,\gamma3 - 2\,x1\,x2\,y\,y1\,y3^2\,\gamma3 + 2\,x2^2\,y\,y1\,y3^2\,\gamma3 - \\
&x^2\,y1^2\,y3^2\,\gamma3 + 2\,x\,x2\,y1^2\,y3^2\,\gamma3 - x2^2\,y1^2\,y3^2\,\gamma3 - 2\,x\,x1\,y\,y2\,y3^2\,\gamma3 + 2\,x1^2\,y\,y2\,y3^2\,\gamma3 + \\
&2\,x\,x2\,y\,y2\,y3^2\,\gamma3 - 2\,x1\,x2\,y\,y2\,y3^2\,\gamma3 + 2\,x^2\,y1\,y2\,y3^2\,\gamma3 - 2\,x\,x1\,y1\,y2\,y3^2\,\gamma3 - \\
&2\,x\,x2\,y1\,y2\,y3^2\,\gamma3 + 2\,x1\,x2\,y1\,y2\,y3^2\,\gamma3 - x^2\,y2^2\,y3^2\,\gamma3 + 2\,x\,x1\,y2^2\,y3^2\,\gamma3 - x1^2\,y2^2\,y3^2\,\gamma3, \\
&0, -x^2\,x1^2\,\beta + x^2\,x1\,x2\,\beta + x\,x1^2\,x2\,\beta - x^2\,x2^2\,\beta + x\,x1\,x2^2\,\beta - x1^2\,x2^2\,\beta + x^2\,x1\,x3\,\beta + \\
&x\,x1^2\,x3\,\beta + x^2\,x2\,x3\,\beta - 6\,x\,x1\,x2\,x3\,\beta + x1^2\,x2\,x3\,\beta + x\,x2^2\,x3\,\beta + x1\,x2^2\,x3\,\beta - x^2\,x3^2\,\beta + \\
&x\,x1\,x3^2\,\beta - x1^2\,x3^2\,\beta + x\,x2\,x3^2\,\beta + x1\,x2\,x3^2\,\beta - x2^2\,x3^2\,\beta - 2\,x1^2\,y^2\,\beta + 2\,x1\,x2\,y^2\,\beta - \\
&2\,x2^2\,y^2\,\beta + 2\,x1\,x3\,y^2\,\beta + 2\,x2\,x3\,y^2\,\beta - 2\,x3^2\,y^2\,\beta + 2\,x\,x1\,y\,y1\,\beta - x\,x2\,y\,y1\,\beta - \\
&x1\,x2\,y\,y1\,\beta + 2\,x2^2\,y\,y1\,\beta - x\,x3\,y\,y1\,\beta - x1\,x3\,y\,y1\,\beta - 2\,x2\,x3\,y\,y1\,\beta + 2\,x3^2\,y\,y1\,\beta - \\
&2\,x^2\,y1^2\,\beta + 2\,x\,x2\,y1^2\,\beta - x2^2\,y1^2\,\beta + 2\,x\,x3\,y1^2\,\beta - x3^2\,y1^2\,\beta - y^2\,y1^2\,\beta - x\,x1\,y\,y2\,\beta + \\
&2\,x1^2\,y\,y2\,\beta + 2\,x\,x2\,y\,y2\,\beta - x1\,x2\,y\,y2\,\beta - x\,x3\,y\,y2\,\beta - 2\,x1\,x3\,y\,y2\,\beta - x2\,x3\,y\,y2\,\beta + \\
&2\,x3^2\,y\,y2\,\beta + 2\,x^2\,y1\,y2\,\beta - x\,x1\,y1\,y2\,\beta - x\,x2\,y1\,y2\,\beta - 2\,x\,x3\,y1\,y2\,\beta + x1\,x3\,y1\,y2\,\beta + \\
&x2\,x3\,y1\,y2\,\beta + y^2\,y1\,y2\,\beta + y\,y1^2\,y2\,\beta - 2\,x^2\,y2^2\,\beta + 2\,x\,x1\,y2^2\,\beta - x1^2\,y2^2\,\beta + 2\,x\,x3\,y2^2\,\beta - \\
&x3^2\,y2^2\,\beta - y^2\,y2^2\,\beta + y\,y1\,y2^2\,\beta - y1^2\,y2^2\,\beta - x\,x1\,y\,y3\,\beta + 2\,x1^2\,y\,y3\,\beta - x\,x2\,y\,y3\,\beta - \\
&2\,x1\,x2\,y\,y3\,\beta + 2\,x2^2\,y\,y3\,\beta + 2\,x\,x3\,y\,y3\,\beta - x1\,x3\,y\,y3\,\beta - x2\,x3\,y\,y3\,\beta + 2\,x^2\,y1\,y3\,\beta - \\
&x\,x1\,y1\,y3\,\beta - 2\,x\,x2\,y1\,y3\,\beta + x1\,x2\,y1\,y3\,\beta - x\,x3\,y1\,y3\,\beta + x2\,x3\,y1\,y3\,\beta + y^2\,y1\,y3\,\beta + \\
&y\,y1^2\,y3\,\beta + 2\,x^2\,y2\,y3\,\beta - 2\,x\,x1\,y2\,y3\,\beta - x\,x2\,y2\,y3\,\beta + x1\,x2\,y2\,y3\,\beta - x\,x3\,y2\,y3\,\beta + \\
&x1\,x3\,y2\,y3\,\beta + y^2\,y2\,y3\,\beta - 6\,y\,y1\,y2\,y3\,\beta + y1^2\,y2\,y3\,\beta + y\,y2^2\,y3\,\beta + y1\,y2^2\,y3\,\beta - \\
&2\,x^2\,y3^2\,\beta + 2\,x\,x1\,y3^2\,\beta - x1^2\,y3^2\,\beta + 2\,x\,x2\,y3^2\,\beta - x2^2\,y3^2\,\beta - y^2\,y3^2\,\beta + y\,y1\,y3^2\,\beta - \\
&y1^2\,y3^2\,\beta + y\,y2\,y3^2\,\beta + y1\,y2\,y3^2\,\beta - y2^2\,y3^2\,\beta - x^2\,x2^2\,\gamma1 + 2\,x\,x1\,x2^2\,\gamma1 - x1^2\,x2^2\,\gamma1 + \\
&2\,x^2\,x2\,x3\,\gamma1 - 4\,x\,x1\,x2\,x3\,\gamma1 + 2\,x1^2\,x2\,x3\,\gamma1 - x^2\,x3^2\,\gamma1 + 2\,x\,x1\,x3^2\,\gamma1 - x1^2\,x3^2\,\gamma1 - \\
&2\,x2^2\,y^2\,\gamma1 + 4\,x2\,x3\,y^2\,\gamma1 - 2\,x3^2\,y^2\,\gamma1 + 2\,x2^2\,y\,y1\,\gamma1 - 4\,x2\,x3\,y\,y1\,\gamma1 + 2\,x3^2\,y\,y1\,\gamma1 - \\
&x2^2\,y1^2\,\gamma1 + 2\,x2\,x3\,y1^2\,\gamma1 - x3^2\,y1^2\,\gamma1 + 2\,x\,x2\,y\,y2\,\gamma1 - 2\,x\,x3\,y\,y2\,\gamma1 - 2\,x2\,x3\,y\,y2\,\gamma1 + \\
&2\,x3^2\,y\,y2\,\gamma1 - 2\,x^2\,y2^2\,\gamma1 + 2\,x\,x1\,y2^2\,\gamma1 - x1^2\,y2^2\,\gamma1 + 2\,x\,x3\,y2^2\,\gamma1 - x3^2\,y2^2\,\gamma1 - y^2\,y2^2\,\gamma1 + \\
&2\,y\,y1\,y2^2\,\gamma1 - y1^2\,y2^2\,\gamma1 - 2\,x\,x2\,y\,y3\,\gamma1 + 2\,x2^2\,y\,y3\,\gamma1 + 2\,x\,x3\,y\,y3\,\gamma1 - 2\,x2\,x3\,y\,y3\,\gamma1 + \\
&4\,x^2\,y2\,y3\,\gamma1 - 4\,x\,x1\,y2\,y3\,\gamma1 + 2\,x1^2\,y2\,y3\,\gamma1 - 2\,x\,x2\,y2\,y3\,\gamma1 - 2\,x\,x3\,y2\,y3\,\gamma1 + \\
&2\,x2\,x3\,y2\,y3\,\gamma1 + 2\,y^2\,y2\,y3\,\gamma1 - 4\,y\,y1\,y2\,y3\,\gamma1 + 2\,y1^2\,y2\,y3\,\gamma1 - 2\,x^2\,y3^2\,\gamma1 + 2\,x\,x1\,y3^2\,\gamma1 - \\
&x1^2\,y3^2\,\gamma1 + 2\,x\,x2\,y3^2\,\gamma1 - x2^2\,y3^2\,\gamma1 - y^2\,y3^2\,\gamma1 + 2\,y\,y1\,y3^2\,\gamma1 - y1^2\,y3^2\,\gamma1 - x^2\,x1^2\,\gamma2 + \\
&2\,x\,x1^2\,x2\,\gamma2 - x1^2\,x2^2\,\gamma2 + 2\,x^2\,x1\,x3\,\gamma2 - 4\,x\,x1\,x2\,x3\,\gamma2 + 2\,x1\,x2^2\,x3\,\gamma2 - x^2\,x3^2\,\gamma2 + \\
&2\,x\,x2\,x3^2\,\gamma2 - x2^2\,x3^2\,\gamma2 - 2\,x1^2\,y^2\,\gamma2 + 4\,x1\,x3\,y^2\,\gamma2 - 2\,x3^2\,y^2\,\gamma2 + 2\,x\,x1\,y\,y1\,\gamma2 - \\
&2\,x\,x3\,y\,y1\,\gamma2 - 2\,x1\,x3\,y\,y1\,\gamma2 + 2\,x3^2\,y\,y1\,\gamma2 - 2\,x^2\,y1^2\,\gamma2 + 2\,x\,x2\,y1^2\,\gamma2 - x2^2\,y1^2\,\gamma2 + \\
&2\,x\,x3\,y1^2\,\gamma2 - x3^2\,y1^2\,\gamma2 - y^2\,y1^2\,\gamma2 + 2\,x1^2\,y\,y2\,\gamma2 - 4\,x1\,x3\,y\,y2\,\gamma2 + 2\,x3^2\,y\,y2\,\gamma2 + \\
&2\,y\,y1^2\,y2\,\gamma2 - x1^2\,y2^2\,\gamma2 + 2\,x1\,x3\,y2^2\,\gamma2 - x3^2\,y2^2\,\gamma2 - y1^2\,y2^2\,\gamma2 - 2\,x\,x1\,y\,y3\,\gamma2 + \\
&2\,x1^2\,y\,y3\,\gamma2 + 2\,x\,x3\,y\,y3\,\gamma2 - 2\,x1\,x3\,y\,y3\,\gamma2 + 4\,x^2\,y1\,y3\,\gamma2 - 2\,x\,x1\,y1\,y3\,\gamma2 - \\
&4\,x\,x2\,y1\,y3\,\gamma2 + 2\,x2^2\,y1\,y3\,\gamma2 - 2\,x\,x3\,y1\,y3\,\gamma2 + 2\,x1\,x3\,y1\,y3\,\gamma2 + 2\,y^2\,y1\,y3\,\gamma2 -
\end{aligned}
$$

$$4\,y\,y1\,y2\,y3\,\gamma2 + 2\,y1\,y2^2\,y3\,\gamma2 - 2\,x^2\,y3^2\,\gamma2 + 2\,x\,x1\,y3^2\,\gamma2 - x1^2\,y3^2\,\gamma2 + 2\,x\,x2\,y3^2\,\gamma2 - x2^2\,y3^2\,\gamma2 - y^2\,y3^2\,\gamma2 + 2\,y\,y2\,y3^2\,\gamma2 - y2^2\,y3^2\,\gamma2 - x^2\,x1^2\,\gamma3 + 2\,x^2\,x1\,x2\,\gamma3 - x^2\,x2^2\,\gamma3 + 2\,x\,x1^2\,x3\,\gamma3 - 4\,x\,x1\,x2\,x3\,\gamma3 + 2\,x\,x2^2\,x3\,\gamma3 - x1^2\,x3^2\,\gamma3 + 2\,x1\,x2\,x3^2\,\gamma3 - x2^2\,x3^2\,\gamma3 - 2\,x1^2\,y^2\,\gamma3 + 4\,x1\,x2\,y^2\,\gamma3 - 2\,x2^2\,y^2\,\gamma3 + 2\,x\,x1\,y\,y1\,\gamma3 - 2\,x\,x2\,y\,y1\,\gamma3 - 2\,x1\,x2\,y\,y1\,\gamma3 + 2\,x2^2\,y\,y1\,\gamma3 - 2\,x^2\,y1^2\,\gamma3 + 2\,x\,x2\,y1^2\,\gamma3 - x2^2\,y1^2\,\gamma3 + 2\,x\,x3\,y1^2\,\gamma3 - x3^2\,y1^2\,\gamma3 - y^2\,y1^2\,\gamma3 - 2\,x\,x1\,y\,y2\,\gamma3 + 2\,x1^2\,y\,y2\,\gamma3 + 2\,x\,x2\,y\,y2\,\gamma3 - 2\,x1\,x2\,y\,y2\,\gamma3 + 4\,x^2\,y1\,y2\,\gamma3 - 2\,x\,x1\,y1\,y2\,\gamma3 - 2\,x\,x2\,y1\,y2\,\gamma3 + 2\,x1\,x2\,y1\,y2\,\gamma3 - 4\,x\,x3\,y1\,y2\,\gamma3 + 2\,x3^2\,y1\,y2\,\gamma3 + 2\,y^2\,y1\,y2\,\gamma3 - 2\,x^2\,y2^2\,\gamma3 + 2\,x\,x1\,y2^2\,\gamma3 - x1^2\,y2^2\,\gamma3 + 2\,x\,x3\,y2^2\,\gamma3 - x3^2\,y2^2\,\gamma3 - y^2\,y2^2\,\gamma3 + 2\,x1^2\,y\,y3\,\gamma3 - 4\,x1\,x2\,y\,y3\,\gamma3 + 2\,x2^2\,y\,y3\,\gamma3 + 2\,y\,y1^2\,y3\,\gamma3 - 4\,y\,y1\,y2\,y3\,\gamma3 + 2\,y\,y2^2\,y3\,\gamma3 - x1^2\,y3^2\,\gamma3 + 2\,x1\,x2\,y3^2\,\gamma3 - x2^2\,y3^2\,\gamma3 - y1^2\,y3^2\,\gamma3 + 2\,y1\,y2\,y3^2\,\gamma3 - y2^2\,y3^2\,\gamma3,\ 0,$$

$$-x1^2\,\beta + x1\,x2\,\beta - x2^2\,\beta + x1\,x3\,\beta + x2\,x3\,\beta - x3^2\,\beta - y1^2\,\beta + y1\,y2\,\beta - y2^2\,\beta + y1\,y3\,\beta + y2\,y3\,\beta - y3^2\,\beta - x2^2\,\gamma1 + 2\,x2\,x3\,\gamma1 - x3^2\,\gamma1 - y2^2\,\gamma1 + 2\,y2\,y3\,\gamma1 - y3^2\,\gamma1 - x1^2\,\gamma2 + 2\,x1\,x3\,\gamma2 - x3^2\,\gamma2 - y1^2\,\gamma2 + 2\,y1\,y3\,\gamma2 - y3^2\,\gamma2 - x1^2\,\gamma3 + 2\,x1\,x2\,\gamma3 - x2^2\,\gamma3 - y1^2\,\gamma3 + 2\,y1\,y2\,\gamma3 - y2^2\,\gamma3\},$$

$$\{0,\ 0,\ (x2\,y1 - x3\,y1 - x1\,y2 + x3\,y2 + x1\,y3 - x2\,y3)^2\}\}$$

In[249]:=
```
{Length[%[[1]]], Length[%[[2]]]}
```
Out[249]= {5, 3}

In[250]:=
```
ratio = Factor[ Factor[ %%[[1]][[5]] z^2 / %%[[2]][[3]] ] /. ySubs]
```
Out[250]=
$$\left(z^2\,(-3\,\beta + x1\,x2\,\beta + x1\,x3\,\beta + x2\,x3\,\beta + y1\,y2\,\beta + y1\,y3\,\beta + y2\,y3\,\beta - 2\,\gamma1 + 2\,x2\,x3\,\gamma1 + 2\,y2\,y3\,\gamma1 - 2\,\gamma2 + 2\,x1\,x3\,\gamma2 + 2\,y1\,y3\,\gamma2 - 2\,\gamma3 + 2\,x1\,x2\,\gamma3 + 2\,y1\,y2\,\gamma3)\right) \Big/ (x2\,y1 - x3\,y1 - x1\,y2 + x3\,y2 + x1\,y3 - x2\,y3)^2$$

In[251]:=
```
ratio2 =
 Factor[Factor[Factor[ -4 ( ( (9 - XH^2 - YH^2) / 2) β + D1 γ1 + D2 γ2 + D3 γ3 ) z^2 /
          (D1 D2 D3)  /. unXYΔSubs] /. DSubs] //. ySubs]
```
Out[251]=
$$-\left(\left(z^2\,(-3\,\beta + x1\,x2\,\beta + x1\,x3\,\beta + x2\,x3\,\beta + y1\,y2\,\beta + y1\,y3\,\beta + y2\,y3\,\beta - 2\,\gamma1 + 2\,x2\,x3\,\gamma1 + 2\,y2\,y3\,\gamma1 - 2\,\gamma2 + 2\,x1\,x3\,\gamma2 + 2\,y1\,y3\,\gamma2 - 2\,\gamma3 + 2\,x1\,x2\,\gamma3 + 2\,y1\,y2\,\gamma3)\right) \Big/ (2\,(-1 + x1\,x2 + y1\,y2)\,(-1 + x1\,x3 + y1\,y3)\,(-1 + x2\,x3 + y2\,y3))\right)$$

In[252]:=
```
Factor[%% / %]
```
Out[252]=
$$-\frac{2\,(-1 + x1\,x2 + y1\,y2)\,(-1 + x1\,x3 + y1\,y3)\,(-1 + x2\,x3 + y2\,y3)}{(-x2\,y1 + x3\,y1 + x1\,y2 - x3\,y2 - x1\,y3 + x2\,y3)^2}$$

In[253]:=
```
Factor[% /. tSubsSp]
```
Out[253]= 1

```
(*    Lemma 11 Check.   *)
```

In[254]:=
```
ι = 3 - x2 x3 - x3 x1 - x1 x2 - y2 y3 - y3 y1 - y1 y2
```
Out[254]= 3 − x1 x2 − x1 x3 − x2 x3 − y1 y2 − y1 y3 − y2 y3

In[255]:=
```
Factor[Factor[2 ι - D1 - D2 - D3 /. DSubs] //. ySubs]
```
Out[255]=
```
0
```

```
(* So H is of form (4) and satisfies (5) *)
```

In[257]:=
```
LL = (2 / H) ((1 - x1) y1 (c1^2 - 1) +
    (1 - x2) y2 (c2^2 - 1) + (1 - x3) y3 (c3^2 - 1) + (y1 + y2 + y3) (1 - c1 c2 c3))
```
Out[257]=
$$\left(2 \left(\left(-1+c1^2\right) (1-x1)\, y1+\left(-1+c2^2\right) (1-x2)\, y2+\left(-1+c3^2\right) (1-x3)\, y3+(1-c1\, c2\, c3)\, (y1+y2+y3)\right)\right) \Big/ \left(1-c1^2-c2^2+2\, c1\, c2\, c3-c3^2\right)$$

In[258]:=
```
Factor[CoefficientList[Numerator[%], {c3, c2, c1}]]
```
Out[258]=
```
{{{2 (x1 y1 + x2 y2 + x3 y3), 0, -2 (-1 + x1) y1}, {0, 0, 0}, {-2 (-1 + x2) y2, 0, 0}},
 {{0, 0, 0}, {0, -2 (y1 + y2 + y3), 0}, {0, 0, 0}},
 {{-2 (-1 + x3) y3, 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

In[259]:=
```
Factor[
 Factor[Factor[%[[2]][[2]][[2]] ι + %[[1]][[1]][[3]] D1 + %[[1]][[3]][[1]] D2 +
      %[[3]][[1]][[1]] D3] /. DSubs] //. tSubsSp]
```
Out[259]=
```
0
```

In[260]:=
```
RR = (2 / H) ((1 + x1) x1 (c1^2 - 1) + (1 + x2) x2 (c2^2 - 1) +
    (1 + x3) x3 (c3^2 - 1) + (1 + x1 + x2 + x3) (1 - c1 c2 c3))
```
Out[260]=
$$\left(2 \left(\left(-1+c1^2\right) x1\, (1+x1)+\left(-1+c2^2\right) x2\, (1+x2)+\left(-1+c3^2\right) x3\, (1+x3)+(1-c1\, c2\, c3)\, (1+x1+x2+x3)\right)\right) \Big/ \left(1-c1^2-c2^2+2\, c1\, c2\, c3-c3^2\right)$$

In[261]:=
```
Factor[CoefficientList[Numerator[%], {c3, c2, c1}]]
```
Out[261]=
```
{{{-2 (-1 + x1^2 + x2^2 + x3^2), 0, 2 x1 (1 + x1)}, {0, 0, 0}, {2 x2 (1 + x2), 0, 0}},
 {{0, 0, 0}, {0, -2 (1 + x1 + x2 + x3), 0}, {0, 0, 0}},
 {{2 x3 (1 + x3), 0, 0}, {0, 0, 0}, {0, 0, 0}}}
```

In[262]:=
```
Factor[
 Factor[Factor[%[[2]][[2]][[2]] ι + %[[1]][[1]][[3]] D1 + %[[1]][[3]][[1]] D2 +
      %[[3]][[1]][[1]] D3] /. DSubs] //. tSubsSp]
```
Out[262]=
```
0
```

```
(*   Theorem 1 Check.   *)
```

In[263]:=
```
H = 1 - c1^2 - c2^2 - c3^2 + 2 c1 c2 c3   (*  H = η^2  *)
```
Out[263]=
$$1-c1^2-c2^2+2\, c1\, c2\, c3-c3^2$$

In[264]:=
```
LL = (2 / H) ((1 - x1) y1 (c1^2 - 1) +
    (1 - x2) y2 (c2^2 - 1) + (1 - x3) y3 (c3^2 - 1) + (y1 + y2 + y3) (1 - c1 c2 c3))
```

Out[264]=
$$\left(2 \left(\left(-1+c1^2\right) (1-x1)\, y1+\left(-1+c2^2\right) (1-x2)\, y2+\left(-1+c3^2\right) (1-x3)\, y3+ (1-c1\, c2\, c3) (y1+y2+y3)\right)\right) \Big/ \left(1-c1^2-c2^2+2\, c1\, c2\, c3-c3^2\right)$$

In[265]:=
```
RR = (2 / H) ((1 + x1) x1 (c1^2 - 1) + (1 + x2) x2 (c2^2 - 1) +
    (1 + x3) x3 (c3^2 - 1) + (1 + x1 + x2 + x3) (1 - c1 c2 c3))
```

Out[265]=
$$\left(2 \left(\left(-1+c1^2\right) x1\, (1+x1)+\left(-1+c2^2\right) x2\, (1+x2)+\left(-1+c3^2\right) x3\, (1+x3)+ (1-c1\, c2\, c3) (1+x1+x2+x3)\right)\right) \Big/ \left(1-c1^2-c2^2+2\, c1\, c2\, c3-c3^2\right)$$

In[266]:=
```
Factor[
 Factor[Factor[Factor[Factor[Factor[Factor[Factor[ LL /. cSubs] /. r2RSubs] /.
              DRSubs] //. tSubsSp] /. untSubs] //. ySubs] //. ySubs] /. Z → z^2]
```

Out[266]=
$$\frac{1}{z^2}\left(-y-2\, x\, y+x^2\, y+2\, x^3\, y+x2\, y-x^2\, x2\, y+x3\, y-x^2\, x3\, y+x2\, x3\, y-x^2\, x2\, x3\, y+y^3+2\, x\, y^3-x2\, y^3- x3\, y^3-x2\, x3\, y^3+y2+x\, y2-x^2\, y2-x^3\, y2-x3\, y2-x\, x3\, y2+x^2\, x3\, y2+x^3\, x3\, y2-y^2\, y2- x\, y^2\, y2+x3\, y^2\, y2+x\, x3\, y^2\, y2+y3+x\, y3-x^2\, y3-x^3\, y3-x2\, y3-x\, x2\, y3+x^2\, x2\, y3+x^3\, x2\, y3- y^2\, y3-x\, y^2\, y3+x2\, y^2\, y3+x\, x2\, y^2\, y3-y\, y2\, y3+x^2\, y\, y2\, y3+y^3\, y2\, y3+2\, y\, z^2+2\, x\, y\, z^2\right)$$

In[267]:=
```
Factor[ XYΔReduce[ % ] /. tweak3]
```

Out[267]=
$$\frac{1}{z^2}\left(-y-2\, x\, y+x^2\, y+2\, x^3\, y+XH\, y-x^2\, XH\, y+y^3+ 2\, x\, y^3-XH\, y^3+YH+x\, YH-x^2\, YH-x^3\, YH-y^2\, YH-x\, y^2\, YH+2\, y\, z^2+2\, x\, y\, z^2\right)$$

In[268]:=
```
{Numerator[%], Denominator[%]}
```

Out[268]=
$$\left\{-y-2\, x\, y+x^2\, y+2\, x^3\, y+XH\, y-x^2\, XH\, y+y^3+2\, x\, y^3- XH\, y^3+YH+x\, YH-x^2\, YH-x^3\, YH-y^2\, YH-x\, y^2\, YH+2\, y\, z^2+2\, x\, y\, z^2,\ z^2\right\}$$

In[269]:=
```
Factor[CoefficientList[%[[1]], z]]
```

Out[269]=
$$\left\{\left(-1+x^2+y^2\right) (y+2\, x\, y-XH\, y-YH-x\, YH),\ 0,\ 2\, (1+x)\, y\right\}$$

In[270]:=
```
LLL = UnCoefficientList[ % , z] / z^2
```

Out[270]=
$$\frac{\left(-1+x^2+y^2\right) (y+2\, x\, y-XH\, y-YH-x\, YH)+2\, (1+x)\, y\, z^2}{z^2}$$

In[271]:=
```
Factor[
 Factor[Factor[Factor[Factor[Factor[Factor[ RR  /. cSubs] /. r2RSubs] /.
              DRSubs] //. tSubsSp] /. untSubs] //. ySubs] //. ySubs] /. Z → z^2]
```

Out[271]=
$$-\frac{1}{z^2}\left(x - x^2 - x^3 + x^4 - x2 + x\,x2 + x^2\,x2 - x^3\,x2 - x3 + x\,x3 + x^2\,x3 - x^3\,x3 - x2\,x3 + x\,x2\,x3 + x^2\,x2\,x3 - x^3\,x2\,x3 + y^2 - x\,y^2 + x2\,y^2 - x\,x2\,y^2 + x3\,y^2 - x\,x3\,y^2 + x2\,x3\,y^2 - x\,x2\,x3\,y^2 - y^4 - y\,y2 + x^2\,y\,y2 + x3\,y\,y2 - x^2\,x3\,y\,y2 + y^3\,y2 - x3\,y^3\,y2 - y\,y3 + x^2\,y\,y3 + x2\,y\,y3 - x^2\,x2\,y\,y3 + y^3\,y3 - x2\,y^3\,y3 + y2\,y3 - x\,y2\,y3 - x^2\,y2\,y3 + x^3\,y2\,y3 - y^2\,y2\,y3 + x\,y^2\,y2\,y3 + z^2 - 2\,x\,z^2 + x^2\,z^2 - y^2\,z^2\right)$$

In[272]:=
```
Factor[ XYΔReduce[ % ] /. tweak3]
```

Out[272]=
$$-\frac{1}{z^2}\left(x - x^2 - x^3 + x^4 - XH + x\,XH + x^2\,XH - x^3\,XH + y^2 - x\,y^2 + XH\,y^2 - x\,XH\,y^2 - y^4 - y\,YH + x^2\,y\,YH + y^3\,YH + z^2 - 2\,x\,z^2 + x^2\,z^2 - y^2\,z^2\right)$$

In[273]:=
```
{Numerator[%], Denominator[%]}
```

Out[273]=
$$\left\{-x + x^2 + x^3 - x^4 + XH - x\,XH - x^2\,XH + x^3\,XH - y^2 + x\,y^2 - XH\,y^2 + x\,XH\,y^2 + y^4 + y\,YH - x^2\,y\,YH - y^3\,YH - z^2 + 2\,x\,z^2 - x^2\,z^2 + y^2\,z^2,\ z^2\right\}$$

In[274]:=
```
Factor[CoefficientList[%[[1]], z]]
```

Out[274]=
$$\left\{-\left(-1 + x^2 + y^2\right)\left(-x + x^2 + XH - x\,XH - y^2 + y\,YH\right),\ 0,\ -(-1 + x - y)(-1 + x + y)\right\}$$

In[275]:=
```
RRR = UnCoefficientList[ % , z] / z^2
```

Out[275]=
$$\frac{-\left(-1 + x^2 + y^2\right)\left(-x + x^2 + XH - x\,XH - y^2 + y\,YH\right) - (-1 + x - y)(-1 + x + y)\,z^2}{z^2}$$

```
(*  Corollary 1 Check.  *)
```

In[276]:=
```
Factor[ H * ((1 + x3) LL + y3 RR + 2 (1 + x3) y3) / (y1 - y2)]
```

Out[276]=
$$-\frac{1}{y1 - y2}2\left(-c1^2\,y1 + c1\,c2\,c3\,y1 - x1\,y1 + c1^2\,x1\,y1 - c1^2\,x3\,y1 + c1\,c2\,c3\,x3\,y1 - x1\,x3\,y1 + c1^2\,x1\,x3\,y1 - c2^2\,y2 + c1\,c2\,c3\,y2 - x2\,y2 + c2^2\,x2\,y2 - c2^2\,x3\,y2 + c1\,c2\,c3\,x3\,y2 - x2\,x3\,y2 + c2^2\,x2\,x3\,y2 - 2\,y3 + c1^2\,y3 + c2^2\,y3 - c1^2\,x1\,y3 + c1\,c2\,c3\,x1\,y3 + x1^2\,y3 - c1^2\,x1^2\,y3 - c2^2\,x2\,y3 + c1\,c2\,c3\,x2\,y3 + x2^2\,y3 - c2^2\,x2^2\,y3 - 2\,x3\,y3 + c1^2\,x3\,y3 + c2^2\,x3\,y3\right)$$

In[277]:=
```
Factor[Factor[CoefficientList[%, {c3, c2, c1}]] //. special]
```

Out[277]=
$$\left\{\left\{\left\{\frac{2\,(x1\,y1 - x2\,y1 - x1\,y2 + x2\,y2 + 2\,y3 - 2\,x1^2\,y3 - 2\,x2^2\,y3 + 2\,x3\,y3)}{y1 - y2},\right.\right.\right.$$
$$\left.\left.\left. 0, -\frac{2\,(-y1 + x1\,y1 + y2 - x1\,y2 + y3 - 2\,x1^2\,y3 + x3\,y3)}{y1 - y2}\right\},\right.$$
$$\left.\left\{0, 0, 0\right\}, \left\{\frac{2\,(-y1 + x2\,y1 + y2 - x2\,y2 - y3 + 2\,x2^2\,y3 - x3\,y3)}{y1 - y2}, 0, 0\right\}\right\},$$
$$\left.\left\{\{0, 0, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\right\}\right\}$$

In[278]:=
```
Factor[% //. tSubsSp]
```

Out[278]=
$$\left\{\left\{\left\{-\frac{4\,t3\,(-2\,t2 - t3 + t2^2\,t3)\,(-3 + t3^2)}{(1 + t2^2)\,(1 + t3^2)^2}, 0, \frac{4\,(-t2 - 2\,t3 + t2\,t3^2)^2}{(1 + t2^2)\,(1 + t3^2)^2}\right\},\right.\right.$$
$$\left.\left.\left\{0, 0, 0\right\}, \left\{-\frac{4\,(t2 - t3)^2}{(1 + t2^2)\,(1 + t3^2)}, 0, 0\right\}\right\}, \left\{\{0, 0, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\right\}\right\}$$

In[279]:=
```
%[[1]]
```

Out[279]=
$$\left\{\left\{-\frac{4\,t3\,(-2\,t2 - t3 + t2^2\,t3)\,(-3 + t3^2)}{(1 + t2^2)\,(1 + t3^2)^2}, 0, \frac{4\,(-t2 - 2\,t3 + t2\,t3^2)^2}{(1 + t2^2)\,(1 + t3^2)^2}\right\},\right.$$
$$\left.\left\{0, 0, 0\right\}, \left\{-\frac{4\,(t2 - t3)^2}{(1 + t2^2)\,(1 + t3^2)}, 0, 0\right\}\right\}$$

In[280]:=
```
Factor[D1 (1 - c2^2) - D2 (1 - c1^2) /. DSubs]
```

Out[280]=
$$-x1^2 + c1^2\,x1^2 + x2^2 - c2^2\,x2^2 + 2\,x1\,x3 - 2\,c1^2\,x1\,x3 - 2\,x2\,x3 + 2\,c2^2\,x2\,x3 + c1^2\,x3^2 - c2^2\,x3^2 -$$
$$y1^2 + c1^2\,y1^2 + y2^2 - c2^2\,y2^2 + 2\,y1\,y3 - 2\,c1^2\,y1\,y3 - 2\,y2\,y3 + 2\,c2^2\,y2\,y3 + c1^2\,y3^2 - c2^2\,y3^2$$

In[281]:=
```
Factor[Factor[Factor[CoefficientList[%, {c3, c2, c1}]] //. ySubs] //. special]
```

Out[281]=
$$\{\{\{2\,(x1 - x2)\,(1 + 2\,x3), 0, 2\,(1 + x2 - 2\,x1\,x3)\}, \{0, 0, 0\}, \{-2\,(1 + x1 - 2\,x2\,x3), 0, 0\}\}\}$$

In[282]:=
```
Factor[% //. tSubsSp]
```

Out[282]=
$$\left\{\left\{\left\{-\frac{4\,t3\,(-2\,t2 - t3 + t2^2\,t3)\,(-3 + t3^2)}{(1 + t2^2)\,(1 + t3^2)^2}, 0, \frac{4\,(-t2 - 2\,t3 + t2\,t3^2)^2}{(1 + t2^2)\,(1 + t3^2)^2}\right\},\right.\right.$$
$$\left.\left.\left\{0, 0, 0\right\}, \left\{-\frac{4\,(t2 - t3)^2}{(1 + t2^2)\,(1 + t3^2)}, 0, 0\right\}\right\}\right\}$$

In[283]:= **%[[1]]**

Out[283]=
$$\left\{\left\{-\frac{4\,t3\,\left(-2\,t2-t3+t2^2\,t3\right)\,\left(-3+t3^2\right)}{\left(1+t2^2\right)\,\left(1+t3^2\right)^2},\ 0,\ \frac{4\,\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)\,\left(1+t3^2\right)^2}\right\},\right.$$
$$\left.\{0,\ 0,\ 0\},\ \left\{-\frac{4\,(t2-t3)^2}{\left(1+t2^2\right)\,\left(1+t3^2\right)},\ 0,\ 0\right\}\right\}$$

In[284]:= **Factor[%%%%% - %]**

Out[284]= {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}

In[290]:= **Factor[((1 + x3) LLL + y3 RRR + 2 (1 + x3) y3) / (y1 - y2) /. unXYΔSubs]**

Out[290]=
$$-\frac{1}{(y1-y2)\,z^2}$$
$$\left(y+2\,x\,y-x^2\,y-2\,x^3\,y-x1\,y+x^2\,x1\,y-x2\,y+x^2\,x2\,y+2\,x\,x3\,y-2\,x^3\,x3\,y-x1\,x3\,y+x^2\,x1\,x3\,y-\right.$$
$$x2\,x3\,y+x^2\,x2\,x3\,y-x3^2\,y+x^2\,x3^2\,y-y^3-2\,x\,y^3+x1\,y^3+x2\,y^3-2\,x\,x3\,y^3+x1\,x3\,y^3+$$
$$x2\,x3\,y^3+x3^2\,y^3-y1-x\,y1+x^2\,y1+x^3\,y1-x3\,y1-x\,x3\,y1+x^2\,x3\,y1+x^3\,x3\,y1+$$
$$y^2\,y1+x\,y^2\,y1+x3\,y^2\,y1+x\,x3\,y^2\,y1-y2-x\,y2+x^2\,y2+x^3\,y2-x3\,y2-x\,x3\,y2+$$
$$x^2\,x3\,y2+x^3\,x3\,y2+y^2\,y2+x\,y^2\,y2+x3\,y^2\,y2+x\,x3\,y^2\,y2-y3+x^4\,y3-x1\,y3+x\,x1\,y3+$$
$$x^2\,x1\,y3-x^3\,x1\,y3-x2\,y3+x\,x2\,y3+x^2\,x2\,y3-x^3\,x2\,y3-2\,x3\,y3+2\,x^2\,x3\,y3+$$
$$2\,y^2\,y3+x1\,y^2\,y3-x\,x1\,y^2\,y3+x2\,y^2\,y3-x\,x2\,y^2\,y3+2\,x3\,y^2\,y3-y^4\,y3-y\,y1\,y3+$$
$$x^2\,y\,y1\,y3+y^3\,y1\,y3-y\,y2\,y3+x^2\,y\,y2\,y3+y^3\,y2\,y3-y\,y3^2+x^2\,y\,y3^2+y^3\,y3^2-2\,y\,z^2-$$
$$\left.2\,x\,y\,z^2-2\,x3\,y\,z^2-2\,x\,x3\,y\,z^2-y3\,z^2-2\,x\,y3\,z^2+x^2\,y3\,z^2-2\,x3\,y3\,z^2-y^2\,y3\,z^2\right)$$

In[291]:= **Factor[% //. tSubsSp]**

Out[291]=
$$-\left(\left(-3\,t3-3\,t2^2\,t3+t3^3+t2^2\,t3^3+4\,t3\,x-4\,t2^2\,t3\,x-8\,t2\,t3^2\,x+2\,t3\,x^2+2\,t2^2\,t3\,x^2-2\,t3^3\,x^2-\right.\right.$$
$$2\,t2^2\,t3^3\,x^2-4\,t3\,x^3+4\,t2^2\,t3\,x^3+8\,t2\,t3^2\,x^3+t3\,x^4+t2^2\,t3\,x^4+t3^3\,x^4+t2^2\,t3^3\,x^4-$$
$$2\,y+2\,t2^2\,y+4\,t2\,t3\,y+2\,t3^2\,y-2\,t2^2\,t3^2\,y-4\,t2\,t3^3\,y+2\,x\,y+2\,t2^2\,x\,y+2\,t3^2\,x\,y+$$
$$2\,t2^2\,t3^2\,x\,y+2\,x^2\,y-2\,t2^2\,x^2\,y-4\,t2\,t3\,x^2\,y-2\,t3^2\,x^2\,y+2\,t2^2\,t3^2\,x^2\,y+4\,t2\,t3^3\,x^2\,y-$$
$$2\,x^3\,y-2\,t2^2\,x^3\,y-2\,t3^2\,x^3\,y-2\,t2^2\,t3^2\,x^3\,y+4\,t3\,y^2+4\,t2^2\,t3\,y^2-4\,t3\,x\,y^2+$$
$$4\,t2^2\,t3\,x\,y^2+8\,t2\,t3^2\,x\,y^2+2\,y^3-2\,t2^2\,y^3-4\,t2\,t3\,y^3-2\,t3^2\,y^3+2\,t2^2\,t3^2\,y^3+$$
$$4\,t2\,t3^3\,y^3-2\,x\,y^3-2\,t2^2\,x\,y^3-2\,t3^2\,x\,y^3-2\,t2^2\,t3^2\,x\,y^3-t3\,y^4-t2^2\,t3\,y^4-t3^3\,y^4-$$
$$t2^2\,t3^3\,y^4-3\,t3\,z^2-3\,t2^2\,t3\,z^2+t3^3\,z^2+t2^2\,t3^3\,z^2-2\,t3\,x\,z^2-2\,t2^2\,t3\,x\,z^2-2\,t3^3\,x\,z^2-$$
$$2\,t2^2\,t3^3\,x\,z^2+t3\,x^2\,z^2+t2^2\,t3\,x^2\,z^2+t3^3\,x^2\,z^2+t2^2\,t3^3\,x^2\,z^2-2\,y\,z^2-2\,t2^2\,y\,z^2-$$
$$2\,t3^2\,y\,z^2-2\,t2^2\,t3^2\,y\,z^2-2\,x\,y\,z^2-2\,t2^2\,x\,y\,z^2-2\,t3^2\,x\,y\,z^2-2\,t2^2\,t3^2\,x\,y\,z^2-$$
$$\left.\left.t3\,y^2\,z^2-t2^2\,t3\,y^2\,z^2-t3^3\,y^2\,z^2-t2^2\,t3^3\,y^2\,z^2\right)\Big/\left(\left(-2\,t2-t3+t2^2\,t3\right)\,\left(1+t3^2\right)\,z^2\right)\right)$$

In[292]:=
```
Factor[
 2 (1 + x3) y3 / (y1 - y2) +
  (y3 / (y1 - y2)) (y^2 - (x - 1)^2) + ((1 + x3) / (y1 - y2)) (2 (x + 1) y) +
  ( (1 + 2 x3) y3 / (y1 - y2) - (y3 / (y1 - y2)) (y^2 - x^2 ) - ((1 + x3) / (y1 - y2)) (2 x y) +
     (2 (x1 + x2) x3 / (y1 - y2)) y - (2 (x1 + x2) y3 / (y1 - y2)) x ) (1 - x^2 - y^2) / z^2
]
```

Out[292]=
$$-\frac{1}{(y1-y2)\,z^2}\left(2\,x\,y-2\,x^3\,y+2\,x\,x3\,y-2\,x^3\,x3\,y-2\,x1\,x3\,y+2\,x^2\,x1\,x3\,y-2\,x2\,x3\,y+2\,x^2\,x2\,x3\,y-2\,x\,y^3-2\,x\,x3\,y^3+2\,x1\,x3\,y^3+2\,x2\,x3\,y^3-y3+x^4\,y3+2\,x\,x1\,y3-2\,x^3\,x1\,y3+2\,x\,x2\,y3-2\,x^3\,x2\,y3-2\,x3\,y3+2\,x^2\,x3\,y3+2\,y^2\,y3-2\,x\,x1\,y^2\,y3-2\,x\,x2\,y^2\,y3+2\,x3\,y^2\,y3-y^4\,y3-2\,y\,z^2-2\,x\,y\,z^2-2\,x3\,y\,z^2-2\,x\,x3\,y\,z^2-y3\,z^2-2\,x\,y3\,z^2+x^2\,y3\,z^2-2\,x3\,y3\,z^2-y^2\,y3\,z^2\right)$$

In[293]:=
```
Factor[% //. tSubsSp]
```

Out[293]=
$$-\left(\left(-3\,t3-3\,t2^2\,t3+t3^3+t2^2\,t3^3+4\,t3\,x-4\,t2^2\,t3\,x-8\,t2\,t3^2\,x+2\,t3\,x^2+2\,t2^2\,t3\,x^2-2\,t3^3\,x^2-2\,t2^2\,t3^3\,x^2-4\,t3\,x^3+4\,t2^2\,t3\,x^3+8\,t2\,t3^2\,x^3+t3\,x^4+t2^2\,t3\,x^4+t3^3\,x^4+t2^2\,t3^3\,x^4-2\,y+2\,t2^2\,y+4\,t2\,t3\,y+2\,t3^2\,y-2\,t2^2\,t3^2\,y-4\,t2\,t3^3\,y+2\,x\,y+2\,t2^2\,x\,y+2\,t3^2\,x\,y+2\,t2^2\,t3^2\,x\,y+2\,x^2\,y-2\,t2^2\,x^2\,y-4\,t2\,t3\,x^2\,y-2\,t3^2\,x^2\,y+2\,t2^2\,t3^2\,x^2\,y+4\,t2\,t3^3\,x^2\,y-2\,x^3\,y-2\,t2^2\,x^3\,y-2\,t3^2\,x^3\,y-2\,t2^2\,t3^2\,x^3\,y+4\,t3\,y^2+4\,t2^2\,t3\,y^2-4\,t3\,x\,y^2+4\,t2^2\,t3\,x\,y^2+8\,t2\,t3^2\,x\,y^2+2\,y^3-2\,t2^2\,y^3-4\,t2\,t3\,y^3-2\,t3^2\,y^3+2\,t2^2\,t3^2\,y^3+4\,t2\,t3^3\,y^3-2\,x\,y^3-2\,t2^2\,x\,y^3-2\,t3^2\,x\,y^3-2\,t2^2\,t3^2\,x\,y^3-t3\,y^4-t2^2\,t3\,y^4-t3^3\,y^4-t2^2\,t3^3\,y^4-3\,t3\,z^2-3\,t2^2\,t3\,z^2+t3^3\,z^2+t2^2\,t3^3\,z^2-2\,t3\,x\,z^2-2\,t2^2\,t3\,x\,z^2-2\,t3^3\,x\,z^2-2\,t2^2\,t3^3\,x\,z^2+t3\,x^2\,z^2+t2^2\,t3\,x^2\,z^2+t3^3\,x^2\,z^2+t2^2\,t3^3\,x^2\,z^2-2\,y\,z^2-2\,t2^2\,y\,z^2-2\,t3^2\,y\,z^2-2\,t2^2\,t3^2\,y\,z^2-2\,x\,y\,z^2-2\,t2^2\,x\,y\,z^2-2\,t3^2\,x\,y\,z^2-2\,t2^2\,t3^2\,x\,y\,z^2-t3\,y^2\,z^2-t2^2\,t3\,y^2\,z^2-t3^3\,y^2\,z^2-t2^2\,t3^3\,y^2\,z^2\right)\Big/\left(\left(-2\,t2-t3+t2^2\,t3\right)\left(1+t3^2\right)z^2\right)\right)$$

In[294]:=
```
Factor[ %%% - % ]
```

Out[294]=
```
0
```

In[295]:=
```
temp = Factor[ Factor[ Factor[

      2 (1 + x3) * {(1 - x1) y1, (1 - x2) y2, (1 - x3) y3, -(y1 + y2 + y3)} +

       2 y3 * {(1 + x1) x1, (1 + x2) x2, (1 + x3) x3, -(1 + x1 + x2 + x3)} +

       2 (1 + x3) y3 {-1, -1, -1, 2}

     ] //. ySubs] //. ySubs]
```

Out[295]=
$$\left\{-2\left(-y1+x1\,y1-x3\,y1+x1\,x3\,y1+y3-x1\,y3-x1^2\,y3+x3\,y3\right),\right.$$
$$-2\left(-y2+x2\,y2-x3\,y2+x2\,x3\,y2+y3-x2\,y3-x2^2\,y3+x3\,y3\right),$$
$$\left.0,\,-2\,(y1+x3\,y1+y2+x3\,y2+x1\,y3+x2\,y3)\right\}$$

In[296]:=
```
Factor[Factor[(y1 - y2) {D2, -D1, 0, 0} /. DSubs //. ySubs] //. ySubs]
```

Out[296]=
$$\{-2\,(y1-y2)\,(-1+x1\,x3+y1\,y3),\ 2\,(y1-y2)\,(-1+x2\,x3+y2\,y3),\ 0,\ 0\}$$

In[297]:=
```
Factor[temp - % //. tSubsSp]
```

Out[297]=
$$\{0, 0, 0, 0\}$$

In[298]:=
```
Factor[temp -
  2 {
    y1 - y3 - x1 y1 - x3 y3 + x1 y3 + x3 y1 + x1^2 y3 - x1 x3 y1,
    y2 - y3 - x2 y2 - x3 y3 + x2 y3 + x3 y2 + x2^2 y3 - x2 x3 y2,
    0,
    -(y1 + y2 + x1 y3 + x2 y3 + x3 y1 + x3 y2)
   } ]
```

Out[298]=
$$\{0, 0, 0, 0\}$$

In[299]:=
```
A3 = ( (y1 + y2 - y3) ((y + y3)^2 - (x + x3)^2) -
        2 (1 + x1 + x2 + x3) (x + x3) (y + y3) )  /
     (-y1 + y2 + y3 + 2 x1 y2)
```

Out[299]=
$$\frac{-2\,(x+x3)\,(1+x1+x2+x3)\,(y+y3)+(y1+y2-y3)\left(-(x+x3)^2+(y+y3)^2\right)}{-y1+y2+2\,x1\,y2+y3}$$

In[300]:=
```
B3 = (x1 - x2) (1 + 2 x3) / 2  +

     ( (y3 - y1 - y2) ((y - (y1 + y2) / 2)^2 - (x - (x1 + x2) / 2)^2)  +
        2 (1 + x1 + x2 + x3) (x - (x1 + x2) / 2) (y - (y1 + y2) / 2) )  /
     (-y1 + y2 + y3 + 2 x1 y2)
```

Out[300]=
$$\frac{1}{2}\,(x1-x2)\,(1+2\,x3)+\frac{1}{-y1+y2+2\,x1\,y2+y3}$$
$$\left(2\left(x+\frac{1}{2}\,(-x1-x2)\right)(1+x1+x2+x3)\left(y+\frac{1}{2}\,(-y1-y2)\right)+\right.$$
$$\left.\left(-\left(x+\frac{1}{2}\,(-x1-x2)\right)^2+\left(y+\frac{1}{2}\,(-y1-y2)\right)^2\right)(-y1-y2+y3)\right)$$

In[301]:=
```
T3 = Factor[
  Factor[Factor[Factor[Factor[A3 + B3 (1 - x^2 - y^2) / z^2] //. ySubs] //. ySubs] //.
     special] //. special]
```

Out[301]=
$$\frac{1}{2\,(y1 - y2 - 2\,x1\,y2 - y3)\,z^2}$$
$$\Big(-4\,y - 4\,x\,y + 4\,x^2\,y + 4\,x^3\,y - 4\,x\,x1\,y + 4\,x^3\,x1\,y + 4\,x1^2\,y - 4\,x^2\,x1^2\,y - 4\,x\,x2\,y + 4\,x^3\,x2\,y + 4\,x2^2\,y -$$
$$4\,x^2\,x2^2\,y + 4\,x3\,y - 4\,x\,x3\,y - 4\,x^2\,x3\,y + 4\,x^3\,x3\,y + 4\,x1\,x3\,y - 4\,x^2\,x1\,x3\,y + 4\,x2\,x3\,y -$$
$$4\,x^2\,x2\,x3\,y + 4\,y^3 + 4\,x\,y^3 + 4\,x\,x1\,y^3 - 4\,x1^2\,y^3 + 4\,x\,x2\,y^3 - 4\,x2^2\,y^3 - 4\,x3\,y^3 + 4\,x\,x3\,y^3 -$$
$$4\,x1\,x3\,y^3 - 4\,x2\,x3\,y^3 + 2\,y1 - 4\,x^2\,y1 + 2\,x^4\,y1 + 4\,x1\,y1 + 4\,x\,x1\,y1 - 4\,x^2\,x1\,y1 - 4\,x^3\,x1\,y1 +$$
$$2\,x1^2\,y1 - 2\,x^2\,x1^2\,y1 + 4\,x1\,x2\,y1 - 4\,x^2\,x1\,x2\,y1 - 3\,x2^2\,y1 + 3\,x^2\,x2^2\,y1 - 4\,x1\,y^2\,y1 -$$
$$4\,x\,x1\,y^2\,y1 - 2\,x1^2\,y^2\,y1 - 4\,x1\,x2\,y^2\,y1 + 3\,x2^2\,y^2\,y1 - 2\,y^4\,y1 + 14\,y2 - 16\,x^2\,y2 +$$
$$2\,x^4\,y2 - 14\,x1^2\,y2 + 14\,x^2\,x1^2\,y2 + 4\,x2\,y2 + 4\,x\,x2\,y2 - 4\,x^2\,x2\,y2 - 4\,x^3\,x2\,y2 - x1\,x2\,y2 +$$
$$x^2\,x1\,x2\,y2 - 10\,x2^2\,y2 + 10\,x^2\,x2^2\,y2 + 16\,x1\,x2\,x3\,y2 - 16\,x^2\,x1\,x2\,x3\,y2 - 12\,y^2\,y2 +$$
$$14\,x1^2\,y^2\,y2 - 4\,x2\,y^2\,y2 - 4\,x\,x2\,y^2\,y2 + x1\,x2\,y^2\,y2 + 10\,x2^2\,y^2\,y2 - 16\,x1\,x2\,x3\,y^2\,y2 -$$
$$2\,y^4\,y2 - 12\,y1^2\,y2 + 12\,x^2\,y1^2\,y2 + 12\,y^2\,y1^2\,y2 + 2\,y3 - 4\,x\,y3 + 4\,x^3\,y3 - 2\,x^4\,y3 + 2\,x1\,y3 -$$
$$4\,x\,x1\,y3 - 2\,x^2\,x1\,y3 + 4\,x^3\,x1\,y3 + x2\,y3 - 4\,x\,x2\,y3 - x^2\,x2\,y3 + 4\,x^3\,x2\,y3 + 8\,x1\,x2\,y3 -$$
$$8\,x^2\,x1\,x2\,y3 + 4\,x1^2\,x2\,y3 - 4\,x^2\,x1^2\,x2\,y3 + 4\,x1\,x2^2\,y3 - 4\,x^2\,x1\,x2^2\,y3 - 2\,x1\,x3\,y3 +$$
$$2\,x^2\,x1\,x3\,y3 + 2\,x2\,x3\,y3 - 2\,x^2\,x2\,x3\,y3 - 4\,y^2\,y3 + 4\,x\,y^2\,y3 - 2\,x1\,y^2\,y3 + 4\,x\,x1\,y^2\,y3 -$$
$$x2\,y^2\,y3 + 4\,x\,x2\,y^2\,y3 - 8\,x1\,x2\,y^2\,y3 - 4\,x1^2\,x2\,y^2\,y3 - 4\,x1\,x2^2\,y^2\,y3 + 2\,x1\,x3\,y^2\,y3 -$$
$$2\,x2\,x3\,y^2\,y3 + 2\,y^4\,y3 - 8\,x2\,y1\,y2\,y3 + 8\,x^2\,x2\,y1\,y2\,y3 + 8\,x2\,y^2\,y1\,y2\,y3 + 4\,y\,z^2 + 4\,x\,y\,z^2 +$$
$$4\,x1\,y\,z^2 + 4\,x\,x1\,y\,z^2 + 4\,x2\,y\,z^2 + 4\,x\,x2\,y\,z^2 + 4\,x3\,y\,z^2 + 4\,x\,x3\,y\,z^2 - 2\,y1\,z^2 - 4\,x\,y1\,z^2 +$$
$$2\,x^2\,y1\,z^2 + 4\,x3^2\,y1\,z^2 - 2\,y^2\,y1\,z^2 - 2\,y2\,z^2 - 4\,x\,y2\,z^2 + 2\,x^2\,y2\,z^2 + 4\,x3^2\,y2\,z^2 - 2\,y^2\,y2\,z^2 +$$
$$2\,y3\,z^2 + 4\,x\,y3\,z^2 - 2\,x^2\,y3\,z^2 + 4\,x3\,y3\,z^2 + 4\,x1\,x3\,y3\,z^2 + 4\,x2\,x3\,y3\,z^2 + 2\,y^2\,y3\,z^2\Big)$$

In[302]:=
```
TT3 =
 Factor[Factor[Factor[Factor[Factor[Factor[Factor[ (D1 (1 - c2^2) - D2 (1 - c1^2)) /
                    H /. cSubs] /. r2RSubs] /. DRSubs /.
               Z → z^2] //. ySubs] //. ySubs] //. special] //. special]
```

Out[302]=
$$-\Big(\Big(2\,\Big(-2\,x\,x1 + 2\,x^3\,x1 + x1^2 + 2\,x\,x1^2 - 2\,x^2\,x1^2 - 2\,x^3\,x1^2 + x^4\,x1^2 - x1^3 + 2\,x\,x1^3 - 2\,x^3\,x1^3 + x^4\,x1^3 -$$
$$2\,x2 - 2\,x\,x2 + 2\,x^2\,x2 + 2\,x^3\,x2 + 6\,x\,x1^2\,x2 - 6\,x^3\,x1^2\,x2 + 2\,x\,x1^3\,x2 - 2\,x^3\,x1^3\,x2 -$$
$$x2^2 - 2\,x\,x2^2 + 2\,x^2\,x2^2 + 2\,x^3\,x2^2 - x^4\,x2^2 - 2\,x1\,x2^2 - 4\,x\,x1\,x2^2 + 2\,x^2\,x1\,x2^2 +$$
$$4\,x^3\,x1\,x2^2 + x2^3 + 2\,x\,x2^3 - 2\,x^3\,x2^3 - x^4\,x2^3 - 2\,x\,x1\,x2^3 + 2\,x^3\,x1\,x2^3 - 2\,x\,x1\,x3 +$$
$$2\,x^2\,x1\,x3 + 2\,x^3\,x1\,x3 - 2\,x^4\,x1\,x3 - 4\,x\,x1^2\,x3 + 2\,x^2\,x1^2\,x3 + 4\,x^3\,x1^2\,x3 - 2\,x^4\,x1^2\,x3 -$$
$$2\,x\,x1^3\,x3 + 2\,x^3\,x1^3\,x3 + 4\,x\,x2\,x3 - 2\,x^2\,x2\,x3 - 4\,x^3\,x2\,x3 + 2\,x^4\,x2\,x3 + 4\,x1^2\,x2\,x3 -$$
$$4\,x\,x1^2\,x2\,x3 - 2\,x^2\,x1^2\,x2\,x3 + 4\,x^3\,x1^2\,x2\,x3 - 2\,x^4\,x1^2\,x2\,x3 + 4\,x\,x2^2\,x3 - 2\,x^2\,x2^2\,x3 -$$
$$4\,x^3\,x2^2\,x3 + 2\,x^4\,x2^2\,x3 - 2\,x\,x1\,x2^2\,x3 - 2\,x^2\,x1\,x2^2\,x3 + 2\,x^3\,x1\,x2^2\,x3 + 2\,x^4\,x1\,x2^2\,x3 -$$
$$x1\,x3^2 + x^4\,x1\,x3^2 + 2\,x1^2\,x3^2 - 2\,x^2\,x1^2\,x3^2 + 2\,x1^3\,x3^2 - 2\,x^2\,x1^3\,x3^2 + x2\,x3^2 +$$
$$2\,x\,x2\,x3^2 - 2\,x^3\,x2\,x3^2 - x^4\,x2\,x3^2 - 2\,x1^2\,x2\,x3^2 - 4\,x\,x1^2\,x2\,x3^2 + 2\,x^2\,x1^2\,x2\,x3^2 +$$
$$4\,x^3\,x1^2\,x2\,x3^2 - 2\,x2^2\,x3^2 + 2\,x^2\,x2^2\,x3^2 + 2\,x1\,x2^2\,x3^2 + 6\,x\,x1\,x2^2\,x3^2 - 2\,x^2\,x1\,x2^2\,x3^2 -$$
$$6\,x^3\,x1\,x2^2\,x3^2 - 2\,x2^3\,x3^2 + 2\,x^2\,x2^3\,x3^2 - 4\,x1\,x3^3 + 2\,x\,x1\,x3^3 + 2\,x^2\,x1\,x3^3 -$$
$$2\,x^3\,x1\,x3^3 + 2\,x^4\,x1\,x3^3 + 4\,x\,x1^2\,x3^3 - 4\,x^3\,x1^2\,x3^3 + 2\,x2\,x3^3 - 2\,x\,x2\,x3^3 + 2\,x^3\,x2\,x3^3 -$$
$$2\,x^4\,x2\,x3^3 - 4\,x1^2\,x2\,x3^3 + 4\,x^2\,x1^2\,x2\,x3^3 - 4\,x\,x2^2\,x3^3 + 4\,x^3\,x2^2\,x3^3 + 6\,x1\,x2^2\,x3^3 -$$
$$6\,x^2\,x1\,x2^2\,x3^3 - 2\,x1\,x3^4 + 2\,x^2\,x1\,x3^4 + 2\,x2\,x3^4 - 2\,x^2\,x2\,x3^4 + 2\,x\,x1\,y^2 - 2\,x\,x1^2\,y^2 +$$

$$2\,x1^3\,y^2 - 2\,x\,x1^3\,y^2 + 2\,x2\,y^2 + 2\,x\,x2\,y^2 - 6\,x\,x1^2\,x2\,y^2 - 2\,x\,x1^3\,x2\,y^2 + 2\,x\,x2^2\,y^2 +$$
$$2\,x1\,x2^2\,y^2 + 4\,x\,x1\,x2^2\,y^2 - 2\,x2^3\,y^2 - 2\,x\,x2^3\,y^2 + 2\,x\,x1\,x2^3\,y^2 - 2\,x1\,x3\,y^2 +$$
$$2\,x\,x1\,x3\,y^2 - 2\,x1^2\,x3\,y^2 + 4\,x\,x1^2\,x3\,y^2 + 2\,x\,x1^3\,x3\,y^2 + 2\,x2\,x3\,y^2 - 4\,x\,x2\,x3\,y^2 -$$
$$6\,x1^2\,x2\,x3\,y^2 + 4\,x\,x1^2\,x2\,x3\,y^2 + 2\,x2^2\,x3\,y^2 - 4\,x\,x2^2\,x3\,y^2 + 2\,x1\,x2^2\,x3\,y^2 +$$
$$2\,x\,x1\,x2^2\,x3\,y^2 + 2\,x1\,x3^2\,y^2 - 2\,x1^2\,x3^2\,y^2 - 2\,x1^3\,x3^2\,y^2 - 2\,x2\,x3^2\,y^2 - 2\,x\,x2\,x3^2\,y^2 +$$
$$2\,x1^2\,x2\,x3^2\,y^2 + 4\,x\,x1^2\,x2\,x3^2\,y^2 + 2\,x2^2\,x3^2\,y^2 - 2\,x1\,x2^2\,x3^2\,y^2 - 6\,x\,x1\,x2^2\,x3^2\,y^2 +$$
$$2\,x2^3\,x3^2\,y^2 + 6\,x1\,x3^3\,y^2 - 2\,x\,x1\,x3^3\,y^2 - 4\,x\,x1^2\,x3^3\,y^2 - 4\,x2\,x3^3\,y^2 + 2\,x\,x2\,x3^3\,y^2 +$$
$$4\,x1^2\,x2\,x3^3\,y^2 + 4\,x\,x2^2\,x3^3\,y^2 - 6\,x1\,x2^2\,x3^3\,y^2 + 2\,x1\,x3^4\,y^2 - 2\,x2\,x3^4\,y^2 - x1^2\,y^4 -$$
$$x1^3\,y^4 + x2^2\,y^4 + x2^3\,y^4 + 2\,x1\,x3\,y^4 + 2\,x1^2\,x3\,y^4 - 2\,x2\,x3\,y^4 + 2\,x1^2\,x2\,x3\,y^4 - 2\,x2^2\,x3\,y^4 -$$
$$2\,x1\,x2^2\,x3\,y^4 - x1\,x3^2\,y^4 + x2\,x3^2\,y^4 - 2\,x1\,x3^3\,y^4 + 2\,x2\,x3^3\,y^4 - 2\,y\,y1 + 6\,x\,y\,y1 +$$
$$2\,x^2\,y\,y1 - 6\,x^3\,y\,y1 - 2\,x1\,y\,y1 - 2\,x\,x1\,y\,y1 + 2\,x^2\,x1\,y\,y1 + 2\,x^3\,x1\,y\,y1 + 2\,x1^2\,y\,y1 -$$
$$8\,x\,x1^2\,y\,y1 - 2\,x^2\,x1^2\,y\,y1 + 8\,x^3\,x1^2\,y\,y1 - 2\,x\,x1\,x2\,y\,y1 + 2\,x^3\,x1\,x2\,y\,y1 + 4\,x2^2\,y\,y1 -$$
$$6\,x\,x2^2\,y\,y1 - 4\,x^2\,x2^2\,y\,y1 + 6\,x^3\,x2^2\,y\,y1 - 4\,x1\,x2\,x3\,y\,y1 + 8\,x\,x1\,x2\,x3\,y\,y1 +$$
$$4\,x^2\,x1\,x2\,x3\,y\,y1 - 8\,x^3\,x1\,x2\,x3\,y\,y1 + 2\,x2^2\,x3^2\,y\,y1 - 2\,x^2\,x2^2\,x3^2\,y\,y1 + 2\,x\,x3^3\,y\,y1 -$$
$$2\,x^3\,x3^3\,y\,y1 - 2\,x1\,x3^3\,y\,y1 + 2\,x^2\,x1\,x3^3\,y\,y1 + 2\,y^3\,y1 - 6\,x\,y^3\,y1 + 2\,x1\,y^3\,y1 +$$
$$2\,x\,x1\,y^3\,y1 - 2\,x1^2\,y^3\,y1 + 8\,x\,x1^2\,y^3\,y1 + 2\,x\,x1\,x2\,y^3\,y1 - 4\,x2^2\,y^3\,y1 + 6\,x\,x2^2\,y^3\,y1 +$$
$$4\,x1\,x2\,x3\,y^3\,y1 - 8\,x\,x1\,x2\,x3\,y^3\,y1 - 2\,x2^2\,x3^2\,y^3\,y1 - 2\,x\,x3^3\,y^3\,y1 + 2\,x1\,x3^3\,y^3\,y1 +$$
$$2\,x1\,y1^2 + 2\,x\,x1\,y1^2 - 2\,x^2\,x1\,y1^2 - 2\,x^3\,x1\,y1^2 - 2\,x\,x2\,x3\,y1^2 + 2\,x^3\,x2\,x3\,y1^2 -$$
$$2\,x1\,y^2\,y1^2 - 2\,x\,x1\,y^2\,y1^2 + 2\,x\,x2\,x3\,y^2\,y1^2 + 2\,y\,y1^3 - 6\,x\,y\,y1^3 - 2\,x^2\,y\,y1^3 +$$
$$6\,x^3\,y\,y1^3 - 2\,y^3\,y1^3 + 6\,x\,y^3\,y1^3 + 2\,y\,y2 - 6\,x\,y\,y2 - 2\,x^2\,y\,y2 + 6\,x^3\,y\,y2 - 4\,x1\,y\,y2 -$$
$$4\,x\,x1\,y\,y2 + 4\,x^2\,x1\,y\,y2 + 4\,x^3\,x1\,y\,y2 - 4\,x1^2\,y\,y2 + 4\,x\,x1^2\,y\,y2 + 4\,x^2\,x1^2\,y\,y2 -$$
$$4\,x^3\,x1^2\,y\,y2 + 2\,x1^3\,y\,y2 - 2\,x^2\,x1^3\,y\,y2 + 2\,x2\,y\,y2 + 2\,x\,x2\,y\,y2 - 2\,x^2\,x2\,y\,y2 -$$
$$2\,x^3\,x2\,y\,y2 - 2\,x\,x1\,x2\,y\,y2 + 2\,x^3\,x1\,x2\,y\,y2 - 2\,x2^2\,y\,y2 + 6\,x\,x2^2\,y\,y2 + 2\,x^2\,x2^2\,y\,y2 -$$
$$6\,x^3\,x2^2\,y\,y2 + 2\,x1\,x2^2\,y\,y2 - 2\,x^2\,x1\,x2^2\,y\,y2 + 4\,x1\,x2\,x3\,y\,y2 - 8\,x\,x1\,x2\,x3\,y\,y2 -$$
$$4\,x^2\,x1\,x2\,x3\,y\,y2 + 8\,x^3\,x1\,x2\,x3\,y\,y2 + 8\,x\,x1\,x3^2\,y\,y2 - 8\,x^3\,x1\,x3^2\,y\,y2 - 4\,x1^2\,x3^2\,y\,y2 +$$
$$4\,x^2\,x1^2\,x3^2\,y\,y2 - 2\,x1\,x2\,x3^2\,y\,y2 + 2\,x^2\,x1\,x2\,x3^2\,y\,y2 - 2\,x\,x3^3\,y\,y2 + 2\,x^3\,x3^3\,y\,y2 +$$
$$2\,x2\,x3^3\,y\,y2 - 2\,x^2\,x2\,x3^3\,y\,y2 - 2\,y^3\,y2 + 6\,x\,y^3\,y2 + 4\,x1\,y^3\,y2 + 4\,x\,x1\,y^3\,y2 +$$
$$4\,x1^2\,y^3\,y2 - 4\,x\,x1^2\,y^3\,y2 - 2\,x1^3\,y^3\,y2 - 2\,x2\,y^3\,y2 - 2\,x\,x2\,y^3\,y2 + 2\,x\,x1\,x2\,y^3\,y2 +$$
$$2\,x2^2\,y^3\,y2 - 6\,x\,x2^2\,y^3\,y2 - 2\,x1\,x2^2\,y^3\,y2 - 4\,x1\,x2\,x3\,y^3\,y2 + 8\,x\,x1\,x2\,x3\,y^3\,y2 -$$
$$8\,x\,x1\,x3^2\,y^3\,y2 + 4\,x1^2\,x3^2\,y^3\,y2 + 2\,x1\,x2\,x3^2\,y^3\,y2 + 2\,x\,x3^3\,y^3\,y2 - 2\,x2\,x3^3\,y^3\,y2 -$$
$$2\,y\,y1^2\,y2 + 6\,x\,y\,y1^2\,y2 + 2\,x^2\,y\,y1^2\,y2 - 6\,x^3\,y\,y1^2\,y2 + 2\,y^3\,y1^2\,y2 - 6\,x\,y^3\,y1^2\,y2 +$$
$$2\,x\,x1\,y2^2 - 2\,x^3\,x1\,y2^2 - 2\,x\,x1\,x3^2\,y2^2 + 2\,x^3\,x1\,x3^2\,y2^2 + 2\,x1\,x3^3\,y2^2 - 2\,x^2\,x1\,x3^3\,y2^2 -$$
$$2\,x\,x1\,y^2\,y2^2 + 2\,x\,x1\,x3^2\,y^2\,y2^2 - 2\,x1\,x3^3\,y^2\,y2^2 - 2\,y\,y3 - 2\,x\,y\,y3 + 2\,x^2\,y\,y3 +$$
$$2\,x^3\,y\,y3 - 2\,x1^3\,y\,y3 + 2\,x^2\,x1^3\,y\,y3 - 2\,x\,x2\,y\,y3 + 2\,x^3\,x2\,y\,y3 - 2\,x\,x1^2\,x2\,y\,y3 +$$
$$2\,x^3\,x1^2\,x2\,y\,y3 + 2\,x2^2\,y\,y3 - 2\,x^2\,x2^2\,y\,y3 + 2\,x2^3\,y\,y3 - 2\,x^2\,x2^3\,y\,y3 - 2\,x1\,x3\,y\,y3 +$$
$$2\,x^2\,x1\,x3\,y\,y3 + 2\,x2\,x3\,y\,y3 - 2\,x^2\,x2\,x3\,y\,y3 + 4\,x\,x3^2\,y\,y3 - 4\,x^3\,x3^2\,y\,y3 - 2\,x1\,x3^2\,y\,y3 -$$
$$2\,x\,x1\,x3^2\,y\,y3 + 2\,x^2\,x1\,x3^2\,y\,y3 + 2\,x^3\,x1\,x3^2\,y\,y3 + 2\,x1^2\,x3^2\,y\,y3 - 2\,x^2\,x1^2\,x3^2\,y\,y3 +$$
$$2\,x\,x2\,x3^2\,y\,y3 - 2\,x^3\,x2\,x3^2\,y\,y3 - 2\,x2^2\,x3^2\,y\,y3 + 2\,x^2\,x2^2\,x3^2\,y\,y3 + 2\,y^3\,y3 +$$
$$2\,x\,y^3\,y3 + 2\,x1^3\,y^3\,y3 + 2\,x\,x2\,y^3\,y3 + 2\,x\,x1^2\,x2\,y^3\,y3 - 2\,x2^2\,y^3\,y3 - 2\,x2^3\,y^3\,y3 +$$
$$2\,x1\,x3\,y^3\,y3 - 2\,x2\,x3\,y^3\,y3 - 4\,x\,x3^2\,y^3\,y3 + 2\,x1\,x3^2\,y^3\,y3 + 2\,x\,x1\,x3^2\,y^3\,y3 -$$
$$2\,x1^2\,x3^2\,y^3\,y3 - 2\,x\,x2\,x3^2\,y^3\,y3 + 2\,x2^2\,x3^2\,y^3\,y3 + 2\,x\,x2^2\,y1\,y3 - 2\,x^3\,x2^2\,y1\,y3 -$$
$$2\,x\,x1\,x2\,x3\,y1\,y3 + 2\,x^3\,x1\,x2\,x3\,y1\,y3 - 2\,x\,x2^2\,y^2\,y1\,y3 + 2\,x\,x1\,x2\,x3\,y^2\,y1\,y3 +$$
$$2\,x2\,y\,y1^2\,y3 - 6\,x\,x2\,y\,y1^2\,y3 - 2\,x^2\,x2\,y\,y1^2\,y3 + 6\,x^3\,x2\,y\,y1^2\,y3 - 2\,x2\,y^3\,y1^2\,y3 +$$
$$6\,x\,x2\,y^3\,y1^2\,y3 + 2\,x\,x2^2\,y2\,y3 - 2\,x^3\,x2^2\,y2\,y3 - 2\,x\,x2^2\,y^2\,y2\,y3 - 2\,x2\,y\,y1\,y2\,y3 +$$

$$4\,x\,x2\,y\,y1\,y2\,y3 + 2\,x^2\,x2\,y\,y1\,y2\,y3 - 4\,x^3\,x2\,y\,y1\,y2\,y3 + 2\,x2\,y^3\,y1\,y2\,y3 -$$
$$4\,x\,x2\,y^3\,y1\,y2\,y3 - 2\,x1^2\,y3^2 + 2\,x^2\,x1^2\,y3^2 + 2\,x2\,y3^2 - 2\,x^2\,x2\,y3^2 - 2\,x\,x1^2\,x2\,y3^2 +$$
$$2\,x^3\,x1^2\,x2\,y3^2 + 2\,x2^2\,y3^2 - 2\,x^2\,x2^2\,y3^2 + 4\,x\,x1\,x2^2\,y3^2 - 4\,x^3\,x1\,x2^2\,y3^2 + 2\,x1^2\,y^2\,y3^2 -$$
$$2\,x2\,y^2\,y3^2 + 2\,x\,x1^2\,x2\,y^2\,y3^2 - 2\,x2^2\,y^2\,y3^2 - 4\,x\,x1\,x2^2\,y^2\,y3^2 - 2\,x\,x1\,z^2 + x1^2\,z^2 -$$
$$2\,x\,x1^2\,z^2 + x^2\,x1^2\,z^2 - x1^3\,z^2 + 2\,x\,x1^3\,z^2 + x^2\,x1^3\,z^2 - 2\,x2\,z^2 + 2\,x\,x2\,z^2 - 4\,x\,x1^2\,x2\,z^2 -$$
$$x2^2\,z^2 + 2\,x\,x2^2\,z^2 - x^2\,x2^2\,z^2 - 2\,x1\,x2^2\,z^2 + 2\,x\,x1\,x2^2\,z^2 + x2^3\,z^2 + 2\,x\,x2^3\,z^2 -$$
$$x^2\,x2^3\,z^2 + 2\,x\,x1\,x3\,z^2 - 2\,x^2\,x1\,x3\,z^2 + 4\,x\,x1^2\,x3\,z^2 - 2\,x^2\,x1^2\,x3\,z^2 + 2\,x\,x1^3\,x3\,z^2 +$$
$$2\,x^2\,x2\,x3\,z^2 + 4\,x1^2\,x2\,x3\,z^2 - 4\,x\,x1^2\,x2\,x3\,z^2 - 2\,x^2\,x1^2\,x2\,x3\,z^2 - 4\,x\,x2^2\,x3\,z^2 +$$
$$2\,x^2\,x2^2\,x3\,z^2 - 2\,x\,x1\,x2^2\,x3\,z^2 + 2\,x^2\,x1\,x2^2\,x3\,z^2 - 4\,x\,x2^3\,x3\,z^2 - x1\,x3^2\,z^2 -$$
$$2\,x\,x1\,x3^2\,z^2 + x^2\,x1\,x3^2\,z^2 + 2\,x1^2\,x3^2\,z^2 + 2\,x1^3\,x3^2\,z^2 + x2\,x3^2\,z^2 - x^2\,x2\,x3^2\,z^2 -$$
$$2\,x1^2\,x2\,x3^2\,z^2 - 2\,x2^2\,x3^2\,z^2 + 2\,x1\,x2^2\,x3^2\,z^2 + 6\,x\,x1\,x2^2\,x3^2\,z^2 - 2\,x2^3\,x3^2\,z^2 -$$
$$4\,x1\,x3^3\,z^2 - 2\,x\,x1\,x3^3\,z^2 + 2\,x^2\,x1\,x3^3\,z^2 + 2\,x2\,x3^3\,z^2 + 2\,x\,x2\,x3^3\,z^2 - 2\,x^2\,x2\,x3^3\,z^2 -$$
$$4\,x1^2\,x2\,x3^3\,z^2 + 6\,x1\,x2^2\,x3^3\,z^2 - 2\,x1\,x3^4\,z^2 + 2\,x2\,x3^4\,z^2 - x1^2\,y^2\,z^2 - x1^3\,y^2\,z^2 +$$
$$x2^2\,y^2\,z^2 + x2^3\,y^2\,z^2 + 2\,x1\,x3\,y^2\,z^2 + 2\,x1^2\,x3\,y^2\,z^2 - 2\,x2\,x3\,y^2\,z^2 + 2\,x1^2\,x2\,x3\,y^2\,z^2 -$$
$$2\,x2^2\,x3\,y^2\,z^2 - 2\,x1\,x2^2\,x3\,y^2\,z^2 - x1\,x3^2\,y^2\,z^2 + x2\,x3^2\,y^2\,z^2 - 2\,x1\,x3^3\,y^2\,z^2 +$$
$$2\,x2\,x3^3\,y^2\,z^2 - 2\,y\,y1\,z^2 - 6\,x\,y\,y1\,z^2 + 2\,x1\,y\,y1\,z^2 + 2\,x\,x1\,y\,y1\,z^2 + 2\,x1^2\,y\,y1\,z^2 +$$
$$8\,x\,x1^2\,y\,y1\,z^2 + 2\,x\,x1\,x2\,y\,y1\,z^2 + 2\,x2^2\,y\,y1\,z^2 + 6\,x\,x2^2\,y\,y1\,z^2 - 4\,x1\,x2\,x3\,y\,y1\,z^2 -$$
$$8\,x\,x1\,x2\,x3\,y\,y1\,z^2 - 2\,x3^2\,y\,y1\,z^2 + 2\,x2^2\,x3^2\,y\,y1\,z^2 - 2\,x\,x3^3\,y\,y1\,z^2 + 2\,x1\,y1^2\,z^2 +$$
$$2\,x\,x1\,y1^2\,z^2 - 2\,x\,x2\,x3\,y1^2\,z^2 + 2\,y\,y1^3\,z^2 + 6\,x\,y\,y1^3\,z^2 + 2\,y\,y2\,z^2 + 6\,x\,y\,y2\,z^2 -$$
$$4\,x1\,y\,y2\,z^2 + 4\,x\,x1\,y\,y2\,z^2 - 2\,x1^2\,y\,y2\,z^2 - 4\,x\,x1^2\,y\,y2\,z^2 - 2\,x2\,y\,y2\,z^2 -$$
$$2\,x\,x2\,y\,y2\,z^2 + 2\,x\,x1\,x2\,y\,y2\,z^2 - 2\,x2^2\,y\,y2\,z^2 - 6\,x\,x2^2\,y\,y2\,z^2 + 4\,x1\,x2\,x3\,y\,y2\,z^2 +$$
$$8\,x\,x1\,x2\,x3\,y\,y2\,z^2 + 2\,x3^2\,y\,y2\,z^2 - 8\,x\,x1\,x3^2\,y\,y2\,z^2 + 2\,x1\,x2\,x3^2\,y\,y2\,z^2 +$$
$$2\,x\,x3^3\,y\,y2\,z^2 - 2\,y\,y1^2\,y2\,z^2 - 6\,x\,y\,y1^2\,y2\,z^2 - 2\,x\,x1\,y2^2\,z^2 + 2\,x\,x1\,x3^2\,y2^2\,z^2 +$$
$$2\,x1\,x3^3\,y2^2\,z^2 - 2\,y\,y3\,z^2 + 2\,x\,y\,y3\,z^2 - 4\,x1^2\,y\,y3\,z^2 - 2\,x1^3\,y\,y3\,z^2 + 2\,x\,x2\,y\,y3\,z^2 +$$
$$2\,x\,x1^2\,x2\,y\,y3\,z^2 + 4\,x2^2\,y\,y3\,z^2 + 2\,x2^3\,y\,y3\,z^2 + 2\,x1\,x3\,y\,y3\,z^2 - 2\,x2\,x3\,y\,y3\,z^2 +$$
$$4\,x1^2\,x2\,x3\,y\,y3\,z^2 - 4\,x1\,x2^2\,x3\,y\,y3\,z^2 - 4\,x\,x3^2\,y\,y3\,z^2 + 2\,x1\,x3^2\,y\,y3\,z^2 +$$
$$2\,x\,x1\,x3^2\,y\,y3\,z^2 - 2\,x\,x2\,x3^2\,y\,y3\,z^2 + 2\,x\,x2^2\,y1\,y3\,z^2 - 2\,x\,x1\,x2\,x3\,y1\,y3\,z^2 +$$
$$2\,x2\,y\,y1^2\,y3\,z^2 + 6\,x\,x2\,y\,y1^2\,y3\,z^2 + 2\,x\,x2^2\,y2\,y3\,z^2 - 2\,x2\,y\,y1\,y2\,y3\,z^2 -$$
$$4\,x\,x2\,y\,y1\,y2\,y3\,z^2 - 2\,x1^2\,y3^2\,z^2 + 2\,x2\,y3^2\,z^2 + 2\,x\,x1^2\,x2\,y3^2\,z^2 + 2\,x2^2\,y3^2\,z^2\big)\big) \Big/$$
$$\big((y1 - y2 + 2\,x1\,y2 + y3 - 2\,x1\,y3 + 2\,x2\,y3)^2\,z^2\big)\big)$$

In[303]:=
```
Factor[Factor[T3 - TT3] //. tSubsSp]
```

Out[303]= 0

In[304]:=
```
TTT3 = Factor[((1 + x3) LLL + y3 RRR + 2 (1 + x3) y3) / (y1 - y2)  /. unXYΔSubs]
```

Out[304]=
$$-\frac{1}{(y1-y2)\,z^2}\Big(y+2\,x\,y-x^2\,y-2\,x^3\,y-x1\,y+x^2\,x1\,y-x2\,y+x^2\,x2\,y+2\,x\,x3\,y-2\,x^3\,x3\,y-x1\,x3\,y+x^2\,x1\,x3\,y-x2\,x3\,y+x^2\,x2\,x3\,y-x3^2\,y+x^2\,x3^2\,y-y^3-2\,x\,y^3+x1\,y^3+x2\,y^3-2\,x\,x3\,y^3+x1\,x3\,y^3+x2\,x3\,y^3+x3^2\,y^3-y1-x\,y1+x^2\,y1+x^3\,y1-x3\,y1-x\,x3\,y1+x^2\,x3\,y1+x^3\,x3\,y1+y^2\,y1+x\,y^2\,y1+x3\,y^2\,y1+x\,x3\,y^2\,y1-y2-x\,y2+x^2\,y2+x^3\,y2-x3\,y2-x\,x3\,y2+x^2\,x3\,y2+x^3\,x3\,y2+y^2\,y2+x\,y^2\,y2+x3\,y^2\,y2+x\,x3\,y^2\,y2-y3+x^4\,y3-x1\,y3+x\,x1\,y3+x^2\,x1\,y3-x^3\,x1\,y3-x2\,y3+x\,x2\,y3+x^2\,x2\,y3-x^3\,x2\,y3-2\,x3\,y3+2\,x^2\,x3\,y3+2\,y^2\,y3+x1\,y^2\,y3-x\,x1\,y^2\,y3+x2\,y^2\,y3-x\,x2\,y^2\,y3+2\,x3\,y^2\,y3-y^4\,y3-y\,y1\,y3+x^2\,y\,y1\,y3+y^3\,y1\,y3-y\,y2\,y3+x^2\,y\,y2\,y3+y^3\,y2\,y3-y\,y3^2+x^2\,y\,y3^2+y^3\,y3^2-2\,y\,z^2-2\,x\,y\,z^2-2\,x3\,y\,z^2-2\,x\,x3\,y\,z^2-y3\,z^2-2\,x\,y3\,z^2+x^2\,y3\,z^2-2\,x3\,y3\,z^2-y^2\,y3\,z^2\Big)$$

In[305]:=
```
Factor[Factor[T3 - TTT3] //. tSubsSp]
```

Out[305]=
$$0$$

In[306]:=
```
{Numerator[T3], Denominator[T3]}
```

Out[306]=
$$\Big\{-4\,y-4\,x\,y+4\,x^2\,y+4\,x^3\,y-4\,x\,x1\,y+4\,x^3\,x1\,y+4\,x1^2\,y-4\,x^2\,x1^2\,y-4\,x\,x2\,y+4\,x^3\,x2\,y+4\,x2^2\,y-4\,x^2\,x2^2\,y+4\,x3\,y-4\,x\,x3\,y-4\,x^2\,x3\,y+4\,x^3\,x3\,y+4\,x1\,x3\,y-4\,x^2\,x1\,x3\,y+4\,x2\,x3\,y-4\,x^2\,x2\,x3\,y+4\,y^3+4\,x\,y^3+4\,x\,x1\,y^3-4\,x1^2\,y^3+4\,x\,x2\,y^3-4\,x2^2\,y^3-4\,x3\,y^3+4\,x\,x3\,y^3-4\,x1\,x3\,y^3-4\,x2\,x3\,y^3+2\,y1-4\,x^2\,y1+2\,x^4\,y1+4\,x1\,y1+4\,x\,x1\,y1-4\,x^2\,x1\,y1-4\,x^3\,x1\,y1+2\,x1^2\,y1-2\,x^2\,x1^2\,y1+4\,x1\,x2\,y1-4\,x^2\,x1\,x2\,y1-3\,x2^2\,y1+3\,x^2\,x2^2\,y1-4\,x1\,y^2\,y1-4\,x\,x1\,y^2\,y1-2\,x1^2\,y^2\,y1-4\,x1\,x2\,y^2\,y1+3\,x2^2\,y^2\,y1-2\,y^4\,y1+14\,y2-16\,x^2\,y2+2\,x^4\,y2-14\,x1^2\,y2+14\,x^2\,x1^2\,y2+4\,x2\,y2+4\,x\,x2\,y2-4\,x^2\,x2\,y2-4\,x^3\,x2\,y2-x1\,x2\,y2+x^2\,x1\,x2\,y2-10\,x2^2\,y2+10\,x^2\,x2^2\,y2+16\,x1\,x2\,x3\,y2-16\,x^2\,x1\,x2\,x3\,y2-12\,y^2\,y2+14\,x1^2\,y^2\,y2-4\,x2\,y^2\,y2-4\,x\,x2\,y^2\,y2+x1\,x2\,y^2\,y2+10\,x2^2\,y^2\,y2-16\,x1\,x2\,x3\,y^2\,y2-2\,y^4\,y2-12\,y1^2\,y2+12\,x^2\,y1^2\,y2+12\,y^2\,y1^2\,y2+2\,y3-4\,x\,y3+4\,x^3\,y3-2\,x^4\,y3+2\,x1\,y3-4\,x\,x1\,y3-2\,x^2\,x1\,y3+4\,x^3\,x1\,y3+x2\,y3-4\,x\,x2\,y3-x^2\,x2\,y3+4\,x^3\,x2\,y3+8\,x1\,x2\,y3-8\,x^2\,x1\,x2\,y3+4\,x1^2\,x2\,y3-4\,x^2\,x1^2\,x2\,y3+4\,x1\,x2^2\,y3-4\,x^2\,x1\,x2^2\,y3-2\,x1\,x3\,y3+2\,x^2\,x1\,x3\,y3+2\,x2\,x3\,y3-2\,x^2\,x2\,x3\,y3-4\,y^2\,y3+4\,x\,y^2\,y3-2\,x1\,y^2\,y3+4\,x\,x1\,y^2\,y3-x2\,y^2\,y3+4\,x\,x2\,y^2\,y3-8\,x1\,x2\,y^2\,y3-4\,x1^2\,x2\,y^2\,y3-4\,x1\,x2^2\,y^2\,y3+2\,x1\,x3\,y^2\,y3-2\,x2\,x3\,y^2\,y3+2\,y^4\,y3-8\,x2\,y1\,y2\,y3+8\,x^2\,x2\,y1\,y2\,y3+8\,x2\,y^2\,y1\,y2\,y3+4\,y\,z^2+4\,x\,y\,z^2+4\,x1\,y\,z^2+4\,x\,x1\,y\,z^2+4\,x2\,y\,z^2+4\,x\,x2\,y\,z^2+4\,x3\,y\,z^2+4\,x\,x3\,y\,z^2-2\,y1\,z^2-4\,x\,y1\,z^2+2\,x^2\,y1\,z^2+4\,x3^2\,y1\,z^2-2\,y^2\,y1\,z^2-2\,y2\,z^2-4\,x\,y2\,z^2+2\,x^2\,y2\,z^2+4\,x3^2\,y2\,z^2-2\,y^2\,y2\,z^2+2\,y3\,z^2+4\,x\,y3\,z^2-2\,x^2\,y3\,z^2+4\,x3\,y3\,z^2+4\,x1\,x3\,y3\,z^2+4\,x2\,x3\,y3\,z^2+2\,y^2\,y3\,z^2,\ 2\,(y1-y2-2\,x1\,y2-y3)\,z^2\Big\}$$

In[307]:=
```
Factor[ % /. (y1 - y2 - 2 x1 y2 - y3) → (2 (1 + x1) (1 + x2) (y1 - y2)) / (1 + x1 + x2 + x3) ]
```

Out[307]=

$$\Big\{-4\,y-4\,x\,y+4\,x^2\,y+4\,x^3\,y-4\,x\,x1\,y+4\,x^3\,x1\,y+4\,x1^2\,y-4\,x^2\,x1^2\,y-4\,x\,x2\,y+4\,x^3\,x2\,y+$$
$$4\,x2^2\,y-4\,x^2\,x2^2\,y+4\,x3\,y-4\,x\,x3\,y-4\,x^2\,x3\,y+4\,x^3\,x3\,y+4\,x1\,x3\,y-4\,x^2\,x1\,x3\,y+$$
$$4\,x2\,x3\,y-4\,x^2\,x2\,x3\,y+4\,y^3+4\,x\,y^3+4\,x\,x1\,y^3-4\,x1^2\,y^3+4\,x\,x2\,y^3-4\,x2^2\,y^3-$$
$$4\,x3\,y^3+4\,x\,x3\,y^3-4\,x1\,x3\,y^3-4\,x2\,x3\,y^3+2\,y1-4\,x^2\,y1+2\,x^4\,y1+4\,x1\,y1+4\,x\,x1\,y1-$$
$$4\,x^2\,x1\,y1-4\,x^3\,x1\,y1+2\,x1^2\,y1-2\,x^2\,x1^2\,y1+4\,x1\,x2\,y1-4\,x^2\,x1\,x2\,y1-3\,x2^2\,y1+$$
$$3\,x^2\,x2^2\,y1-4\,x1\,y^2\,y1-4\,x\,x1\,y^2\,y1-2\,x1^2\,y^2\,y1-4\,x1\,x2\,y^2\,y1+3\,x2^2\,y^2\,y1-$$
$$2\,y^4\,y1+14\,y2-16\,x^2\,y2+2\,x^4\,y2-14\,x1^2\,y2+14\,x^2\,x1^2\,y2+4\,x2\,y2+4\,x\,x2\,y2-$$
$$4\,x^2\,x2\,y2-4\,x^3\,x2\,y2-x1\,x2\,y2+x^2\,x1\,x2\,y2-10\,x2^2\,y2+10\,x^2\,x2^2\,y2+16\,x1\,x2\,x3\,y2-$$
$$16\,x^2\,x1\,x2\,x3\,y2-12\,y^2\,y2+14\,x1^2\,y^2\,y2-4\,x2\,y^2\,y2-4\,x\,x2\,y^2\,y2+x1\,x2\,y^2\,y2+$$
$$10\,x2^2\,y^2\,y2-16\,x1\,x2\,x3\,y^2\,y2-2\,y^4\,y2-12\,y1^2\,y2+12\,x^2\,y1^2\,y2+12\,y^2\,y1^2\,y2+$$
$$2\,y3-4\,x\,y3+4\,x^3\,y3-2\,x^4\,y3+2\,x1\,y3-4\,x\,x1\,y3-2\,x^2\,x1\,y3+4\,x^3\,x1\,y3+x2\,y3-$$
$$4\,x\,x2\,y3-x^2\,x2\,y3+4\,x^3\,x2\,y3+8\,x1\,x2\,y3-8\,x^2\,x1\,x2\,y3+4\,x1^2\,x2\,y3-4\,x^2\,x1^2\,x2\,y3+$$
$$4\,x1\,x2^2\,y3-4\,x^2\,x1\,x2^2\,y3-2\,x1\,x3\,y3+2\,x^2\,x1\,x3\,y3+2\,x2\,x3\,y3-2\,x^2\,x2\,x3\,y3-$$
$$4\,y^2\,y3+4\,x\,y^2\,y3-2\,x1\,y^2\,y3+4\,x\,x1\,y^2\,y3-x2\,y^2\,y3+4\,x\,x2\,y^2\,y3-8\,x1\,x2\,y^2\,y3-$$
$$4\,x1^2\,x2\,y^2\,y3-4\,x1\,x2^2\,y^2\,y3+2\,x1\,x3\,y^2\,y3-2\,x2\,x3\,y^2\,y3+2\,y^4\,y3-8\,x2\,y1\,y2\,y3+$$
$$8\,x^2\,x2\,y1\,y2\,y3+8\,x2\,y^2\,y1\,y2\,y3+4\,y\,z^2+4\,x\,y\,z^2+4\,x1\,y\,z^2+4\,x\,x1\,y\,z^2+4\,x2\,y\,z^2+$$
$$4\,x\,x2\,y\,z^2+4\,x3\,y\,z^2+4\,x\,x3\,y\,z^2-2\,y1\,z^2-4\,x\,y1\,z^2+2\,x^2\,y1\,z^2+4\,x3^2\,y1\,z^2-2\,y^2\,y1\,z^2-$$
$$2\,y2\,z^2-4\,x\,y2\,z^2+2\,x^2\,y2\,z^2+4\,x3^2\,y2\,z^2-2\,y^2\,y2\,z^2+2\,y3\,z^2+4\,x\,y3\,z^2-2\,x^2\,y3\,z^2+$$
$$4\,x3\,y3\,z^2+4\,x1\,x3\,y3\,z^2+4\,x2\,x3\,y3\,z^2+2\,y^2\,y3\,z^2,\ \frac{4\,(1+x1)\,(1+x2)\,(y1-y2)\,z^2}{1+x1+x2+x3}\Big\}$$

In[308]:=
```
T30 = Factor[%[[1]] / %[[2]]]
```

Out[308]=
$$-\frac{1}{4\,(1+x1)\,(1+x2)\,(y1-y2)\,z^2}$$
$$(1+x1+x2+x3)\,\left(4\,y+4\,x\,y-4\,x^2\,y-4\,x^3\,y+4\,x\,x1\,y-4\,x^3\,x1\,y-4\,x1^2\,y+4\,x^2\,x1^2\,y+4\,x\,x2\,y-\right.$$
$$4\,x^3\,x2\,y-4\,x2^2\,y+4\,x^2\,x2^2\,y-4\,x3\,y+4\,x\,x3\,y+4\,x^2\,x3\,y-4\,x^3\,x3\,y-4\,x1\,x3\,y+4\,x^2\,x1\,x3\,y-$$
$$4\,x2\,x3\,y+4\,x^2\,x2\,x3\,y-4\,y^3-4\,x\,y^3-4\,x\,x1\,y^3+4\,x1^2\,y^3-4\,x\,x2\,y^3+4\,x2^2\,y^3+4\,x3\,y^3-$$
$$4\,x\,x3\,y^3+4\,x1\,x3\,y^3+4\,x2\,x3\,y^3-2\,y1+4\,x^2\,y1-2\,x^4\,y1-4\,x1\,y1-4\,x\,x1\,y1+4\,x^2\,x1\,y1+$$
$$4\,x^3\,x1\,y1-2\,x1^2\,y1+2\,x^2\,x1^2\,y1-4\,x1\,x2\,y1+4\,x^2\,x1\,x2\,y1+3\,x2^2\,y1-3\,x^2\,x2^2\,y1+$$
$$4\,x1\,y^2\,y1+4\,x\,x1\,y^2\,y1+2\,x1^2\,y^2\,y1+4\,x1\,x2\,y^2\,y1-3\,x2^2\,y^2\,y1+2\,y^4\,y1-14\,y2+16\,x^2\,y2-$$
$$2\,x^4\,y2+14\,x1^2\,y2-14\,x^2\,x1^2\,y2-4\,x2\,y2-4\,x\,x2\,y2+4\,x^2\,x2\,y2+4\,x^3\,x2\,y2+x1\,x2\,y2-$$
$$x^2\,x1\,x2\,y2+10\,x2^2\,y2-10\,x^2\,x2^2\,y2-16\,x1\,x2\,x3\,y2+16\,x^2\,x1\,x2\,x3\,y2+12\,y^2\,y2-$$
$$14\,x1^2\,y^2\,y2+4\,x2\,y^2\,y2+4\,x\,x2\,y^2\,y2-x1\,x2\,y^2\,y2-10\,x2^2\,y^2\,y2+16\,x1\,x2\,x3\,y^2\,y2+$$
$$2\,y^4\,y2+12\,y1^2\,y2-12\,x^2\,y1^2\,y2-12\,y^2\,y1^2\,y2-2\,y3+4\,x\,y3-4\,x^3\,y3+2\,x^4\,y3-2\,x1\,y3+$$
$$4\,x\,x1\,y3+2\,x^2\,x1\,y3-4\,x^3\,x1\,y3-x2\,y3+4\,x\,x2\,y3+x^2\,x2\,y3-4\,x^3\,x2\,y3-8\,x1\,x2\,y3+$$
$$8\,x^2\,x1\,x2\,y3-4\,x1^2\,x2\,y3+4\,x^2\,x1^2\,x2\,y3-4\,x1\,x2^2\,y3+4\,x^2\,x1\,x2^2\,y3+2\,x1\,x3\,y3-$$
$$2\,x^2\,x1\,x3\,y3-2\,x2\,x3\,y3+2\,x^2\,x2\,x3\,y3+4\,y^2\,y3-4\,x\,y^2\,y3+2\,x1\,y^2\,y3-4\,x\,x1\,y^2\,y3+$$
$$x2\,y^2\,y3-4\,x\,x2\,y^2\,y3+8\,x1\,x2\,y^2\,y3+4\,x1^2\,x2\,y^2\,y3+4\,x1\,x2^2\,y^2\,y3-2\,x1\,x3\,y^2\,y3+$$
$$2\,x2\,x3\,y^2\,y3-2\,y^4\,y3+8\,x2\,y1\,y2\,y3-8\,x^2\,x2\,y1\,y2\,y3-8\,x2\,y^2\,y1\,y2\,y3-4\,y\,z^2-4\,x\,y\,z^2-$$
$$4\,x1\,y\,z^2-4\,x\,x1\,y\,z^2-4\,x2\,y\,z^2-4\,x\,x2\,y\,z^2-4\,x3\,y\,z^2-4\,x\,x3\,y\,z^2+2\,y1\,z^2+4\,x\,y1\,z^2-$$
$$2\,x^2\,y1\,z^2-4\,x3^2\,y1\,z^2+2\,y^2\,y1\,z^2+2\,y2\,z^2+4\,x\,y2\,z^2-2\,x^2\,y2\,z^2-4\,x3^2\,y2\,z^2+2\,y^2\,y2\,z^2-$$
$$\left.2\,y3\,z^2-4\,x\,y3\,z^2+2\,x^2\,y3\,z^2-4\,x3\,y3\,z^2-4\,x1\,x3\,y3\,z^2-4\,x2\,x3\,y3\,z^2-2\,y^2\,y3\,z^2\right)$$

In[309]:=
```
Factor[Factor[T3 - T30] //. tSubsSp]
```

Out[309]=
```
0
```

In[310]:=
```
Factor[Factor[TTT3 - T30] //. tSubsSp]
```

Out[310]=
```
0
```

In[311]:=
```
thing1 =
 (1 + x1 + x2 + x3) (4 y + 4 x y - 4 x^2 y - 4 x^3 y + 4 x x1 y - 4 x^3 x1 y - 4 x1^2 y + 4 x^2 x1^2 y + 4 x x2 y -
    4 x^3 x2 y - 4 x2^2 y + 4 x^2 x2^2 y - 4 x3 y + 4 x x3 y + 4 x^2 x3 y - 4 x^3 x3 y - 4 x1 x3 y +
    4 x^2 x1 x3 y - 4 x2 x3 y + 4 x^2 x2 x3 y - 4 y^3 - 4 x y^3 - 4 x x1 y^3 + 4 x1^2 y^3 - 4 x x2 y^3 +
    4 x2^2 y^3 + 4 x3 y^3 - 4 x x3 y^3 + 4 x1 x3 y^3 + 4 x2 x3 y^3 - 2 y1 + 4 x^2 y1 - 2 x^4 y1 - 4 x1 y1 -
    4 x x1 y1 + 4 x^2 x1 y1 + 4 x^3 x1 y1 - 2 x1^2 y1 + 2 x^2 x1^2 y1 - 4 x1 x2 y1 + 4 x^2 x1 x2 y1 +
    3 x2^2 y1 - 3 x^2 x2^2 y1 + 4 x1 y^2 y1 + 4 x x1 y^2 y1 + 2 x1^2 y^2 y1 + 4 x1 x2 y^2 y1 -
    3 x2^2 y^2 y1 + 2 y^4 y1 - 14 y2 + 16 x^2 y2 - 2 x^4 y2 + 14 x1^2 y2 - 14 x^2 x1^2 y2 - 4 x2 y2 -
    4 x x2 y2 + 4 x^2 x2 y2 + 4 x^3 x2 y2 + x1 x2 y2 - x^2 x1 x2 y2 + 10 x2^2 y2 - 10 x^2 x2^2 y2 -
    16 x1 x2 x3 y2 + 16 x^2 x1 x2 x3 y2 + 12 y^2 y2 - 14 x1^2 y^2 y2 + 4 x2 y^2 y2 + 4 x x2 y^2 y2 -
    x1 x2 y^2 y2 - 10 x2^2 y^2 y2 + 16 x1 x2 x3 y^2 y2 + 2 y^4 y2 + 12 y1^2 y2 - 12 x^2 y1^2 y2 -
    12 y^2 y1^2 y2 - 2 y3 + 4 x y3 - 4 x^3 y3 + 2 x^4 y3 - 2 x1 y3 + 4 x x1 y3 + 2 x^2 x1 y3 - 4 x^3 x1 y3 -
    x2 y3 + 4 x x2 y3 + x^2 x2 y3 - 4 x^3 x2 y3 - 8 x1 x2 y3 + 8 x^2 x1 x2 y3 - 4 x1^2 x2 y3 +
    4 x^2 x1^2 x2 y3 - 4 x1 x2^2 y3 + 4 x^2 x1 x2^2 y3 + 2 x1 x3 y3 - 2 x^2 x1 x3 y3 - 2 x2 x3 y3 +
    2 x^2 x2 x3 y3 + 4 y^2 y3 - 4 x y^2 y3 + 2 x1 y^2 y3 - 4 x x1 y^2 y3 + x2 y^2 y3 - 4 x x2 y^2 y3 +
    8 x1 x2 y^2 y3 + 4 x1^2 x2 y^2 y3 + 4 x1 x2^2 y^2 y3 - 2 x1 x3 y^2 y3 + 2 x2 x3 y^2 y3 - 2 y^4 y3 +
    8 x2 y1 y2 y3 - 8 x^2 x2 y1 y2 y3 - 8 x2 y^2 y1 y2 y3 - 4 y z^2 - 4 x y z^2 - 4 x1 y z^2 -
    4 x x1 y z^2 - 4 x2 y z^2 - 4 x x2 y z^2 - 4 x3 y z^2 - 4 x x3 y z^2 + 2 y1 z^2 + 4 x y1 z^2 -
    2 x^2 y1 z^2 - 4 x3^2 y1 z^2 + 2 y^2 y1 z^2 + 2 y2 z^2 + 4 x y2 z^2 - 2 x^2 y2 z^2 - 4 x3^2 y2 z^2 + 2 y^2 y2 z^2 -
    2 y3 z^2 - 4 x y3 z^2 + 2 x^2 y3 z^2 - 4 x3 y3 z^2 - 4 x1 x3 y3 z^2 - 4 x2 x3 y3 z^2 - 2 y^2 y3 z^2)
```

Out[311]=
```
(1 + x1 + x2 + x3)
 (4 y + 4 x y - 4 x^2 y - 4 x^3 y + 4 x x1 y - 4 x^3 x1 y - 4 x1^2 y + 4 x^2 x1^2 y + 4 x x2 y - 4 x^3 x2 y -
    4 x2^2 y + 4 x^2 x2^2 y - 4 x3 y + 4 x x3 y + 4 x^2 x3 y - 4 x^3 x3 y - 4 x1 x3 y + 4 x^2 x1 x3 y -
    4 x2 x3 y + 4 x^2 x2 x3 y - 4 y^3 - 4 x y^3 - 4 x x1 y^3 + 4 x1^2 y^3 - 4 x x2 y^3 + 4 x2^2 y^3 + 4 x3 y^3 -
    4 x x3 y^3 + 4 x1 x3 y^3 + 4 x2 x3 y^3 - 2 y1 + 4 x^2 y1 - 2 x^4 y1 - 4 x1 y1 - 4 x x1 y1 + 4 x^2 x1 y1 +
    4 x^3 x1 y1 - 2 x1^2 y1 + 2 x^2 x1^2 y1 - 4 x1 x2 y1 + 4 x^2 x1 x2 y1 + 3 x2^2 y1 - 3 x^2 x2^2 y1 +
    4 x1 y^2 y1 + 4 x x1 y^2 y1 + 2 x1^2 y^2 y1 + 4 x1 x2 y^2 y1 - 3 x2^2 y^2 y1 + 2 y^4 y1 - 14 y2 + 16 x^2 y2 -
    2 x^4 y2 + 14 x1^2 y2 - 14 x^2 x1^2 y2 - 4 x2 y2 - 4 x x2 y2 + 4 x^2 x2 y2 + 4 x^3 x2 y2 + x1 x2 y2 -
    x^2 x1 x2 y2 + 10 x2^2 y2 - 10 x^2 x2^2 y2 - 16 x1 x2 x3 y2 + 16 x^2 x1 x2 x3 y2 + 12 y^2 y2 -
    14 x1^2 y^2 y2 + 4 x2 y^2 y2 + 4 x x2 y^2 y2 - x1 x2 y^2 y2 - 10 x2^2 y^2 y2 + 16 x1 x2 x3 y^2 y2 +
    2 y^4 y2 + 12 y1^2 y2 - 12 x^2 y1^2 y2 - 12 y^2 y1^2 y2 - 2 y3 + 4 x y3 - 4 x^3 y3 + 2 x^4 y3 - 2 x1 y3 +
    4 x x1 y3 + 2 x^2 x1 y3 - 4 x^3 x1 y3 - x2 y3 + 4 x x2 y3 + x^2 x2 y3 - 4 x^3 x2 y3 - 8 x1 x2 y3 +
    8 x^2 x1 x2 y3 - 4 x1^2 x2 y3 + 4 x^2 x1^2 x2 y3 - 4 x1 x2^2 y3 + 4 x^2 x1 x2^2 y3 + 2 x1 x3 y3 -
    2 x^2 x1 x3 y3 - 2 x2 x3 y3 + 2 x^2 x2 x3 y3 + 4 y^2 y3 - 4 x y^2 y3 + 2 x1 y^2 y3 - 4 x x1 y^2 y3 +
    x2 y^2 y3 - 4 x x2 y^2 y3 + 8 x1 x2 y^2 y3 + 4 x1^2 x2 y^2 y3 + 4 x1 x2^2 y^2 y3 - 2 x1 x3 y^2 y3 +
    2 x2 x3 y^2 y3 - 2 y^4 y3 + 8 x2 y1 y2 y3 - 8 x^2 x2 y1 y2 y3 - 8 x2 y^2 y1 y2 y3 - 4 y z^2 - 4 x y z^2 -
    4 x1 y z^2 - 4 x x1 y z^2 - 4 x2 y z^2 - 4 x x2 y z^2 - 4 x3 y z^2 - 4 x x3 y z^2 + 2 y1 z^2 + 4 x y1 z^2 -
    2 x^2 y1 z^2 - 4 x3^2 y1 z^2 + 2 y^2 y1 z^2 + 2 y2 z^2 + 4 x y2 z^2 - 2 x^2 y2 z^2 - 4 x3^2 y2 z^2 + 2 y^2 y2 z^2 -
    2 y3 z^2 - 4 x y3 z^2 + 2 x^2 y3 z^2 - 4 x3 y3 z^2 - 4 x1 x3 y3 z^2 - 4 x2 x3 y3 z^2 - 2 y^2 y3 z^2)
```

In[312]:=
```
thing2 =
 4 (1 + x1) (1 + x2)
  (y + 2 x y - x^2 y - 2 x^3 y - x1 y + x^2 x1 y - x2 y + x^2 x2 y + 2 x x3 y - 2 x^3 x3 y - x1 x3 y +
    x^2 x1 x3 y - x2 x3 y + x^2 x2 x3 y - x3^2 y + x^2 x3^2 y - y^3 - 2 x y^3 + x1 y^3 + x2 y^3 - 2 x x3 y^3 +
    x1 x3 y^3 + x2 x3 y^3 + x3^2 y^3 - y1 - x y1 + x^2 y1 + x^3 y1 - x3 y1 - x x3 y1 + x^2 x3 y1 +
    x^3 x3 y1 + y^2 y1 + x y^2 y1 + x3 y^2 y1 + x x3 y^2 y1 - y2 - x y2 + x^2 y2 + x^3 y2 - x3 y2 -
    x x3 y2 + x^2 x3 y2 + x^3 x3 y2 + y^2 y2 + x y^2 y2 + x3 y^2 y2 + x x3 y^2 y2 - y3 + x^4 y3 - x1 y3 +
    x x1 y3 + x^2 x1 y3 - x^3 x1 y3 - x2 y3 + x x2 y3 + x^2 x2 y3 - x^3 x2 y3 - 2 x3 y3 + 2 x^2 x3 y3 +
    2 y^2 y3 + x1 y^2 y3 - x x1 y^2 y3 + x2 y^2 y3 - x x2 y^2 y3 + 2 x3 y^2 y3 - y^4 y3 - y y1 y3 +
    x^2 y y1 y3 + y^3 y1 y3 - y y2 y3 + x^2 y y2 y3 + y^3 y2 y3 - y y3^2 + x^2 y y3^2 + y^3 y3^2 - 2 y z^2 -
    2 x y z^2 - 2 x3 y z^2 - 2 x x3 y z^2 - y3 z^2 - 2 x y3 z^2 + x^2 y3 z^2 - 2 x3 y3 z^2 - y^2 y3 z^2)
```

Out[312]=
```
4 (1 + x1) (1 + x2)
 (y + 2 x y - x^2 y - 2 x^3 y - x1 y + x^2 x1 y - x2 y + x^2 x2 y + 2 x x3 y - 2 x^3 x3 y - x1 x3 y +
   x^2 x1 x3 y - x2 x3 y + x^2 x2 x3 y - x3^2 y + x^2 x3^2 y - y^3 - 2 x y^3 + x1 y^3 + x2 y^3 - 2 x x3 y^3 +
   x1 x3 y^3 + x2 x3 y^3 + x3^2 y^3 - y1 - x y1 + x^2 y1 + x^3 y1 - x3 y1 - x x3 y1 + x^2 x3 y1 +
   x^3 x3 y1 + y^2 y1 + x y^2 y1 + x3 y^2 y1 + x x3 y^2 y1 - y2 - x y2 + x^2 y2 + x^3 y2 - x3 y2 -
   x x3 y2 + x^2 x3 y2 + x^3 x3 y2 + y^2 y2 + x y^2 y2 + x3 y^2 y2 + x x3 y^2 y2 - y3 + x^4 y3 - x1 y3 +
   x x1 y3 + x^2 x1 y3 - x^3 x1 y3 - x2 y3 + x x2 y3 + x^2 x2 y3 - x^3 x2 y3 - 2 x3 y3 + 2 x^2 x3 y3 +
   2 y^2 y3 + x1 y^2 y3 - x x1 y^2 y3 + x2 y^2 y3 - x x2 y^2 y3 + 2 x3 y^2 y3 - y^4 y3 - y y1 y3 +
   x^2 y y1 y3 + y^3 y1 y3 - y y2 y3 + x^2 y y2 y3 + y^3 y2 y3 - y y3^2 + x^2 y y3^2 + y^3 y3^2 - 2 y z^2 -
   2 x y z^2 - 2 x3 y z^2 - 2 x x3 y z^2 - y3 z^2 - 2 x y3 z^2 + x^2 y3 z^2 - 2 x3 y3 z^2 - y^2 y3 z^2)
```

In[313]:=
```
Factor[XYΔReduce[thing1] //. tweak3]
```

Out[313]=
```
2/3 (1 + XH)
 (3 y + 6 x y - 3 x^2 y - 6 x^3 y + 6 x XH y - 6 x^3 XH y - 3 XH^2 y + 3 x^2 XH^2 y - 3 y^3 - 6 x y^3 - 6 x XH y^3 +
   3 XH^2 y^3 - 2 x Y1 + 2 x^2 Y1 + 2 x^3 Y1 - 2 x^4 Y1 + 2 XH Y1 - 2 x XH Y1 - 2 x^2 XH Y1 + 2 x^3 XH Y1 -
   2 y^2 Y1 + 2 x y^2 Y1 - 2 XH y^2 Y1 + 2 x XH y^2 Y1 + 2 y^4 Y1 - 2 x Y2 + 2 x^2 Y2 + 2 x^3 Y2 - 2 x^4 Y2 +
   2 XH Y2 - 2 x XH Y2 - 2 x^2 XH Y2 + 2 x^3 XH Y2 - 2 y^2 Y2 + 2 x y^2 Y2 - 2 XH y^2 Y2 + 2 x XH y^2 Y2 +
   2 y^4 Y2 - 3 YH - 4 x YH + 4 x^2 YH + 4 x^3 YH - x^4 YH - 2 XH YH - 4 x XH YH + 2 x^2 XH YH +
   4 x^3 XH YH + 2 y^2 YH + 4 x y^2 YH + 2 XH y^2 YH + 4 x XH y^2 YH + y^4 YH + 2 y Y1 YH - 2 x^2 y Y1 YH -
   2 y^3 Y1 YH + 2 y Y2 YH - 2 x^2 y Y2 YH - 2 y^3 Y2 YH + y YH^2 - x^2 y YH^2 - y^3 YH^2 - 6 y z^2 -
   6 x y z^2 - 6 XH y z^2 - 6 x XH y z^2 + 4 x Y1 z^2 - 2 x^2 Y1 z^2 + 2 XH Y1 z^2 + 2 y^2 Y1 z^2 + 4 x Y2 z^2 -
   2 x^2 Y2 z^2 + 2 XH Y2 z^2 + 2 y^2 Y2 z^2 - 3 YH z^2 + 2 x YH z^2 - x^2 YH z^2 - 2 XH YH z^2 + y^2 YH z^2)
```

In[314]:=
```
Factor[XYΔReduce[thing2] //. tweak3]
```

Out[314]=
$$\frac{2}{3}(1+XH)$$
$$\begin{aligned}&\big(3\,y+6\,x\,y-3\,x^2\,y-6\,x^3\,y+6\,x\,XH\,y-6\,x^3\,XH\,y-3\,XH^2\,y+3\,x^2\,XH^2\,y-3\,y^3-6\,x\,y^3-6\,x\,XH\,y^3+\\&\quad 3\,XH^2\,y^3-2\,x\,Y1+2\,x^2\,Y1+2\,x^3\,Y1-2\,x^4\,Y1+2\,XH\,Y1-2\,x\,XH\,Y1-2\,x^2\,XH\,Y1+2\,x^3\,XH\,Y1-\\&\quad 2\,y^2\,Y1+2\,x\,y^2\,Y1-2\,XH\,y^2\,Y1+2\,x\,XH\,y^2\,Y1+2\,y^4\,Y1-2\,x\,Y2+2\,x^2\,Y2+2\,x^3\,Y2-2\,x^4\,Y2+\\&\quad 2\,XH\,Y2-2\,x\,XH\,Y2-2\,x^2\,XH\,Y2+2\,x^3\,XH\,Y2-2\,y^2\,Y2+2\,x\,y^2\,Y2-2\,XH\,y^2\,Y2+2\,x\,XH\,y^2\,Y2+\\&\quad 2\,y^4\,Y2-3\,YH-4\,x\,YH+4\,x^2\,YH+4\,x^3\,YH-x^4\,YH-2\,XH\,YH-4\,x\,XH\,YH+2\,x^2\,XH\,YH+\\&\quad 4\,x^3\,XH\,YH+2\,y^2\,YH+4\,x\,y^2\,YH+2\,XH\,y^2\,YH+4\,x\,XH\,y^2\,YH+y^4\,YH+2\,y\,Y1\,YH-2\,x^2\,y\,Y1\,YH-\\&\quad 2\,y^3\,Y1\,YH+2\,y\,Y2\,YH-2\,x^2\,y\,Y2\,YH-2\,y^3\,Y2\,YH+y\,YH^2-x^2\,y\,YH^2-y^3\,YH^2-6\,y\,z^2-\\&\quad 6\,x\,y\,z^2-6\,XH\,y\,z^2-6\,x\,XH\,y\,z^2+4\,x\,Y1\,z^2-2\,x^2\,Y1\,z^2+2\,XH\,Y1\,z^2+2\,y^2\,Y1\,z^2+4\,x\,Y2\,z^2-\\&\quad 2\,x^2\,Y2\,z^2+2\,XH\,Y2\,z^2+2\,y^2\,Y2\,z^2-3\,YH\,z^2+2\,x\,YH\,z^2-x^2\,YH\,z^2-2\,XH\,YH\,z^2+y^2\,YH\,z^2\big)\end{aligned}$$

In[315]:=
```
Factor[%% - %]
```

Out[315]=
```
0
```

In[316]:=
```
list1 = Factor[CoefficientList[thing1, {z, y, x}]]
```

Out[316]=
$$\begin{aligned}&\Big\{\Big\{\Big\{-(1+x1+x2+x3)\,\big(2\,y1+4\,x1\,y1+2\,x1^2\,y1+4\,x1\,x2\,y1-3\,x2^2\,y1+14\,y2-14\,x1^2\,y2+\\&\qquad 4\,x2\,y2-x1\,x2\,y2-10\,x2^2\,y2+16\,x1\,x2\,x3\,y2-12\,y1^2\,y2+2\,y3+2\,x1\,y3+x2\,y3+\\&\qquad 8\,x1\,x2\,y3+4\,x1^2\,x2\,y3+4\,x1\,x2^2\,y3-2\,x1\,x3\,y3+2\,x2\,x3\,y3-8\,x2\,y1\,y2\,y3\big),\\&\quad -4\,(1+x1+x2+x3)\,(x1\,y1+x2\,y2-y3-x1\,y3-x2\,y3),\\&\quad (1+x1+x2+x3)\,\big(4\,y1+4\,x1\,y1+2\,x1^2\,y1+4\,x1\,x2\,y1-3\,x2^2\,y1+16\,y2-14\,x1^2\,y2+\\&\qquad 4\,x2\,y2-x1\,x2\,y2-10\,x2^2\,y2+16\,x1\,x2\,x3\,y2-12\,y1^2\,y2+2\,x1\,y3+x2\,y3+\\&\qquad 8\,x1\,x2\,y3+4\,x1^2\,x2\,y3+4\,x1\,x2^2\,y3-2\,x1\,x3\,y3+2\,x2\,x3\,y3-8\,x2\,y1\,y2\,y3\big),\\&\quad 4\,(1+x1+x2+x3)\,(x1\,y1+x2\,y2-y3-x1\,y3-x2\,y3),\\&\quad -2\,(1+x1+x2+x3)\,(y1+y2-y3)\Big\},\\&\Big\{-4\,(1+x1+x2+x3)\,\big(-1+x1^2+x2^2+x3+x1\,x3+x2\,x3\big),\,4\,(1+x1+x2+x3)^2,\\&\quad 4\,(1+x1+x2+x3)\,\big(-1+x1^2+x2^2+x3+x1\,x3+x2\,x3\big),\,-4\,(1+x1+x2+x3)^2,\,0\Big\},\\&\Big\{(1+x1+x2+x3)\,\big(4\,x1\,y1+2\,x1^2\,y1+4\,x1\,x2\,y1-3\,x2^2\,y1+12\,y2-14\,x1^2\,y2+\\&\qquad 4\,x2\,y2-x1\,x2\,y2-10\,x2^2\,y2+16\,x1\,x2\,x3\,y2-12\,y1^2\,y2+4\,y3+2\,x1\,y3+x2\,y3+\\&\qquad 8\,x1\,x2\,y3+4\,x1^2\,x2\,y3+4\,x1\,x2^2\,y3-2\,x1\,x3\,y3+2\,x2\,x3\,y3-8\,x2\,y1\,y2\,y3\big),\\&\quad 4\,(1+x1+x2+x3)\,(x1\,y1+x2\,y2-y3-x1\,y3-x2\,y3),\,0,\,0,\,0\Big\},\\&\Big\{4\,(1+x1+x2+x3)\,\big(-1+x1^2+x2^2+x3+x1\,x3+x2\,x3\big),\,-4\,(1+x1+x2+x3)^2,\,0,\,0,\,0\Big\},\\&\Big\{2\,(1+x1+x2+x3)\,(y1+y2-y3),\,0,\,0,\,0,\,0\Big\}\Big\},\\&\big\{\{0,\,0,\,0,\,0,\,0\},\,\{0,\,0,\,0,\,0,\,0\},\,\{0,\,0,\,0,\,0,\,0\},\\&\quad \{0,\,0,\,0,\,0,\,0\},\,\{0,\,0,\,0,\,0,\,0\}\big\},\\&\Big\{\Big\{-2\,(1+x1+x2+x3)\,\big(-y1+2\,x3^2\,y1-y2+2\,x3^2\,y2+y3+2\,x3\,y3+2\,x1\,x3\,y3+2\,x2\,x3\,y3\big),\\&\quad 4\,(1+x1+x2+x3)\,(y1+y2-y3),\,-2\,(1+x1+x2+x3)\,(y1+y2-y3),\,0,\,0\Big\},\\&\Big\{-4\,(1+x1+x2+x3)^2,\,-4\,(1+x1+x2+x3)^2,\,0,\,0,\,0\Big\},\\&\Big\{2\,(1+x1+x2+x3)\,(y1+y2-y3),\,0,\,0,\,0,\,0\Big\},\\&\big\{0,\,0,\,0,\,0,\,0\big\},\,\big\{0,\,0,\,0,\,0,\,0\big\}\Big\}\Big\}\end{aligned}$$

In[317]:=
```
list2 = Factor[CoefficientList[thing2, {z, y, x}]]
```

Out[317]=
```
{{{-4 (1 + x1) (1 + x2) (y1 + x3 y1 + y2 + x3 y2 + y3 + x1 y3 + x2 y3 + 2 x3 y3),
    4 (1 + x1) (1 + x2) (-y1 - x3 y1 - y2 - x3 y2 + x1 y3 + x2 y3),
    4 (1 + x1) (1 + x2) (y1 + x3 y1 + y2 + x3 y2 + x1 y3 + x2 y3 + 2 x3 y3),
    -4 (1 + x1) (1 + x2) (-y1 - x3 y1 - y2 - x3 y2 + x1 y3 + x2 y3), 4 (1 + x1) (1 + x2) y3},
   {-4 (1 + x1) (1 + x2) (-1 + x1 + x2 + x1 x3 + x2 x3 + x3^2 + y1 y3 + y2 y3 + y3^2),
    8 (1 + x1) (1 + x2) (1 + x3), 4 (1 + x1) (1 + x2)
     (-1 + x1 + x2 + x1 x3 + x2 x3 + x3^2 + y1 y3 + y2 y3 + y3^2), -8 (1 + x1) (1 + x2) (1 + x3), 0},
   {4 (1 + x1) (1 + x2) (y1 + x3 y1 + y2 + x3 y2 + 2 y3 + x1 y3 + x2 y3 + 2 x3 y3),
    -4 (1 + x1) (1 + x2) (-y1 - x3 y1 - y2 - x3 y2 + x1 y3 + x2 y3), 0, 0, 0},
   {4 (1 + x1) (1 + x2) (-1 + x1 + x2 + x1 x3 + x2 x3 + x3^2 + y1 y3 + y2 y3 + y3^2),
    -8 (1 + x1) (1 + x2) (1 + x3), 0, 0, 0}, {-4 (1 + x1) (1 + x2) y3, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
  {{-4 (1 + x1) (1 + x2) (1 + 2 x3) y3, -8 (1 + x1) (1 + x2) y3, 4 (1 + x1) (1 + x2) y3, 0, 0},
   {-8 (1 + x1) (1 + x2) (1 + x3), -8 (1 + x1) (1 + x2) (1 + x3), 0, 0, 0},
   {-4 (1 + x1) (1 + x2) y3, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}}
```

In[318]:=
```
Factor[ list1 - list2 //. tSubsSp]
```

Out[318]=
```
{{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}}
```

In[319]:=
```
Factor[XYΔReduce[list1] //. tweak3]
```

Out[319]=
$$\left\{\left\{\left\{\frac{2}{3}(1+XH)(2\,XH\,Y1+2\,XH\,Y2-3\,YH-2\,XH\,YH),\ -\frac{4}{3}(1+XH)^2(Y1+Y2+2\,YH),\right.\right.\right.$$
$$\left.-\frac{4}{3}(1+XH)(-Y1+XH\,Y1-Y2+XH\,Y2-2\,YH-XH\,YH),\ \frac{4}{3}(1+XH)^2(Y1+Y2+2\,YH),\right.$$
$$\left.\left.-\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH)\right\},\left\{-\frac{2}{3}(1+XH)\left(-3+3\,XH^2-2\,Y1\,YH-2\,Y2\,YH-YH^2\right),\right.$$
$$\left.4(1+XH)^2,\ \frac{2}{3}(1+XH)\left(-3+3\,XH^2-2\,Y1\,YH-2\,Y2\,YH-YH^2\right),\ -4(1+XH)^2,\ 0\right\},$$
$$\left\{-\frac{4}{3}(1+XH)^2(Y1+Y2-YH),\ \frac{4}{3}(1+XH)^2(Y1+Y2+2\,YH),\ 0,\ 0,\ 0\right\},$$
$$\left\{\frac{2}{3}(1+XH)\left(-3+3\,XH^2-2\,Y1\,YH-2\,Y2\,YH-YH^2\right),\ -4(1+XH)^2,\ 0,\ 0,\ 0\right\},$$
$$\left.\left\{\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH),\ 0,\ 0,\ 0,\ 0\right\}\right\},$$
$$\{\{0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0\}\},$$
$$\left\{\left\{\frac{2}{3}(1+XH)(2\,XH\,Y1+2\,XH\,Y2-3\,YH-2\,XH\,YH),\ \frac{4}{3}(1+XH)(2\,Y1+2\,Y2+YH),\right.\right.$$
$$\left.\left.-\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH),\ 0,\ 0\right\},\left\{-4(1+XH)^2,\ -4(1+XH)^2,\ 0,\ 0,\ 0\right\},\right.$$
$$\left.\left\{\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH),\ 0,\ 0,\ 0,\ 0\right\},\ \{0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0\}\right\}\right\}$$

In[320]:=
```
Factor[XYΔReduce[list2] //. tweak3]
```

Out[320]=
$$\left\{\left\{\left\{\frac{2}{3}(1+XH)(2\,XH\,Y1+2\,XH\,Y2-3\,YH-2\,XH\,YH),\ -\frac{4}{3}(1+XH)^2(Y1+Y2+2\,YH),\right.\right.\right.$$
$$-\frac{4}{3}(1+XH)(-Y1+XH\,Y1-Y2+XH\,Y2-2\,YH-XH\,YH),\ \frac{4}{3}(1+XH)^2(Y1+Y2+2\,YH),$$
$$\left.\left.-\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH)\right\},\left\{-\frac{2}{3}(1+XH)\left(-3+3\,XH^2-2\,Y1\,YH-2\,Y2\,YH-YH^2\right),\right.\right.$$
$$\left.4(1+XH)^2,\ \frac{2}{3}(1+XH)\left(-3+3\,XH^2-2\,Y1\,YH-2\,Y2\,YH-YH^2\right),\ -4(1+XH)^2,\ 0\right\},$$
$$\left\{-\frac{4}{3}(1+XH)^2(Y1+Y2-YH),\ \frac{4}{3}(1+XH)^2(Y1+Y2+2\,YH),\ 0,\ 0,\ 0\right\},$$
$$\left\{\frac{2}{3}(1+XH)\left(-3+3\,XH^2-2\,Y1\,YH-2\,Y2\,YH-YH^2\right),\ -4(1+XH)^2,\ 0,\ 0,\ 0\right\},$$
$$\left.\left\{\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH),\ 0,\ 0,\ 0,\ 0\right\}\right\},$$
$$\{\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\}\},$$
$$\left\{\left\{\frac{2}{3}(1+XH)(2\,XH\,Y1+2\,XH\,Y2-3\,YH-2\,XH\,YH),\ \frac{4}{3}(1+XH)(2\,Y1+2\,Y2+YH),\right.\right.$$
$$\left.-\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH),\ 0,\ 0\right\},\left\{-4(1+XH)^2,\ -4(1+XH)^2,\ 0,\ 0,\ 0\right\},$$
$$\left.\left\{\frac{2}{3}(1+XH)(2\,Y1+2\,Y2+YH),\ 0,\ 0,\ 0,\ 0\right\},\{0,0,0,0,0\},\{0,0,0,0,0\}\right\}\}$$

In[321]:=
```
Factor[%% - %]
```

Out[321]=
$$\{\{\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\},\{0,0,0,0,0\}\}\}$$

```
(* Lemma 12 Check.  *)
```

In[322]:=
```
deltoid = (L^2 + (R + 1)^2)^2 + 8 (R + 1) ((R + 1)^2 - 3 L^2) + 18 (L^2 + (R + 1)^2) - 27
```

Out[322]=
$$-27+8(1+R)\left(-3L^2+(1+R)^2\right)+18\left(L^2+(1+R)^2\right)+\left(L^2+(1+R)^2\right)^2$$

In[323]:=
```
Factor[deltoid /. {L → 2 (x + 1) y, R → y^2 - (x - 1)^2}]
```

Out[323]=
$$\left(-1+x^2+y^2\right)^2\left(-27+18\,x^2-8\,x^3+x^4+18\,y^2+24\,x\,y^2+2\,x^2\,y^2+y^4\right)$$

In[324]:=
```
Factor[(x^2 + y^2 - 1)^2 ((x^2 + y^2)^2 + 8 x (3 y^2 - x^2) + 18 (x^2 + y^2) - 27) ]
```

Out[324]=
$$\left(-1+x^2+y^2\right)^2\left(-27+18\,x^2-8\,x^3+x^4+18\,y^2+24\,x\,y^2+2\,x^2\,y^2+y^4\right)$$

In[325]:=
```
Factor[%% / %]
```

Out[325]=
$$1$$

In[326]:= **LLL**

Out[326]=
$$\frac{\left(-1+x^2+y^2\right)\ (y+2\,x\,y-XH\,y-YH-x\,YH)+2\ (1+x)\ y\,z^2}{z^2}$$

In[327]:= **RRR**

Out[327]=
$$\frac{-\left(-1+x^2+y^2\right)\ \left(-x+x^2+XH-x\,XH-y^2+y\,YH\right)-(-1+x-y)\ (-1+x+y)\ z^2}{z^2}$$

In[328]:= **Factor[y (LLL - 2 (x + 1) y) - (x + 1) (RRR - y^2 + (x - 1)^2)]**

Out[328]=
$$\frac{(x-XH)\ \left(-1+x^2+y^2\right)^2}{z^2}$$

In[329]:= **Factor[Factor[deltoid /. {L → LLL, R → RRR}] /. z → Sqrt[Z]]**

Out[329]=
$$\frac{1}{Z^4}\left(-1+x^2+y^2\right)^2$$
$$\left(x^4-4\,x^5+4\,x^6+4\,x^7-10\,x^8+4\,x^9+4\,x^{10}-4\,x^{11}+x^{12}-4\,x^3\,XH+16\,x^4\,XH-16\,x^5\,XH-16\,x^6\,XH+\right.$$
$$40\,x^7\,XH-16\,x^8\,XH-16\,x^9\,XH+16\,x^{10}\,XH-4\,x^{11}\,XH+6\,x^2\,XH^2-24\,x^3\,XH^2+24\,x^4\,XH^2+$$
$$24\,x^5\,XH^2-60\,x^6\,XH^2+24\,x^7\,XH^2+24\,x^8\,XH^2-24\,x^9\,XH^2+6\,x^{10}\,XH^2-4\,x\,XH^3+16\,x^2\,XH^3-$$
$$16\,x^3\,XH^3-16\,x^4\,XH^3+40\,x^5\,XH^3-16\,x^6\,XH^3-16\,x^7\,XH^3+16\,x^8\,XH^3-4\,x^9\,XH^3+XH^4-4\,x\,XH^4+$$
$$4\,x^2\,XH^4+4\,x^3\,XH^4-10\,x^4\,XH^4+4\,x^5\,XH^4+4\,x^6\,XH^4-4\,x^7\,XH^4+x^8\,XH^4+2\,x^2\,y^2+8\,x^3\,y^2-$$
$$24\,x^4\,y^2-4\,x^5\,y^2+32\,x^6\,y^2-16\,x^8\,y^2-4\,x^9\,y^2+6\,x^{10}\,y^2-4\,x\,XH\,y^2-24\,x^2\,XH\,y^2+64\,x^3\,XH\,y^2+$$
$$8\,x^4\,XH\,y^2-72\,x^5\,XH\,y^2-8\,x^6\,XH\,y^2+32\,x^7\,XH\,y^2+24\,x^8\,XH\,y^2-20\,x^9\,XH\,y^2+2\,XH^2\,y^2+$$
$$24\,x\,XH^2\,y^2-56\,x^2\,XH^2\,y^2+36\,x^4\,XH^2\,y^2+24\,x^5\,XH^2\,y^2-8\,x^6\,XH^2\,y^2-48\,x^7\,XH^2\,y^2+26\,x^8\,XH^2\,y^2-$$
$$8\,XH^3\,y^2+16\,x\,XH^3\,y^2-8\,x^2\,XH^3\,y^2+16\,x^3\,XH^3\,y^2-24\,x^4\,XH^3\,y^2-16\,x^5\,XH^3\,y^2+40\,x^6\,XH^3\,y^2-$$
$$16\,x^7\,XH^3\,y^2+4\,x\,XH^4\,y^2-12\,x^2\,XH^4\,y^2+8\,x^3\,XH^4\,y^2+8\,x^4\,XH^4\,y^2-12\,x^5\,XH^4\,y^2+4\,x^6\,XH^4\,y^2+$$
$$y^4+12\,x\,y^4+36\,x^2\,y^4-20\,x^3\,y^4-36\,x^4\,y^4-24\,x^5\,y^4-8\,x^6\,y^4+24\,x^7\,y^4+15\,x^8\,y^4-8\,XH\,y^4-$$
$$48\,x\,XH\,y^4+32\,x^2\,XH\,y^4-8\,x^3\,XH\,y^4+40\,x^4\,XH\,y^4+64\,x^5\,XH\,y^4-32\,x^6\,XH\,y^4-40\,x^7\,XH\,y^4+$$
$$16\,XH^2\,y^4-24\,x\,XH^2\,y^4+60\,x^2\,XH^2\,y^4-24\,x^3\,XH^2\,y^4-72\,x^4\,XH^2\,y^4+44\,x^6\,XH^2\,y^4+8\,XH^3\,y^4-$$
$$24\,x\,XH^3\,y^4+16\,x^3\,XH^3\,y^4+24\,x^4\,XH^3\,y^4-24\,x^5\,XH^3\,y^4-2\,XH^4\,y^4+4\,x\,XH^4\,y^4+4\,x^2\,XH^4\,y^4-$$
$$12\,x^3\,XH^4\,y^4+6\,x^4\,XH^4\,y^4-12\,x\,y^6-80\,x^2\,y^6-32\,x^3\,y^6+48\,x^4\,y^6+56\,x^5\,y^6+20\,x^6\,y^6+8\,XH\,y^6+$$
$$104\,x\,XH\,y^6+40\,x^2\,XH\,y^6-32\,x^3\,XH\,y^6-80\,x^4\,XH\,y^6-40\,x^5\,XH\,y^6-36\,XH^2\,y^6-24\,x\,XH^2\,y^6-$$
$$24\,x^2\,XH^2\,y^6+48\,x^3\,XH^2\,y^6+36\,x^4\,XH^2\,y^6+8\,XH^3\,y^6+16\,x\,XH^3\,y^6-8\,x^2\,XH^3\,y^6-16\,x^3\,XH^3\,y^6-$$
$$4\,x\,XH^4\,y^6+4\,x^2\,XH^4\,y^6-2\,y^8-12\,x\,y^8+36\,x^2\,y^8+44\,x^3\,y^8+15\,x^4\,y^8+8\,XH\,y^8-48\,x\,XH\,y^8-$$
$$48\,x^2\,XH\,y^8-20\,x^3\,XH\,y^8+16\,XH^2\,y^8+24\,x\,XH^2\,y^8+14\,x^2\,XH^2\,y^8-8\,XH^3\,y^8-4\,x\,XH^3\,y^8+$$
$$XH^4\,y^8+12\,x\,y^{10}+6\,x^2\,y^{10}-8\,XH\,y^{10}-4\,x\,XH\,y^{10}+2\,XH^2\,y^{10}+y^{12}-4\,x^2\,y\,YH-8\,x^3\,y\,YH+$$
$$32\,x^4\,y\,YH+8\,x^5\,y\,YH-56\,x^6\,y\,YH+8\,x^7\,y\,YH+32\,x^8\,y\,YH-8\,x^9\,y\,YH-4\,x^{10}\,y\,YH+8\,x\,XH\,y\,YH+$$
$$24\,x^2\,XH\,y\,YH-80\,x^3\,XH\,y\,YH-24\,x^4\,XH\,y\,YH+144\,x^5\,XH\,y\,YH-24\,x^6\,XH\,y\,YH-80\,x^7\,XH\,y\,YH+$$
$$24\,x^8\,XH\,y\,YH+8\,x^9\,XH\,y\,YH-4\,XH^2\,y\,YH-24\,x\,XH^2\,y\,YH+64\,x^2\,XH^2\,y\,YH+24\,x^3\,XH^2\,y\,YH-$$
$$120\,x^4\,XH^2\,y\,YH+24\,x^5\,XH^2\,y\,YH+64\,x^6\,XH^2\,y\,YH-24\,x^7\,XH^2\,y\,YH-4\,x^8\,XH^2\,y\,YH+8\,XH^3\,y\,YH-$$
$$16\,x\,XH^3\,y\,YH-8\,x^2\,XH^3\,y\,YH+32\,x^3\,XH^3\,y\,YH-8\,x^4\,XH^3\,y\,YH-16\,x^5\,XH^3\,y\,YH+8\,x^6\,XH^3\,y\,YH-$$
$$4\,y^3\,YH-40\,x\,y^3\,YH-96\,x^2\,y^3\,YH+48\,x^3\,y^3\,YH+152\,x^4\,y^3\,YH+56\,x^5\,y^3\,YH-32\,x^6\,y^3\,YH-$$
$$64\,x^7\,y^3\,YH-20\,x^8\,y^3\,YH+24\,XH\,y^3\,YH+112\,x\,XH\,y^3\,YH-48\,x^2\,XH\,y^3\,YH-96\,x^3\,XH\,y^3\,YH-$$
$$72\,x^4\,XH\,y^3\,YH-48\,x^5\,XH\,y^3\,YH+96\,x^6\,XH\,y^3\,YH+32\,x^7\,XH\,y^3\,YH-32\,XH^2\,y^3\,YH+24\,x\,XH^2\,y^3\,YH-$$

$$48\,x^2\,XH^2\,y^3\,YH + 48\,x^3\,XH^2\,y^3\,YH + 96\,x^4\,XH^2\,y^3\,YH - 72\,x^5\,XH^2\,y^3\,YH - 16\,x^6\,XH^2\,y^3\,YH -$$
$$8\,XH^3\,y^3\,YH + 32\,x\,XH^3\,y^3\,YH - 16\,x^2\,XH^3\,y^3\,YH - 32\,x^3\,XH^3\,y^3\,YH + 24\,x^4\,XH^3\,y^3\,YH + 40\,x\,y^5\,YH +$$
$$216\,x^2\,y^5\,YH + 88\,x^3\,y^5\,YH - 160\,x^4\,y^5\,YH - 144\,x^5\,y^5\,YH - 40\,x^6\,y^5\,YH - 24\,XH\,y^5\,YH -$$
$$240\,x\,XH\,y^5\,YH - 72\,x^2\,XH\,y^5\,YH + 144\,x^3\,XH\,y^5\,YH + 144\,x^4\,XH\,y^5\,YH + 48\,x^5\,XH\,y^5\,YH +$$
$$72\,XH^2\,y^5\,YH + 24\,x\,XH^2\,y^5\,YH - 72\,x^3\,XH^2\,y^5\,YH - 24\,x^4\,XH^2\,y^5\,YH - 8\,XH^3\,y^5\,YH - 16\,x\,XH^3\,y^5\,YH +$$
$$24\,x^2\,XH^3\,y^5\,YH + 8\,y^7\,YH + 40\,x\,y^7\,YH - 96\,x^2\,y^7\,YH - 128\,x^3\,y^7\,YH - 40\,x^4\,y^7\,YH - 24\,XH\,y^7\,YH +$$
$$112\,x\,XH\,y^7\,YH + 96\,x^2\,XH\,y^7\,YH + 32\,x^3\,XH\,y^7\,YH - 32\,XH^2\,y^7\,YH - 24\,x\,XH^2\,y^7\,YH - 16\,x^2\,XH^2\,y^7\,YH +$$
$$8\,XH^3\,y^7\,YH - 40\,x\,y^9\,YH - 20\,x^2\,y^9\,YH + 24\,XH\,y^9\,YH + 8\,x\,XH\,y^9\,YH - 4\,XH^2\,y^9\,YH - 4\,y^{11}\,YH +$$
$$2\,x^2\,YH^2 - 8\,x^4\,YH^2 + 12\,x^6\,YH^2 - 8\,x^8\,YH^2 + 2\,x^{10}\,YH^2 - 4\,x\,XH\,YH^2 + 16\,x^3\,XH\,YH^2 - 24\,x^5\,XH\,YH^2 +$$
$$16\,x^7\,XH\,YH^2 - 4\,x^9\,XH\,YH^2 + 2\,XH^2\,YH^2 - 8\,x^2\,XH^2\,YH^2 + 12\,x^4\,XH^2\,YH^2 - 8\,x^6\,XH^2\,YH^2 +$$
$$2\,x^8\,XH^2\,YH^2 + 6\,y^2\,YH^2 + 48\,x\,y^2\,YH^2 + 88\,x^2\,y^2\,YH^2 - 48\,x^3\,y^2\,YH^2 - 180\,x^4\,y^2\,YH^2 - 48\,x^5\,y^2\,YH^2 +$$
$$72\,x^6\,y^2\,YH^2 + 48\,x^7\,y^2\,YH^2 + 14\,x^8\,y^2\,YH^2 - 24\,XH\,y^2\,YH^2 - 80\,x\,XH\,y^2\,YH^2 + 24\,x^2\,XH\,y^2\,YH^2 +$$
$$144\,x^3\,XH\,y^2\,YH^2 + 24\,x^4\,XH\,y^2\,YH^2 - 48\,x^5\,XH\,y^2\,YH^2 - 24\,x^6\,XH\,y^2\,YH^2 - 16\,x^7\,XH\,y^2\,YH^2 +$$
$$16\,XH^2\,y^2\,YH^2 - 24\,x^2\,XH^2\,y^2\,YH^2 + 8\,x^6\,XH^2\,y^2\,YH^2 - 48\,x\,y^4\,YH^2 - 204\,x^2\,y^4\,YH^2 - 96\,x^3\,y^4\,YH^2 +$$
$$168\,x^4\,y^4\,YH^2 + 144\,x^5\,y^4\,YH^2 + 36\,x^6\,y^4\,YH^2 + 24\,XH\,y^4\,YH^2 + 168\,x\,XH\,y^4\,YH^2 + 48\,x^2\,XH\,y^4\,YH^2 -$$
$$144\,x^3\,XH\,y^4\,YH^2 - 72\,x^4\,XH\,y^4\,YH^2 - 24\,x^5\,XH\,y^4\,YH^2 - 36\,XH^2\,y^4\,YH^2 + 24\,x^2\,XH^2\,y^4\,YH^2 +$$
$$12\,x^4\,XH^2\,y^4\,YH^2 - 12\,y^6\,YH^2 - 48\,x\,y^6\,YH^2 + 88\,x^2\,y^6\,YH^2 + 144\,x^3\,y^6\,YH^2 + 44\,x^4\,y^6\,YH^2 +$$
$$24\,XH\,y^6\,YH^2 - 80\,x\,XH\,y^6\,YH^2 - 72\,x^2\,XH\,y^6\,YH^2 - 16\,x^3\,XH\,y^6\,YH^2 + 16\,XH^2\,y^6\,YH^2 + 8\,x^2\,XH^2\,y^6\,YH^2 +$$
$$48\,x\,y^8\,YH^2 + 26\,x^2\,y^8\,YH^2 - 24\,XH\,y^8\,YH^2 - 4\,x\,XH\,y^8\,YH^2 + 2\,XH^2\,y^8\,YH^2 + 6\,y^{10}\,YH^2 - 4\,y\,YH^3 -$$
$$24\,x\,y\,YH^3 - 32\,x^2\,y\,YH^3 + 24\,x^3\,y\,YH^3 + 72\,x^4\,y\,YH^3 + 24\,x^5\,y\,YH^3 - 32\,x^6\,y\,YH^3 - 24\,x^7\,y\,YH^3 -$$
$$4\,x^8\,y\,YH^3 + 8\,XH\,y\,YH^3 + 16\,x\,XH\,y\,YH^3 - 8\,x^2\,XH\,y\,YH^3 - 32\,x^3\,XH\,y\,YH^3 - 8\,x^4\,XH\,y\,YH^3 +$$
$$16\,x^5\,XH\,y\,YH^3 + 8\,x^6\,XH\,y\,YH^3 + 24\,x\,y^3\,YH^3 + 80\,x^2\,y^3\,YH^3 + 48\,x^3\,y^3\,YH^3 - 64\,x^4\,y^3\,YH^3 -$$
$$72\,x^5\,y^3\,YH^3 - 16\,x^6\,y^3\,YH^3 - 8\,XH\,y^3\,YH^3 - 32\,x\,XH\,y^3\,YH^3 - 16\,x^2\,XH\,y^3\,YH^3 + 32\,x^3\,XH\,y^3\,YH^3 +$$
$$24\,x^4\,XH\,y^3\,YH^3 + 8\,y^5\,YH^3 + 24\,x\,y^5\,YH^3 - 32\,x^2\,y^5\,YH^3 - 72\,x^3\,y^5\,YH^3 - 24\,x^4\,y^5\,YH^3 - 8\,XH\,y^5\,YH^3 +$$
$$16\,x\,XH\,y^5\,YH^3 + 24\,x^2\,XH\,y^5\,YH^3 - 24\,x\,y^7\,YH^3 - 16\,x^2\,y^7\,YH^3 + 8\,XH\,y^7\,YH^3 - 4\,y^9\,YH^3 + YH^4 +$$
$$4\,x\,YH^4 + 4\,x^2\,YH^4 - 4\,x^3\,YH^4 - 10\,x^4\,YH^4 - 4\,x^5\,YH^4 + 4\,x^6\,YH^4 + 4\,x^7\,YH^4 + x^8\,YH^4 - 4\,x\,y^2\,YH^4 -$$
$$12\,x^2\,y^2\,YH^4 - 8\,x^3\,y^2\,YH^4 + 8\,x^4\,y^2\,YH^4 + 12\,x^5\,y^2\,YH^4 + 4\,x^6\,y^2\,YH^4 - 2\,y^4\,YH^4 - 4\,x\,y^4\,YH^4 +$$
$$4\,x^2\,y^4\,YH^4 + 12\,x^3\,y^4\,YH^4 + 6\,x^4\,y^4\,YH^4 + 4\,x\,y^6\,YH^4 + 4\,x^2\,y^6\,YH^4 + y^8\,YH^4 - 8\,x^3\,Z + 16\,x^4\,Z +$$
$$12\,x^5\,Z - 44\,x^6\,Z + 16\,x^7\,Z + 24\,x^8\,Z - 20\,x^9\,Z + 4\,x^{10}\,Z + 24\,x^2\,XH\,Z - 48\,x^3\,XH\,Z - 36\,x^4\,XH\,Z +$$
$$132\,x^5\,XH\,Z - 48\,x^6\,XH\,Z - 72\,x^7\,XH\,Z + 60\,x^8\,XH\,Z - 12\,x^9\,XH\,Z - 24\,x\,XH^2\,Z + 48\,x^2\,XH^2\,Z +$$
$$36\,x^3\,XH^2\,Z - 132\,x^4\,XH^2\,Z + 48\,x^5\,XH^2\,Z + 72\,x^6\,XH^2\,Z - 60\,x^7\,XH^2\,Z + 12\,x^8\,XH^2\,Z + 8\,XH^3\,Z -$$
$$16\,x\,XH^3\,Z - 12\,x^2\,XH^3\,Z + 44\,x^3\,XH^3\,Z - 16\,x^4\,XH^3\,Z - 24\,x^5\,XH^3\,Z + 20\,x^6\,XH^3\,Z - 4\,x^7\,XH^3\,Z +$$
$$24\,x\,y^2\,Z + 32\,x^2\,y^2\,Z - 24\,x^3\,y^2\,Z - 60\,x^4\,y^2\,Z - 16\,x^5\,y^2\,Z + 24\,x^6\,y^2\,Z + 20\,x^8\,y^2\,Z - 24\,XH\,y^2\,Z -$$
$$48\,x\,XH\,y^2\,Z - 24\,x^2\,XH\,y^2\,Z + 120\,x^3\,XH\,y^2\,Z + 72\,x^4\,XH\,y^2\,Z - 72\,x^5\,XH\,y^2\,Z + 24\,x^6\,XH\,y^2\,Z -$$
$$48\,x^7\,XH\,y^2\,Z + 16\,XH^2\,y^2\,Z + 68\,x\,XH^2\,y^2\,Z - 88\,x^2\,XH^2\,y^2\,Z - 64\,x^3\,XH^2\,y^2\,Z + 80\,x^4\,XH^2\,y^2\,Z -$$
$$52\,x^5\,XH^2\,y^2\,Z + 40\,x^6\,XH^2\,y^2\,Z - 20\,XH^3\,y^2\,Z + 28\,x\,XH^3\,y^2\,Z + 8\,x^2\,XH^3\,y^2\,Z - 32\,x^3\,XH^3\,y^2\,Z +$$
$$28\,x^4\,XH^3\,y^2\,Z - 12\,x^5\,XH^3\,y^2\,Z + 16\,y^4\,Z - 36\,x\,y^4\,Z - 180\,x^2\,y^4\,Z - 80\,x^3\,y^4\,Z + 120\,x^4\,y^4\,Z +$$
$$120\,x^5\,y^4\,Z + 40\,x^6\,y^4\,Z + 44\,XH\,y^4\,Z + 180\,x\,XH\,y^4\,Z + 128\,x^2\,XH\,y^4\,Z - 120\,x^3\,XH\,y^4\,Z -$$
$$160\,x^4\,XH\,y^4\,Z - 72\,x^5\,XH\,y^4\,Z - 52\,XH^2\,y^4\,Z - 112\,x\,XH^2\,y^4\,Z + 40\,x^2\,XH^2\,y^4\,Z + 76\,x^3\,XH^2\,y^4\,Z +$$
$$48\,x^4\,XH^2\,y^4\,Z + 24\,XH^3\,y^4\,Z - 8\,x\,XH^3\,y^4\,Z - 4\,x^2\,XH^3\,y^4\,Z - 12\,x^3\,XH^3\,y^4\,Z - 36\,y^6\,Z -$$
$$48\,x\,y^6\,Z + 136\,x^2\,y^6\,Z + 160\,x^3\,y^6\,Z + 40\,x^4\,y^6\,Z + 8\,XH\,y^6\,Z - 120\,x\,XH\,y^6\,Z - 152\,x^2\,XH\,y^6\,Z -$$
$$48\,x^3\,XH\,y^6\,Z + 32\,XH^2\,y^6\,Z + 68\,x\,XH^2\,y^6\,Z + 24\,x^2\,XH^2\,y^6\,Z - 12\,XH^3\,y^6\,Z - 4\,x\,XH^3\,y^6\,Z +$$
$$16\,y^8\,Z + 60\,x\,y^8\,Z + 20\,x^2\,y^8\,Z - 28\,XH\,y^8\,Z - 12\,x\,XH\,y^8\,Z + 4\,XH^2\,y^8\,Z + 4\,y^{10}\,Z - 48\,x\,y\,YH\,Z -$$
$$48\,x^2\,y\,YH\,Z + 96\,x^3\,y\,YH\,Z + 60\,x^4\,y\,YH\,Z - 24\,x^5\,y\,YH\,Z - 24\,x^7\,y\,YH\,Z - 12\,x^8\,y\,YH\,Z +$$

$$48\,XH\,y\,YH\,Z + 64\,x\,XH\,y\,YH\,Z - 88\,x^2\,XH\,y\,YH\,Z - 64\,x^3\,XH\,y\,YH\,Z - 16\,x^4\,XH\,y\,YH\,Z -$$
$$16\,x^5\,XH\,y\,YH\,Z + 56\,x^6\,XH\,y\,YH\,Z + 16\,x^7\,XH\,y\,YH\,Z - 16\,XH^2\,y\,YH\,Z - 8\,x\,XH^2\,y\,YH\,Z +$$
$$4\,x^2\,XH^2\,y\,YH\,Z + 40\,x^3\,XH^2\,y\,YH\,Z + 16\,x^4\,XH^2\,y\,YH\,Z - 32\,x^5\,XH^2\,y\,YH\,Z - 4\,x^6\,XH^2\,y\,YH\,Z -$$
$$48\,y^3\,YH\,Z + 64\,x\,y^3\,YH\,Z + 360\,x^2\,y^3\,YH\,Z + 112\,x^3\,y^3\,YH\,Z - 240\,x^4\,y^3\,YH\,Z - 200\,x^5\,y^3\,YH\,Z -$$
$$48\,x^6\,y^3\,YH\,Z - 88\,XH\,y^3\,YH\,Z - 256\,x\,XH\,y^3\,YH\,Z - 32\,x^2\,XH\,y^3\,YH\,Z + 160\,x^3\,XH\,y^3\,YH\,Z +$$
$$168\,x^4\,XH\,y^3\,YH\,Z + 48\,x^5\,XH\,y^3\,YH\,Z + 52\,XH^2\,y^3\,YH\,Z + 40\,x\,XH^2\,y^3\,YH\,Z - 16\,x^2\,XH^2\,y^3\,YH\,Z -$$
$$64\,x^3\,XH^2\,y^3\,YH\,Z - 12\,x^4\,XH^2\,y^3\,YH\,Z + 108\,y^5\,YH\,Z + 136\,x\,y^5\,YH\,Z - 288\,x^2\,y^5\,YH\,Z -$$
$$328\,x^3\,y^5\,YH\,Z - 72\,x^4\,y^5\,YH\,Z - 16\,XH\,y^5\,YH\,Z + 176\,x\,XH\,y^5\,YH\,Z + 168\,x^2\,XH\,y^5\,YH\,Z +$$
$$48\,x^3\,XH\,y^5\,YH\,Z - 32\,XH^2\,y^5\,YH\,Z - 32\,x\,XH^2\,y^5\,YH\,Z - 12\,x^2\,XH^2\,y^5\,YH\,Z - 48\,y^7\,YH\,Z -$$
$$152\,x\,y^7\,YH\,Z - 48\,x^2\,y^7\,YH\,Z + 56\,XH\,y^7\,YH\,Z + 16\,x\,XH\,y^7\,YH\,Z - 4\,XH^2\,y^7\,YH\,Z - 12\,y^9\,YH\,Z +$$
$$24\,x\,YH^2\,Z + 16\,x^2\,YH^2\,Z - 52\,x^3\,YH^2\,Z - 28\,x^4\,YH^2\,Z + 32\,x^5\,YH^2\,Z + 8\,x^6\,YH^2\,Z - 4\,x^7\,YH^2\,Z +$$
$$4\,x^8\,YH^2\,Z - 24\,XH\,YH^2\,Z - 16\,x\,XH\,YH^2\,Z + 52\,x^2\,XH\,YH^2\,Z + 28\,x^3\,XH\,YH^2\,Z - 32\,x^4\,XH\,YH^2\,Z -$$
$$8\,x^5\,XH\,YH^2\,Z + 4\,x^6\,XH\,YH^2\,Z - 4\,x^7\,XH\,YH^2\,Z + 48\,y^2\,YH^2\,Z - 20\,x\,y^2\,YH^2\,Z - 232\,x^2\,y^2\,YH^2\,Z -$$
$$96\,x^3\,y^2\,YH^2\,Z + 160\,x^4\,y^2\,YH^2\,Z + 116\,x^5\,y^2\,YH^2\,Z + 24\,x^6\,y^2\,YH^2\,Z + 44\,XH\,y^2\,YH^2\,Z +$$
$$76\,x\,XH\,y^2\,YH^2\,Z - 24\,x^2\,XH\,y^2\,YH^2\,Z - 64\,x^3\,XH\,y^2\,YH^2\,Z - 20\,x^4\,XH\,y^2\,YH^2\,Z - 12\,x^5\,XH\,y^2\,YH^2\,Z -$$
$$108\,y^4\,YH^2\,Z - 128\,x\,y^4\,YH^2\,Z + 200\,x^2\,y^4\,YH^2\,Z + 244\,x^3\,y^4\,YH^2\,Z + 48\,x^4\,y^4\,YH^2\,Z + 8\,XH\,y^4\,YH^2\,Z -$$
$$56\,x\,XH\,y^4\,YH^2\,Z - 52\,x^2\,XH\,y^4\,YH^2\,Z - 12\,x^3\,XH\,y^4\,YH^2\,Z + 48\,y^6\,YH^2\,Z + 124\,x\,y^6\,YH^2\,Z +$$
$$40\,x^2\,y^6\,YH^2\,Z - 28\,XH\,y^6\,YH^2\,Z - 4\,x\,XH\,y^6\,YH^2\,Z + 12\,y^8\,YH^2\,Z - 16\,y\,YH^3\,Z - 8\,x\,y\,YH^3\,Z +$$
$$52\,x^2\,y\,YH^3\,Z + 40\,x^3\,y\,YH^3\,Z - 32\,x^4\,y\,YH^3\,Z - 32\,x^5\,y\,YH^3\,Z - 4\,x^6\,y\,YH^3\,Z + 36\,y^3\,YH^3\,Z +$$
$$40\,x\,y^3\,YH^3\,Z - 48\,x^2\,y^3\,YH^3\,Z - 64\,x^3\,y^3\,YH^3\,Z - 12\,x^4\,y^3\,YH^3\,Z - 16\,y^5\,YH^3\,Z - 32\,x\,y^5\,YH^3\,Z -$$
$$12\,x^2\,y^5\,YH^3\,Z - 4\,y^7\,YH^3\,Z + 18\,x^2\,Z^2 + 12\,x^3\,Z^2 - 78\,x^4\,Z^2 + 24\,x^5\,Z^2 + 54\,x^6\,Z^2 - 36\,x^7\,Z^2 +$$
$$6\,x^8\,Z^2 - 36\,x\,XH\,Z^2 - 24\,x^2\,XH\,Z^2 + 156\,x^3\,XH\,Z^2 - 48\,x^4\,XH\,Z^2 - 108\,x^5\,XH\,Z^2 + 72\,x^6\,XH\,Z^2 -$$
$$12\,x^7\,XH\,Z^2 + 18\,XH^2\,Z^2 + 12\,x\,XH^2\,Z^2 - 78\,x^2\,XH^2\,Z^2 + 24\,x^3\,XH^2\,Z^2 + 54\,x^4\,XH^2\,Z^2 - 36\,x^5\,XH^2\,Z^2 +$$
$$6\,x^6\,XH^2\,Z^2 + 18\,y^2\,Z^2 - 36\,x\,y^2\,Z^2 - 156\,x^2\,y^2\,Z^2 - 48\,x^3\,y^2\,Z^2 + 126\,x^4\,y^2\,Z^2 + 36\,x^5\,y^2\,Z^2 +$$
$$24\,x^6\,y^2\,Z^2 + 24\,XH\,y^2\,Z^2 + 156\,x\,XH\,y^2\,Z^2 + 96\,x^2\,XH\,y^2\,Z^2 - 168\,x^3\,XH\,y^2\,Z^2 - 48\,x^4\,XH\,y^2\,Z^2 -$$
$$36\,x^5\,XH\,y^2\,Z^2 - 46\,XH^2\,y^2\,Z^2 - 40\,x\,XH^2\,y^2\,Z^2 + 60\,x^2\,XH^2\,y^2\,Z^2 + 8\,x^3\,XH^2\,y^2\,Z^2 + 14\,x^4\,XH^2\,y^2\,Z^2 -$$
$$78\,y^4\,Z^2 - 72\,x\,y^4\,Z^2 + 186\,x^2\,y^4\,Z^2 + 180\,x^3\,y^4\,Z^2 + 36\,x^4\,y^4\,Z^2 + 16\,XH\,y^4\,Z^2 - 124\,x\,XH\,y^4\,Z^2 -$$
$$152\,x^2\,XH\,y^4\,Z^2 - 36\,x^3\,XH\,y^4\,Z^2 + 22\,XH^2\,y^4\,Z^2 + 44\,x\,XH^2\,y^4\,Z^2 + 10\,x^2\,XH^2\,y^4\,Z^2 + 50\,y^6\,Z^2 +$$
$$108\,x\,y^6\,Z^2 + 24\,x^2\,y^6\,Z^2 - 32\,XH\,y^6\,Z^2 - 12\,x\,XH\,y^6\,Z^2 + 2\,XH^2\,y^6\,Z^2 + 6\,y^8\,Z^2 - 36\,y\,YH\,Z^2 +$$
$$48\,x\,y\,YH\,Z^2 + 156\,x^2\,y\,YH\,Z^2 - 84\,x^4\,y\,YH\,Z^2 - 24\,x^5\,y\,YH\,Z^2 - 12\,x^6\,y\,YH\,Z^2 - 24\,XH\,y\,YH\,Z^2 -$$
$$64\,x\,XH\,y\,YH\,Z^2 - 16\,x^2\,XH\,y\,YH\,Z^2 + 48\,x^3\,XH\,y\,YH\,Z^2 + 32\,x^4\,XH\,y\,YH\,Z^2 + 8\,x^5\,XH\,y\,YH\,Z^2 +$$
$$156\,y^3\,YH\,Z^2 + 128\,x\,y^3\,YH\,Z^2 - 248\,x^2\,y^3\,YH\,Z^2 - 208\,x^3\,y^3\,YH\,Z^2 - 36\,x^4\,y^3\,YH\,Z^2 -$$
$$16\,XH\,y^3\,YH\,Z^2 + 80\,x\,XH\,y^3\,YH\,Z^2 + 64\,x^2\,XH\,y^3\,YH\,Z^2 + 16\,x^3\,XH\,y^3\,YH\,Z^2 - 100\,y^5\,YH\,Z^2 -$$
$$184\,x\,y^5\,YH\,Z^2 - 36\,x^2\,y^5\,YH\,Z^2 + 32\,XH\,y^5\,YH\,Z^2 + 8\,x\,XH\,y^5\,YH\,Z^2 - 12\,y^7\,YH\,Z^2 + 18\,YH^2\,Z^2 -$$
$$12\,x\,YH^2\,Z^2 - 46\,x^2\,YH^2\,Z^2 + 8\,x^3\,YH^2\,Z^2 + 18\,x^4\,YH^2\,Z^2 - 4\,x^5\,YH^2\,Z^2 + 2\,x^6\,YH^2\,Z^2 - 78\,y^2\,YH^2\,Z^2 -$$
$$56\,x\,y^2\,YH^2\,Z^2 + 84\,x^2\,y^2\,YH^2\,Z^2 + 72\,x^3\,y^2\,YH^2\,Z^2 + 10\,x^4\,y^2\,YH^2\,Z^2 + 50\,y^4\,YH^2\,Z^2 + 76\,x\,y^4\,YH^2\,Z^2 +$$
$$14\,x^2\,y^4\,YH^2\,Z^2 + 6\,y^6\,YH^2\,Z^2 - 72\,x^2\,Z^3 + 12\,x^3\,Z^3 + 52\,x^4\,Z^3 - 28\,x^5\,Z^3 + 4\,x^6\,Z^3 + 72\,x\,XH\,Z^3 -$$
$$12\,x^2\,XH\,Z^3 - 52\,x^3\,XH\,Z^3 + 28\,x^4\,XH\,Z^3 - 4\,x^5\,XH\,Z^3 - 72\,y^2\,Z^3 - 36\,x\,y^2\,Z^3 + 104\,x^2\,y^2\,Z^3 +$$
$$56\,x^3\,y^2\,Z^3 + 12\,x^4\,y^2\,Z^3 + 12\,XH\,y^2\,Z^3 - 52\,x\,XH\,y^2\,Z^3 - 48\,x^2\,XH\,y^2\,Z^3 - 8\,x^3\,XH\,y^2\,Z^3 + 52\,y^4\,Z^3 +$$
$$84\,x\,y^4\,Z^3 + 12\,x^2\,y^4\,Z^3 - 12\,XH\,y^4\,Z^3 - 4\,x\,XH\,y^4\,Z^3 + 4\,y^6\,Z^3 + 72\,y\,YH\,Z^3 + 24\,x\,y\,YH\,Z^3 -$$
$$52\,x^2\,y\,YH\,Z^3 - 8\,x^3\,y\,YH\,Z^3 - 4\,x^4\,y\,YH\,Z^3 - 52\,y^3\,YH\,Z^3 - 72\,x\,y^3\,YH\,Z^3 - 8\,x^2\,y^3\,YH\,Z^3 -$$
$$4\,y^5\,YH\,Z^3 - 27\,Z^4 + 18\,x^2\,Z^4 - 8\,x^3\,Z^4 + x^4\,Z^4 + 18\,y^2\,Z^4 + 24\,x\,y^2\,Z^4 + 2\,x^2\,y^2\,Z^4 + y^4\,Z^4\Big)$$

```
(* Lemma 13 Check.  *)

In[330]:= p[XH_, YH_, x_, y_, Z_] :=
 (x^4 - 4 x^5 + 4 x^6 + 4 x^7 - 10 x^8 + 4 x^9 + 4 x^10 - 4 x^11 + x^12 - 4 x^3 XH + 16 x^4 XH - 16 x^5 XH - 16 x^6 XH +
   40 x^7 XH - 16 x^8 XH - 16 x^9 XH + 16 x^10 XH - 4 x^11 XH + 6 x^2 XH^2 - 24 x^3 XH^2 + 24 x^4 XH^2 +
   24 x^5 XH^2 - 60 x^6 XH^2 + 24 x^7 XH^2 + 24 x^8 XH^2 - 24 x^9 XH^2 + 6 x^10 XH^2 - 4 x XH^3 + 16 x^2 XH^3 -
   16 x^3 XH^3 - 16 x^4 XH^3 + 40 x^5 XH^3 - 16 x^6 XH^3 - 16 x^7 XH^3 + 16 x^8 XH^3 - 4 x^9 XH^3 + XH^4 - 4 x XH^4 +
   4 x^2 XH^4 + 4 x^3 XH^4 - 10 x^4 XH^4 + 4 x^5 XH^4 + 4 x^6 XH^4 - 4 x^7 XH^4 + x^8 XH^4 + 2 x^2 y^2 + 8 x^3 y^2 -
   24 x^4 y^2 - 4 x^5 y^2 + 32 x^6 y^2 - 16 x^8 y^2 - 4 x^9 y^2 + 6 x^10 y^2 - 4 x XH y^2 - 24 x^2 XH y^2 + 64 x^3 XH y^2 +
   8 x^4 XH y^2 - 72 x^5 XH y^2 - 8 x^6 XH y^2 + 32 x^7 XH y^2 + 24 x^8 XH y^2 - 20 x^9 XH y^2 + 2 XH^2 y^2 +
   24 x XH^2 y^2 - 56 x^2 XH^2 y^2 + 36 x^4 XH^2 y^2 + 24 x^5 XH^2 y^2 - 8 x^6 XH^2 y^2 - 48 x^7 XH^2 y^2 + 26 x^8 XH^2 y^2 -
   8 XH^3 y^2 + 16 x XH^3 y^2 - 8 x^2 XH^3 y^2 + 16 x^3 XH^3 y^2 - 24 x^4 XH^3 y^2 - 16 x^5 XH^3 y^2 + 40 x^6 XH^3 y^2 -
   16 x^7 XH^3 y^2 + 4 x XH^4 y^2 - 12 x^2 XH^4 y^2 + 8 x^3 XH^4 y^2 + 8 x^4 XH^4 y^2 - 12 x^5 XH^4 y^2 + 4 x^6 XH^4 y^2 +
   y^4 + 12 x y^4 + 36 x^2 y^4 - 20 x^3 y^4 - 36 x^4 y^4 - 24 x^5 y^4 - 8 x^6 y^4 + 24 x^7 y^4 + 15 x^8 y^4 - 8 XH y^4 -
   48 x XH y^4 + 32 x^2 XH y^4 - 8 x^3 XH y^4 + 40 x^4 XH y^4 + 64 x^5 XH y^4 - 32 x^6 XH y^4 - 40 x^7 XH y^4 +
   16 XH^2 y^4 - 24 x XH^2 y^4 + 60 x^2 XH^2 y^4 - 24 x^3 XH^2 y^4 - 72 x^4 XH^2 y^4 + 44 x^6 XH^2 y^4 + 8 XH^3 y^4 -
   24 x XH^3 y^4 + 16 x^3 XH^3 y^4 + 24 x^4 XH^3 y^4 - 24 x^5 XH^3 y^4 - 2 XH^4 y^4 + 4 x XH^4 y^4 + 4 x^2 XH^4 y^4 -
   12 x^3 XH^4 y^4 + 6 x^4 XH^4 y^4 - 12 x y^6 - 80 x^2 y^6 - 32 x^3 y^6 + 48 x^4 y^6 + 56 x^5 y^6 + 20 x^6 y^6 + 8 XH y^6 +
   104 x XH y^6 + 40 x^2 XH y^6 - 32 x^3 XH y^6 - 80 x^4 XH y^6 - 40 x^5 XH y^6 - 36 XH^2 y^6 - 24 x XH^2 y^6 -
   24 x^2 XH^2 y^6 + 48 x^3 XH^2 y^6 + 36 x^4 XH^2 y^6 + 8 XH^3 y^6 + 16 x XH^3 y^6 - 8 x^2 XH^3 y^6 - 16 x^3 XH^3 y^6 -
   4 x XH^4 y^6 + 4 x^2 XH^4 y^6 - 2 y^8 - 12 x y^8 + 36 x^2 y^8 + 44 x^3 y^8 + 15 x^4 y^8 + 8 XH y^8 - 48 x XH y^8 -
   48 x^2 XH y^8 - 20 x^3 XH y^8 + 16 XH^2 y^8 + 24 x XH^2 y^8 + 14 x^2 XH^2 y^8 - 8 XH^3 y^8 - 4 x XH^3 y^8 +
   XH^4 y^8 + 12 x y^10 + 6 x^2 y^10 - 8 XH y^10 - 4 x XH y^10 + 2 XH^2 y^10 + y^12 - 4 x^2 y YH - 8 x^3 y YH +
   32 x^4 y YH + 8 x^5 y YH - 56 x^6 y YH + 8 x^7 y YH + 32 x^8 y YH - 8 x^9 y YH - 4 x^10 y YH + 8 x XH y YH +
   24 x^2 XH y YH - 80 x^3 XH y YH - 24 x^4 XH y YH + 144 x^5 XH y YH - 24 x^6 XH y YH - 80 x^7 XH y YH +
   24 x^8 XH y YH + 8 x^9 XH y YH - 4 XH^2 y YH - 24 x XH^2 y YH + 64 x^2 XH^2 y YH + 24 x^3 XH^2 y YH -
   120 x^4 XH^2 y YH + 24 x^5 XH^2 y YH + 64 x^6 XH^2 y YH - 24 x^7 XH^2 y YH - 4 x^8 XH^2 y YH + 8 XH^3 y YH -
   16 x XH^3 y YH - 8 x^2 XH^3 y YH + 32 x^3 XH^3 y YH - 8 x^4 XH^3 y YH - 16 x^5 XH^3 y YH + 8 x^6 XH^3 y YH -
   4 y^3 YH - 40 x y^3 YH - 96 x^2 y^3 YH + 48 x^3 y^3 YH + 152 x^4 y^3 YH + 56 x^5 y^3 YH - 32 x^6 y^3 YH -
   64 x^7 y^3 YH - 20 x^8 y^3 YH + 24 XH y^3 YH + 112 x XH y^3 YH - 48 x^2 XH y^3 YH - 96 x^3 XH y^3 YH -
   72 x^4 XH y^3 YH - 48 x^5 XH y^3 YH + 96 x^6 XH y^3 YH + 32 x^7 XH y^3 YH - 32 XH^2 y^3 YH + 24 x XH^2 y^3 YH -
   48 x^2 XH^2 y^3 YH + 48 x^3 XH^2 y^3 YH + 96 x^4 XH^2 y^3 YH - 72 x^5 XH^2 y^3 YH - 16 x^6 XH^2 y^3 YH -
   8 XH^3 y^3 YH + 32 x XH^3 y^3 YH - 16 x^2 XH^3 y^3 YH - 32 x^3 XH^3 y^3 YH + 24 x^4 XH^3 y^3 YH + 40 x y^5 YH +
   216 x^2 y^5 YH + 88 x^3 y^5 YH - 160 x^4 y^5 YH - 144 x^5 y^5 YH - 40 x^6 y^5 YH - 24 XH y^5 YH -
   240 x XH y^5 YH - 72 x^2 XH y^5 YH + 144 x^3 XH y^5 YH + 144 x^4 XH y^5 YH + 48 x^5 XH y^5 YH +
   72 XH^2 y^5 YH + 24 x XH^2 y^5 YH - 72 x^3 XH^2 y^5 YH - 24 x^4 XH^2 y^5 YH - 8 XH^3 y^5 YH - 16 x XH^3 y^5 YH +
   24 x^2 XH^3 y^5 YH + 8 y^7 YH + 40 x y^7 YH - 96 x^2 y^7 YH - 128 x^3 y^7 YH - 40 x^4 y^7 YH - 24 XH y^7 YH +
   112 x XH y^7 YH + 96 x^2 XH y^7 YH + 32 x^3 XH y^7 YH - 32 XH^2 y^7 YH - 24 x XH^2 y^7 YH - 16 x^2 XH^2 y^7 YH +
   8 XH^3 y^7 YH - 40 x y^9 YH - 20 x^2 y^9 YH + 24 XH y^9 YH + 8 x XH y^9 YH - 4 XH^2 y^9 YH - 4 y^11 YH +
   2 x^2 YH^2 - 8 x^4 YH^2 + 12 x^6 YH^2 - 8 x^8 YH^2 + 2 x^10 YH^2 - 4 x XH YH^2 + 16 x^3 XH YH^2 - 24 x^5 XH YH^2 +
   16 x^7 XH YH^2 - 4 x^9 XH YH^2 + 2 XH^2 YH^2 - 8 x^2 XH^2 YH^2 + 12 x^4 XH^2 YH^2 - 8 x^6 XH^2 YH^2 +
   2 x^8 XH^2 YH^2 + 6 y^2 YH^2 + 48 x y^2 YH^2 + 88 x^2 y^2 YH^2 - 48 x^3 y^2 YH^2 - 180 x^4 y^2 YH^2 - 48 x^5 y^2 YH^2 +
```

$$72 x^6 y^2 YH^2 + 48 x^7 y^2 YH^2 + 14 x^8 y^2 YH^2 - 24 XH y^2 YH^2 - 80 x XH y^2 YH^2 + 24 x^2 XH y^2 YH^2 + 144 x^3 XH y^2 YH^2 + 24 x^4 XH y^2 YH^2 - 48 x^5 XH y^2 YH^2 - 24 x^6 XH y^2 YH^2 - 16 x^7 XH y^2 YH^2 + 16 XH^2 y^2 YH^2 - 24 x^2 XH^2 y^2 YH^2 + 8 x^6 XH^2 y^2 YH^2 - 48 x y^4 YH^2 - 204 x^2 y^4 YH^2 - 96 x^3 y^4 YH^2 + 168 x^4 y^4 YH^2 + 144 x^5 y^4 YH^2 + 36 x^6 y^4 YH^2 + 24 XH y^4 YH^2 + 168 x XH y^4 YH^2 + 48 x^2 XH y^4 YH^2 - 144 x^3 XH y^4 YH^2 - 72 x^4 XH y^4 YH^2 - 24 x^5 XH y^4 YH^2 - 36 XH^2 y^4 YH^2 + 24 x^2 XH^2 y^4 YH^2 + 12 x^4 XH^2 y^4 YH^2 - 12 y^6 YH^2 - 48 x y^6 YH^2 + 88 x^2 y^6 YH^2 + 144 x^3 y^6 YH^2 + 44 x^4 y^6 YH^2 + 24 XH y^6 YH^2 - 80 x XH y^6 YH^2 - 72 x^2 XH y^6 YH^2 - 16 x^3 XH y^6 YH^2 + 16 XH^2 y^6 YH^2 + 8 x^2 XH^2 y^6 YH^2 + 48 x y^8 YH^2 + 26 x^2 y^8 YH^2 - 24 XH y^8 YH^2 - 4 x XH y^8 YH^2 + 2 XH^2 y^8 YH^2 + 6 y^{10} YH^2 - 4 y YH^3 - 24 x y YH^3 - 32 x^2 y YH^3 + 24 x^3 y YH^3 + 72 x^4 y YH^3 + 24 x^5 y YH^3 - 32 x^6 y YH^3 - 24 x^7 y YH^3 - 4 x^8 y YH^3 + 8 XH y YH^3 + 16 x XH y YH^3 - 8 x^2 XH y YH^3 - 32 x^3 XH y YH^3 - 8 x^4 XH y YH^3 + 16 x^5 XH y YH^3 + 8 x^6 XH y YH^3 + 24 x y^3 YH^3 + 80 x^2 y^3 YH^3 + 48 x^3 y^3 YH^3 - 64 x^4 y^3 YH^3 - 72 x^5 y^3 YH^3 - 16 x^6 y^3 YH^3 - 8 XH y^3 YH^3 - 32 x XH y^3 YH^3 - 16 x^2 XH y^3 YH^3 + 32 x^3 XH y^3 YH^3 + 24 x^4 XH y^3 YH^3 + 8 y^5 YH^3 + 24 x y^5 YH^3 - 32 x^2 y^5 YH^3 - 72 x^3 y^5 YH^3 - 24 x^4 y^5 YH^3 - 8 XH y^5 YH^3 + 16 x XH y^5 YH^3 + 24 x^2 XH y^5 YH^3 - 24 x y^7 YH^3 - 16 x^2 y^7 YH^3 + 8 XH y^7 YH^3 - 4 y^9 YH^3 + YH^4 + 4 x YH^4 + 4 x^2 YH^4 - 4 x^3 YH^4 - 10 x^4 YH^4 - 4 x^5 YH^4 + 4 x^6 YH^4 + 4 x^7 YH^4 + x^8 YH^4 - 4 x y^2 YH^4 - 12 x^2 y^2 YH^4 - 8 x^3 y^2 YH^4 + 8 x^4 y^2 YH^4 + 12 x^5 y^2 YH^4 + 4 x^6 y^2 YH^4 - 2 y^4 YH^4 - 4 x y^4 YH^4 + 4 x^2 y^4 YH^4 + 12 x^3 y^4 YH^4 + 6 x^4 y^4 YH^4 + 4 x y^6 YH^4 + 4 x^2 y^6 YH^4 + y^8 YH^4 - 8 x^3 Z + 16 x^4 Z + 12 x^5 Z - 44 x^6 Z + 16 x^7 Z + 24 x^8 Z - 20 x^9 Z + 4 x^{10} Z + 24 x^2 XH Z - 48 x^3 XH Z - 36 x^4 XH Z + 132 x^5 XH Z - 48 x^6 XH Z - 72 x^7 XH Z + 60 x^8 XH Z - 12 x^9 XH Z - 24 x XH^2 Z + 48 x^2 XH^2 Z + 36 x^3 XH^2 Z - 132 x^4 XH^2 Z + 48 x^5 XH^2 Z + 72 x^6 XH^2 Z - 60 x^7 XH^2 Z + 12 x^8 XH^2 Z + 8 XH^3 Z - 16 x XH^3 Z - 12 x^2 XH^3 Z + 44 x^3 XH^3 Z - 16 x^4 XH^3 Z - 24 x^5 XH^3 Z + 20 x^6 XH^3 Z - 4 x^7 XH^3 Z + 24 x y^2 Z + 32 x^2 y^2 Z - 24 x^3 y^2 Z - 60 x^4 y^2 Z - 16 x^5 y^2 Z + 24 x^6 y^2 Z + 20 x^8 y^2 Z - 24 XH y^2 Z - 48 x XH y^2 Z - 24 x^2 XH y^2 Z + 120 x^3 XH y^2 Z + 72 x^4 XH y^2 Z - 72 x^5 XH y^2 Z + 24 x^6 XH y^2 Z - 48 x^7 XH y^2 Z + 16 XH^2 y^2 Z + 68 x XH^2 y^2 Z - 88 x^2 XH^2 y^2 Z - 64 x^3 XH^2 y^2 Z + 80 x^4 XH^2 y^2 Z - 52 x^5 XH^2 y^2 Z + 40 x^6 XH^2 y^2 Z - 20 XH^3 y^2 Z + 28 x XH^3 y^2 Z + 8 x^2 XH^3 y^2 Z - 32 x^3 XH^3 y^2 Z + 28 x^4 XH^3 y^2 Z - 12 x^5 XH^3 y^2 Z + 16 y^4 Z - 36 x y^4 Z - 180 x^2 y^4 Z - 80 x^3 y^4 Z + 120 x^4 y^4 Z + 120 x^5 y^4 Z + 40 x^6 y^4 Z + 44 XH y^4 Z + 180 x XH y^4 Z + 128 x^2 XH y^4 Z - 120 x^3 XH y^4 Z - 160 x^4 XH y^4 Z - 72 x^5 XH y^4 Z - 52 XH^2 y^4 Z - 112 x XH^2 y^4 Z + 40 x^2 XH^2 y^4 Z + 76 x^3 XH^2 y^4 Z + 48 x^4 XH^2 y^4 Z + 24 XH^3 y^4 Z - 8 x XH^3 y^4 Z - 4 x^2 XH^3 y^4 Z - 12 x^3 XH^3 y^4 Z - 36 y^6 Z - 48 x y^6 Z + 136 x^2 y^6 Z + 160 x^3 y^6 Z + 40 x^4 y^6 Z + 8 XH y^6 Z - 120 x XH y^6 Z - 152 x^2 XH y^6 Z - 48 x^3 XH y^6 Z + 32 XH^2 y^6 Z + 68 x XH^2 y^6 Z + 24 x^2 XH^2 y^6 Z - 12 XH^3 y^6 Z - 4 x XH^3 y^6 Z + 16 y^8 Z + 60 x y^8 Z + 20 x^2 y^8 Z - 28 XH y^8 Z - 12 x XH y^8 Z + 4 XH^2 y^8 Z + 4 y^{10} Z - 48 x y YH Z - 48 x^2 y YH Z + 96 x^3 y YH Z + 60 x^4 y YH Z - 24 x^5 y YH Z - 24 x^7 y YH Z - 12 x^8 y YH Z + 48 XH y YH Z + 64 x XH y YH Z - 88 x^2 XH y YH Z - 64 x^3 XH y YH Z - 16 x^4 XH y YH Z - 16 x^5 XH y YH Z + 56 x^6 XH y YH Z + 16 x^7 XH y YH Z - 16 XH^2 y YH Z - 8 x XH^2 y YH Z + 4 x^2 XH^2 y YH Z + 40 x^3 XH^2 y YH Z + 16 x^4 XH^2 y YH Z - 32 x^5 XH^2 y YH Z - 4 x^6 XH^2 y YH Z - 48 y^3 YH Z + 64 x y^3 YH Z + 360 x^2 y^3 YH Z + 112 x^3 y^3 YH Z - 240 x^4 y^3 YH Z - 200 x^5 y^3 YH Z - 48 x^6 y^3 YH Z - 88 XH y^3 YH Z - 256 x XH y^3 YH Z - 32 x^2 XH y^3 YH Z + 160 x^3 XH y^3 YH Z + 168 x^4 XH y^3 YH Z + 48 x^5 XH y^3 YH Z + 52 XH^2 y^3 YH Z + 40 x XH^2 y^3 YH Z - 16 x^2 XH^2 y^3 YH Z - 64 x^3 XH^2 y^3 YH Z - 12 x^4 XH^2 y^3 YH Z + 108 y^5 YH Z + 136 x y^5 YH Z - 288 x^2 y^5 YH Z - 328 x^3 y^5 YH Z - 72 x^4 y^5 YH Z - 16 XH y^5 YH Z + 176 x XH y^5 YH Z + 168 x^2 XH y^5 YH Z + 48 x^3 XH y^5 YH Z - 32 XH^2 y^5 YH Z - 32 x XH^2 y^5 YH Z - 12 x^2 XH^2 y^5 YH Z - 48 y^7 YH Z - 152 x y^7 YH Z - 48 x^2 y^7 YH Z + 56 XH y^7 YH Z + 16 x XH y^7 YH Z - 4 XH^2 y^7 YH Z - 12 y^9 YH Z + 24 x YH^2 Z + 16 x^2 YH^2 Z - 52 x^3 YH^2 Z - 28 x^4 YH^2 Z + 32 x^5 YH^2 Z + 8 x^6 YH^2 Z - 4 x^7 YH^2 Z +$$

$$4\,x^8\,YH^2\,Z - 24\,XH\,YH^2\,Z - 16\,x\,XH\,YH^2\,Z + 52\,x^2\,XH\,YH^2\,Z + 28\,x^3\,XH\,YH^2\,Z - 32\,x^4\,XH\,YH^2\,Z -$$
$$8\,x^5\,XH\,YH^2\,Z + 4\,x^6\,XH\,YH^2\,Z - 4\,x^7\,XH\,YH^2\,Z + 48\,y^2\,YH^2\,Z - 20\,x\,y^2\,YH^2\,Z - 232\,x^2\,y^2\,YH^2\,Z -$$
$$96\,x^3\,y^2\,YH^2\,Z + 160\,x^4\,y^2\,YH^2\,Z + 116\,x^5\,y^2\,YH^2\,Z + 24\,x^6\,y^2\,YH^2\,Z + 44\,XH\,y^2\,YH^2\,Z +$$
$$76\,x\,XH\,y^2\,YH^2\,Z - 24\,x^2\,XH\,y^2\,YH^2\,Z - 64\,x^3\,XH\,y^2\,YH^2\,Z - 20\,x^4\,XH\,y^2\,YH^2\,Z - 12\,x^5\,XH\,y^2\,YH^2\,Z -$$
$$108\,y^4\,YH^2\,Z - 128\,x\,y^4\,YH^2\,Z + 200\,x^2\,y^4\,YH^2\,Z + 244\,x^3\,y^4\,YH^2\,Z + 48\,x^4\,y^4\,YH^2\,Z + 8\,XH\,y^4\,YH^2\,Z -$$
$$56\,x\,XH\,y^4\,YH^2\,Z - 52\,x^2\,XH\,y^4\,YH^2\,Z - 12\,x^3\,XH\,y^4\,YH^2\,Z + 48\,y^6\,YH^2\,Z + 124\,x\,y^6\,YH^2\,Z +$$
$$40\,x^2\,y^6\,YH^2\,Z - 28\,XH\,y^6\,YH^2\,Z - 4\,x\,XH\,y^6\,YH^2\,Z + 12\,y^8\,YH^2\,Z - 16\,y\,YH^3\,Z - 8\,x\,y\,YH^3\,Z +$$
$$52\,x^2\,y\,YH^3\,Z + 40\,x^3\,y\,YH^3\,Z - 32\,x^4\,y\,YH^3\,Z - 32\,x^5\,y\,YH^3\,Z - 4\,x^6\,y\,YH^3\,Z + 36\,y^3\,YH^3\,Z +$$
$$40\,x\,y^3\,YH^3\,Z - 48\,x^2\,y^3\,YH^3\,Z - 64\,x^3\,y^3\,YH^3\,Z - 12\,x^4\,y^3\,YH^3\,Z - 16\,y^5\,YH^3\,Z - 32\,x\,y^5\,YH^3\,Z -$$
$$12\,x^2\,y^5\,YH^3\,Z - 4\,y^7\,YH^3\,Z + 18\,x^2\,Z^2 + 12\,x^3\,Z^2 - 78\,x^4\,Z^2 + 24\,x^5\,Z^2 + 54\,x^6\,Z^2 - 36\,x^7\,Z^2 +$$
$$6\,x^8\,Z^2 - 36\,x\,XH\,Z^2 - 24\,x^2\,XH\,Z^2 + 156\,x^3\,XH\,Z^2 - 48\,x^4\,XH\,Z^2 - 108\,x^5\,XH\,Z^2 + 72\,x^6\,XH\,Z^2 -$$
$$12\,x^7\,XH\,Z^2 + 18\,XH^2\,Z^2 + 12\,x\,XH^2\,Z^2 - 78\,x^2\,XH^2\,Z^2 + 24\,x^3\,XH^2\,Z^2 + 54\,x^4\,XH^2\,Z^2 - 36\,x^5\,XH^2\,Z^2 +$$
$$6\,x^6\,XH^2\,Z^2 + 18\,y^2\,Z^2 - 36\,x\,y^2\,Z^2 - 156\,x^2\,y^2\,Z^2 - 48\,x^3\,y^2\,Z^2 + 126\,x^4\,y^2\,Z^2 + 36\,x^5\,y^2\,Z^2 +$$
$$24\,x^6\,y^2\,Z^2 + 24\,XH\,y^2\,Z^2 + 156\,x\,XH\,y^2\,Z^2 + 96\,x^2\,XH\,y^2\,Z^2 - 168\,x^3\,XH\,y^2\,Z^2 - 48\,x^4\,XH\,y^2\,Z^2 -$$
$$36\,x^5\,XH\,y^2\,Z^2 - 46\,XH^2\,y^2\,Z^2 - 40\,x\,XH^2\,y^2\,Z^2 + 60\,x^2\,XH^2\,y^2\,Z^2 + 8\,x^3\,XH^2\,y^2\,Z^2 + 14\,x^4\,XH^2\,y^2\,Z^2 -$$
$$78\,y^4\,Z^2 - 72\,x\,y^4\,Z^2 + 186\,x^2\,y^4\,Z^2 + 180\,x^3\,y^4\,Z^2 + 36\,x^4\,y^4\,Z^2 + 16\,XH\,y^4\,Z^2 - 124\,x\,XH\,y^4\,Z^2 -$$
$$152\,x^2\,XH\,y^4\,Z^2 - 36\,x^3\,XH\,y^4\,Z^2 + 22\,XH^2\,y^4\,Z^2 + 44\,x\,XH^2\,y^4\,Z^2 + 10\,x^2\,XH^2\,y^4\,Z^2 + 50\,y^6\,Z^2 +$$
$$108\,x\,y^6\,Z^2 + 24\,x^2\,y^6\,Z^2 - 32\,XH\,y^6\,Z^2 - 12\,x\,XH\,y^6\,Z^2 + 2\,XH^2\,y^6\,Z^2 + 6\,y^8\,Z^2 - 36\,y\,YH\,Z^2 +$$
$$48\,x\,y\,YH\,Z^2 + 156\,x^2\,y\,YH\,Z^2 - 84\,x^4\,y\,YH\,Z^2 - 24\,x^5\,y\,YH\,Z^2 - 12\,x^6\,y\,YH\,Z^2 - 24\,XH\,y\,YH\,Z^2 -$$
$$64\,x\,XH\,y\,YH\,Z^2 - 16\,x^2\,XH\,y\,YH\,Z^2 + 48\,x^3\,XH\,y\,YH\,Z^2 + 32\,x^4\,XH\,y\,YH\,Z^2 + 8\,x^5\,XH\,y\,YH\,Z^2 +$$
$$156\,y^3\,YH\,Z^2 + 128\,x\,y^3\,YH\,Z^2 - 248\,x^2\,y^3\,YH\,Z^2 - 208\,x^3\,y^3\,YH\,Z^2 - 36\,x^4\,y^3\,YH\,Z^2 -$$
$$16\,XH\,y^3\,YH\,Z^2 + 80\,x\,XH\,y^3\,YH\,Z^2 + 64\,x^2\,XH\,y^3\,YH\,Z^2 + 16\,x^3\,XH\,y^3\,YH\,Z^2 - 100\,y^5\,YH\,Z^2 -$$
$$184\,x\,y^5\,YH\,Z^2 - 36\,x^2\,y^5\,YH\,Z^2 + 32\,XH\,y^5\,YH\,Z^2 + 8\,x\,XH\,y^5\,YH\,Z^2 - 12\,y^7\,YH\,Z^2 + 18\,YH^2\,Z^2 -$$
$$12\,x\,YH^2\,Z^2 - 46\,x^2\,YH^2\,Z^2 + 8\,x^3\,YH^2\,Z^2 + 18\,x^4\,YH^2\,Z^2 - 4\,x^5\,YH^2\,Z^2 + 2\,x^6\,YH^2\,Z^2 - 78\,y^2\,YH^2\,Z^2 -$$
$$56\,x\,y^2\,YH^2\,Z^2 + 84\,x^2\,y^2\,YH^2\,Z^2 + 72\,x^3\,y^2\,YH^2\,Z^2 + 10\,x^4\,y^2\,YH^2\,Z^2 + 50\,y^4\,YH^2\,Z^2 + 76\,x\,y^4\,YH^2\,Z^2 +$$
$$14\,x^2\,y^4\,YH^2\,Z^2 + 6\,y^6\,YH^2\,Z^2 - 72\,x^2\,Z^3 + 12\,x^3\,Z^3 + 52\,x^4\,Z^3 - 28\,x^5\,Z^3 + 4\,x^6\,Z^3 + 72\,x\,XH\,Z^3 -$$
$$12\,x^2\,XH\,Z^3 - 52\,x^3\,XH\,Z^3 + 28\,x^4\,XH\,Z^3 - 4\,x^5\,XH\,Z^3 - 72\,y^2\,Z^3 - 36\,x\,y^2\,Z^3 + 104\,x^2\,y^2\,Z^3 +$$
$$56\,x^3\,y^2\,Z^3 + 12\,x^4\,y^2\,Z^3 + 12\,XH\,y^2\,Z^3 - 52\,x\,XH\,y^2\,Z^3 - 48\,x^2\,XH\,y^2\,Z^3 - 8\,x^3\,XH\,y^2\,Z^3 + 52\,y^4\,Z^3 +$$
$$84\,x\,y^4\,Z^3 + 12\,x^2\,y^4\,Z^3 - 12\,XH\,y^4\,Z^3 - 4\,x\,XH\,y^4\,Z^3 + 4\,y^6\,Z^3 + 72\,y\,YH\,Z^3 + 24\,x\,y\,YH\,Z^3 -$$
$$52\,x^2\,y\,YH\,Z^3 - 8\,x^3\,y\,YH\,Z^3 - 4\,x^4\,y\,YH\,Z^3 - 52\,y^3\,YH\,Z^3 - 72\,x\,y^3\,YH\,Z^3 - 8\,x^2\,y^3\,YH\,Z^3 -$$
$$4\,y^5\,YH\,Z^3 - 27\,Z^4 + 18\,x^2\,Z^4 - 8\,x^3\,Z^4 + x^4\,Z^4 + 18\,y^2\,Z^4 + 24\,x\,y^2\,Z^4 + 2\,x^2\,y^2\,Z^4 + y^4\,Z^4\Big)$$

In[331]:=
```
Factor[p[0, 0, x, y, Z]]
```

Out[331]= $x^4 - 4\,x^5 + 4\,x^6 + 4\,x^7 - 10\,x^8 + 4\,x^9 + 4\,x^{10} - 4\,x^{11} + x^{12} + 2\,x^2\,y^2 + 8\,x^3\,y^2 - 24\,x^4\,y^2 - 4\,x^5\,y^2 +$
$32\,x^6\,y^2 - 16\,x^8\,y^2 - 4\,x^9\,y^2 + 6\,x^{10}\,y^2 + y^4 + 12\,x\,y^4 + 36\,x^2\,y^4 - 20\,x^3\,y^4 - 36\,x^4\,y^4 - 24\,x^5\,y^4 -$
$8\,x^6\,y^4 + 24\,x^7\,y^4 + 15\,x^8\,y^4 - 12\,x\,y^6 - 80\,x^2\,y^6 - 32\,x^3\,y^6 + 48\,x^4\,y^6 + 56\,x^5\,y^6 + 20\,x^6\,y^6 -$
$2\,y^8 - 12\,x\,y^8 + 36\,x^2\,y^8 + 44\,x^3\,y^8 + 15\,x^4\,y^8 + 12\,x\,y^{10} + 6\,x^2\,y^{10} + y^{12} - 8\,x^3\,Z + 16\,x^4\,Z +$
$12\,x^5\,Z - 44\,x^6\,Z + 16\,x^7\,Z + 24\,x^8\,Z - 20\,x^9\,Z + 4\,x^{10}\,Z + 24\,x\,y^2\,Z + 32\,x^2\,y^2\,Z - 24\,x^3\,y^2\,Z -$
$60\,x^4\,y^2\,Z - 16\,x^5\,y^2\,Z + 24\,x^6\,y^2\,Z + 20\,x^8\,y^2\,Z + 16\,y^4\,Z - 36\,x\,y^4\,Z - 180\,x^2\,y^4\,Z - 80\,x^3\,y^4\,Z +$
$120\,x^4\,y^4\,Z + 120\,x^5\,y^4\,Z + 40\,x^6\,y^4\,Z - 36\,y^6\,Z - 48\,x\,y^6\,Z + 136\,x^2\,y^6\,Z + 160\,x^3\,y^6\,Z +$
$40\,x^4\,y^6\,Z + 16\,y^8\,Z + 60\,x\,y^8\,Z + 20\,x^2\,y^8\,Z + 4\,y^{10}\,Z + 18\,x^2\,Z^2 + 12\,x^3\,Z^2 - 78\,x^4\,Z^2 + 24\,x^5\,Z^2 +$
$54\,x^6\,Z^2 - 36\,x^7\,Z^2 + 6\,x^8\,Z^2 + 18\,y^2\,Z^2 - 36\,x\,y^2\,Z^2 - 156\,x^2\,y^2\,Z^2 - 48\,x^3\,y^2\,Z^2 + 126\,x^4\,y^2\,Z^2 +$
$36\,x^5\,y^2\,Z^2 + 24\,x^6\,y^2\,Z^2 - 78\,y^4\,Z^2 - 72\,x\,y^4\,Z^2 + 186\,x^2\,y^4\,Z^2 + 180\,x^3\,y^4\,Z^2 + 36\,x^4\,y^4\,Z^2 +$
$50\,y^6\,Z^2 + 108\,x\,y^6\,Z^2 + 24\,x^2\,y^6\,Z^2 + 6\,y^8\,Z^2 - 72\,x^2\,Z^3 + 12\,x^3\,Z^3 + 52\,x^4\,Z^3 - 28\,x^5\,Z^3 +$
$4\,x^6\,Z^3 - 72\,y^2\,Z^3 - 36\,x\,y^2\,Z^3 + 104\,x^2\,y^2\,Z^3 + 56\,x^3\,y^2\,Z^3 + 12\,x^4\,y^2\,Z^3 + 52\,y^4\,Z^3 + 84\,x\,y^4\,Z^3 +$
$12\,x^2\,y^4\,Z^3 + 4\,y^6\,Z^3 - 27\,Z^4 + 18\,x^2\,Z^4 - 8\,x^3\,Z^4 + x^4\,Z^4 + 18\,y^2\,Z^4 + 24\,x\,y^2\,Z^4 + 2\,x^2\,y^2\,Z^4 + y^4\,Z^4$

In[332]:=
```
( (x^2 + y^2)^2 + 8 x (3 y^2 - x^2) + 18 (x^2 + y^2) - 27) Z^4 +

 4 ( (x^2 + y^2)^3 + 7 x (3 y^2 - x^2) (x^2 + y^2) +
     13 (x^2 + y^2)^2 + 3 x (x^2 - 3 y^2) - 18 (x^2 + y^2) ) Z^3 +
  2 ( 3 (x^2 + y^2)^4 + 18 x (3 y^2 - x^2) (x^2 + y^2)^2 +
     (3 x^2 + y^2) (9 x^4 + 18 x^2 y^2 + 25 y^4) + 12 x (x^2 - 3 y^2) (x^2 + y^2) -
     39 (x^2 + y^2)^2 + 6 x (x^2 - 3 y^2) + 9 (x^2 + y^2) ) Z^2  +
  4 (x^2 + y^2 - 1) ( (x^2 + y^2)^4 - 5 x (x^2 - 3 y^2) (x^2 + y^2)^2 +
     (7 x^6 + 3 x^4 y^2 + 33 x^2 y^4 + 5 y^6) -
     x (x^2 - 3 y^2) (x^2 + y^2) - 6 x y^2 - 4 (x^2 + y^2)^2 + 2 x^3 ) Z +

 (x^2 + y^2 - 1)^2 ((x^2 + y^2)^2 + 2 x (3 y^2 - x^2) + (x^2 + y^2))^2
```

Out[332]= $\left(-1 + x^2 + y^2\right)^2 \left(x^2 + y^2 + \left(x^2 + y^2\right)^2 + 2\,x\,\left(-x^2 + 3\,y^2\right)\right)^2 +$
$4\,\left(-1 + x^2 + y^2\right)\,\left(2\,x^3 + 7\,x^6 - 6\,x\,y^2 + 3\,x^4\,y^2 + 33\,x^2\,y^4 + 5\,y^6 -\right.$
$\left.x\,\left(x^2 - 3\,y^2\right)\,\left(x^2 + y^2\right) - 4\,\left(x^2 + y^2\right)^2 - 5\,x\,\left(x^2 - 3\,y^2\right)\,\left(x^2 + y^2\right)^2 + \left(x^2 + y^2\right)^4\right)\,Z +$
$2\,\left(6\,x\,\left(x^2 - 3\,y^2\right) + 9\,\left(x^2 + y^2\right) + 12\,x\,\left(x^2 - 3\,y^2\right)\,\left(x^2 + y^2\right) - 39\,\left(x^2 + y^2\right)^2 + 3\,\left(x^2 + y^2\right)^4 +\right.$
$\left.18\,x\,\left(x^2 + y^2\right)^2\,\left(-x^2 + 3\,y^2\right) + \left(3\,x^2 + y^2\right)\,\left(9\,x^4 + 18\,x^2\,y^2 + 25\,y^4\right)\right)\,Z^2 +$
$4\,\left(3\,x\,\left(x^2 - 3\,y^2\right) - 18\,\left(x^2 + y^2\right) + 13\,\left(x^2 + y^2\right)^2 + \left(x^2 + y^2\right)^3 + 7\,x\,\left(x^2 + y^2\right)\,\left(-x^2 + 3\,y^2\right)\right)\,Z^3 +$
$\left(-27 + 18\,\left(x^2 + y^2\right) + \left(x^2 + y^2\right)^2 + 8\,x\,\left(-x^2 + 3\,y^2\right)\right)\,Z^4$

In[333]:=
```
Factor[%% - %]
```

Out[333]= 0

```
(*  Lemma 14 Check.  *)
```

In[334]:=
```
Factor[{c1^2, c2^2, c3^2, c1 c2 c3} /. cSubs]
```

Out[334]=
$$\left\{\frac{\left(-D1+r2^2+r3^2\right)^2}{4\,r2^2\,r3^2},\ \frac{\left(-D2+r1^2+r3^2\right)^2}{4\,r1^2\,r3^2},\ \frac{\left(-D3+r1^2+r2^2\right)^2}{4\,r1^2\,r2^2},\ \frac{\left(-D3+r1^2+r2^2\right)\left(-D2+r1^2+r3^2\right)\left(-D1+r2^2+r3^2\right)}{8\,r1^2\,r2^2\,r3^2}\right\}$$

In[335]:=
```
{Numerator[%], Denominator[%]}
```

Out[335]=
$$\left\{\left\{\left(-D1+r2^2+r3^2\right)^2,\ \left(-D2+r1^2+r3^2\right)^2,\ \left(-D3+r1^2+r2^2\right)^2,\ \left(-D3+r1^2+r2^2\right)\left(-D2+r1^2+r3^2\right)\left(-D1+r2^2+r3^2\right)\right\},\ \left\{4\,r2^2\,r3^2,\ 4\,r1^2\,r3^2,\ 4\,r1^2\,r2^2,\ 8\,r1^2\,r2^2\,r3^2\right\}\right\}$$

In[336]:=
```
Factor[ % /. {r1^2 → R1, r2^2 → R2, r3^2 → R3,
    -D1 + r2^2 + r3^2 → R1bar, -D2 + r1^2 + r3^2 → R2bar, -D3 + r1^2 + r2^2 → R3bar} ]
```

Out[336]=
$$\left\{\left\{R1bar^2,\ R2bar^2,\ R3bar^2,\ R1bar\ R2bar\ R3bar\right\},\ \left\{4\,R2\,R3,\ 4\,R1\,R3,\ 4\,R1\,R2,\ 8\,R1\,R2\,R3\right\}\right\}$$

In[337]:=
```
Factor[ %[[1]] / %[[2]] ]
```

Out[337]=
$$\left\{\frac{R1bar^2}{4\,R2\,R3},\ \frac{R2bar^2}{4\,R1\,R3},\ \frac{R3bar^2}{4\,R1\,R2},\ \frac{R1bar\ R2bar\ R3bar}{8\,R1\,R2\,R3}\right\}$$

In[338]:=
```
Factor[
  Factor[Factor[{ r1^2, r2^2, r3^2, -D1 + r2^2 + r3^2, -D2 + r1^2 + r3^2, -D3 + r1^2 + r2^2 } /.
      r2RSubs] /. DRSubs] //. ySubs]
```

Out[338]=
$$\left\{1+x^2-2\,x\,x1+y^2-2\,y\,y1+Z,\ 1+x^2-2\,x\,x2+y^2-2\,y\,y2+Z,\ 1+x^2-2\,x\,x3+y^2-2\,y\,y3+Z,\ 2\left(x^2-x\,x2-x\,x3+x2\,x3+y^2-y\,y2-y\,y3+y2\,y3+Z\right),\ 2\left(x^2-x\,x1-x\,x3+x1\,x3+y^2-y\,y1-y\,y3+y1\,y3+Z\right),\ 2\left(x^2-x\,x1-x\,x2+x1\,x2+y^2-y\,y1-y\,y2+y1\,y2+Z\right)\right\}$$

In[339]:=
```
Factor[% /. {x → 0, y → 0}]
```

Out[339]=
$$\{1+Z,\ 1+Z,\ 1+Z,\ 2\,(x2\,x3+y2\,y3+Z),\ 2\,(x1\,x3+y1\,y3+Z),\ 2\,(x1\,x2+y1\,y2+Z)\}$$

In[340]:=
```
Factor[XYΔReduce[%] /. tweak3]
```

Out[340]=
$$\left\{1+Z,\ 1+Z,\ 1+Z,\ \frac{1}{3}\left(-3-2\,X1\,XH+XH^2-2\,Y1\,YH+YH^2+6\,Z\right),\ \frac{1}{3}\left(-3-2\,X2\,XH+XH^2-2\,Y2\,YH+YH^2+6\,Z\right),\ \frac{1}{3}\left(-3+2\,X1\,XH+2\,X2\,XH+XH^2+2\,Y1\,YH+2\,Y2\,YH+YH^2+6\,Z\right)\right\}$$

In[341]:=
```
list = Delete[%, {{1}, {2}}]
```

Out[341]=
$$\left\{1+Z,\ \frac{1}{3}\left(-3-2\,X1\,XH+XH^2-2\,Y1\,YH+YH^2+6\,Z\right),\ \frac{1}{3}\left(-3-2\,X2\,XH+XH^2-2\,Y2\,YH+YH^2+6\,Z\right),\right.$$
$$\left.\frac{1}{3}\left(-3+2\,X1\,XH+2\,X2\,XH+XH^2+2\,Y1\,YH+2\,Y2\,YH+YH^2+6\,Z\right)\right\}$$

In[342]:=
```
newlist = Factor[ XYΔReduce[ { list[[1]], list[[2]] + list[[3]] + list[[4]],
     list[[3]] list[[4]] + list[[4]] list[[2]] + list[[2]] list[[3]],
     list[[2]] list[[3]] list[[4]] } ]  /. tweak3]
```

Out[342]=
$$\left\{1+Z,\ -3+XH^2+YH^2+6\,Z,\ 3-5\,XH^2+2\,XH^3-5\,YH^2-6\,XH\,YH^2-12\,Z+4\,XH^2\,Z+4\,YH^2\,Z+12\,Z^2,\right.$$
$$\left.-1+4\,XH^2-4\,XH^3+XH^4+4\,YH^2+12\,XH\,YH^2+2\,XH^2\,YH^2+YH^4+6\,Z-10\,XH^2\,Z+4\,XH^3\,Z-10\,YH^2\,Z-12\,XH\,YH^2\,Z-12\,Z^2+4\,XH^2\,Z^2+4\,YH^2\,Z^2+8\,Z^3\right\}$$

```
(* Poly's are scaled differently in paper - will change there *)
```

In[343]:=
```
{Q0, Q1, Q2, Q3} - %
```

Out[343]=
$$\left\{-1+Q0-Z,\ 3+Q1-XH^2-YH^2-6\,Z,\right.$$
$$-3+Q2+5\,XH^2-2\,XH^3+5\,YH^2+6\,XH\,YH^2+12\,Z-4\,XH^2\,Z-4\,YH^2\,Z-12\,Z^2,$$
$$1+Q3-4\,XH^2+4\,XH^3-XH^4-4\,YH^2-12\,XH\,YH^2-2\,XH^2\,YH^2-YH^4-6\,Z+10\,XH^2\,Z-4\,XH^3\,Z+10\,YH^2\,Z+12\,XH\,YH^2\,Z+12\,Z^2-4\,XH^2\,Z^2-4\,YH^2\,Z^2-8\,Z^3\Big\}$$

In[344]:=
```
GroebnerBasis[ {-1 + Q0 - Z, 3 + Q1 - XH^2 - YH^2 - 6 Z,
  -3 + Q2 + 5 XH^2 - 2 XH^3 + 5 YH^2 + 6 XH YH^2 + 12 Z - 4 XH^2 Z - 4 YH^2 Z - 12 Z^2,
  1 + Q3 - 4 XH^2 + 4 XH^3 - XH^4 - 4 YH^2 - 12 XH YH^2 - 2 XH^2 YH^2 - YH^4 - 6 Z + 10 XH^2 Z - 4 XH^3 Z +
   10 YH^2 Z + 12 XH YH^2 Z + 12 Z^2 - 4 XH^2 Z^2 - 4 YH^2 Z^2 - 8 Z^3}, {Z, YH, XH, Q3, Q2, Q1, Q0}]
```

Out[344]=
$$\left\{12\,Q0^2-8\,Q0^3-4\,Q0\,Q1+4\,Q0^2\,Q1-Q1^2+4\,Q2-2\,Q0\,Q2+Q3,\right.$$
$$-54+54\,Q0-12\,Q0^2-9\,Q1+4\,Q0\,Q1-Q2-54\,XH+36\,Q0\,XH-6\,Q1\,XH+8\,XH^3,$$
$$\left.-9+6\,Q0-Q1+XH^2+YH^2,\ 1-Q0+Z\right\}$$

In[345]:=
```
poly = %[[1]]
```

Out[345]=
$$12\,Q0^2-8\,Q0^3-4\,Q0\,Q1+4\,Q0^2\,Q1-Q1^2+4\,Q2-2\,Q0\,Q2+Q3$$

```
(*
   This generates ideal of polynomials in Q0, Q1, Q2, Q3
   (independent of XH and YH) such that substitutions yield zero.
*)
```

```
(*
    Theorem 2 Check.
    Mathematica becomes overwhelmed trying to compute the
    multi-polynomial resultant.
    Let us here start instead by obtaining the discriminant
    of the quartic. Start with Grunert's system:
*)
```

In[346]:=
```
p1 = r2^2 + r3^2 - 2 r2 r3 c1 - D1
p2 = r3^2 + r1^2 - 2 r3 r1 c2 - D2
p3 = r1^2 + r2^2 - 2 r1 r2 c3 - D3
```

Out[346]= $-D1 + r2^2 - 2\,c1\,r2\,r3 + r3^2$

Out[347]= $-D2 + r1^2 - 2\,c2\,r1\,r3 + r3^2$

Out[348]= $-D3 + r1^2 - 2\,c3\,r1\,r2 + r2^2$

In[349]:=
```
p4 = Factor[Resultant[p1, p2, r3]]
```

Out[349]=
$$D1^2 - 2\,D1\,D2 + D2^2 + 2\,D1\,r1^2 - 4\,c2^2\,D1\,r1^2 - 2\,D2\,r1^2 + r1^4 + 4\,c1\,c2\,D1\,r1\,r2 + 4\,c1\,c2\,D2\,r1\,r2 - 4\,c1\,c2\,r1^3\,r2 - 2\,D1\,r2^2 + 2\,D2\,r2^2 - 4\,c1^2\,D2\,r2^2 - 2\,r1^2\,r2^2 + 4\,c1^2\,r1^2\,r2^2 + 4\,c2^2\,r1^2\,r2^2 - 4\,c1\,c2\,r1\,r2^3 + r2^4$$

In[350]:=
```
p5 = Factor[Factor[Resultant[p3, p4, r2]] /. r1 → Sqrt[R1]]
```

Out[350]=
$$
\begin{aligned}
&D1^4 - 4\,D1^3\,D2 + 6\,D1^2\,D2^2 - 4\,D1\,D2^3 + D2^4 - 4\,D1^3\,D3 + 12\,D1^2\,D2\,D3 - 8\,c1^2\,D1^2\,D2\,D3 - 12\,D1\,D2^2\,D3 + \\
&16\,c1^2\,D1\,D2^2\,D3 + 4\,D2^3\,D3 - 8\,c1^2\,D2^3\,D3 + 6\,D1^2\,D3^2 - 12\,D1\,D2\,D3^2 + 16\,c1^2\,D1\,D2\,D3^2 + \\
&6\,D2^2\,D3^2 - 16\,c1^2\,D2^2\,D3^2 + 16\,c1^4\,D2^2\,D3^2 - 4\,D1\,D3^3 + 4\,D2\,D3^3 - 8\,c1^2\,D2\,D3^3 + D3^4 + \\
&8\,D1^3\,R1 - 8\,c2^2\,D1^3\,R1 + 8\,c1\,c2\,c3\,D1^3\,R1 - 8\,c3^2\,D1^3\,R1 - 24\,D1^2\,D2\,R1 + 8\,c1^2\,D1^2\,D2\,R1 + \\
&16\,c2^2\,D1^2\,D2\,R1 - 8\,c1\,c2\,c3\,D1^2\,D2\,R1 + 24\,c3^2\,D1^2\,D2\,R1 - 16\,c1^2\,c3^2\,D1^2\,D2\,R1 + 24\,D1\,D2^2\,R1 - \\
&16\,c1^2\,D1\,D2^2\,R1 - 8\,c2^2\,D1\,D2^2\,R1 - 8\,c1\,c2\,c3\,D1\,D2^2\,R1 - 24\,c3^2\,D1\,D2^2\,R1 + 32\,c1^2\,c3^2\,D1\,D2^2\,R1 - \\
&8\,D2^3\,R1 + 8\,c1^2\,D2^3\,R1 + 8\,c1\,c2\,c3\,D2^3\,R1 + 8\,c3^2\,D2^3\,R1 - 16\,c1^2\,c3^2\,D2^3\,R1 - 24\,D1^2\,D3\,R1 + \\
&8\,c1^2\,D1^2\,D3\,R1 + 24\,c2^2\,D1^2\,D3\,R1 - 16\,c1^2\,c2^2\,D1^2\,D3\,R1 - 8\,c1\,c2\,c3\,D1^2\,D3\,R1 + 16\,c3^2\,D1^2\,D3\,R1 + \\
&48\,D1\,D2\,D3\,R1 - 64\,c1^2\,D1\,D2\,D3\,R1 - 32\,c2^2\,D1\,D2\,D3\,R1 + 48\,c1\,c2\,c3\,D1\,D2\,D3\,R1 + \\
&32\,c1^3\,c2\,c3\,D1\,D2\,D3\,R1 - 32\,c3^2\,D1\,D2\,D3\,R1 - 24\,D2^2\,D3\,R1 + 56\,c1^2\,D2^2\,D3\,R1 - 32\,c1^4\,D2^2\,D3\,R1 + \\
&8\,c2^2\,D2^2\,D3\,R1 - 16\,c1^2\,c2^2\,D2^2\,D3\,R1 - 40\,c1\,c2\,c3\,D2^2\,D3\,R1 + 32\,c1^3\,c2\,c3\,D2^2\,D3\,R1 + \\
&16\,c3^2\,D2^2\,D3\,R1 + 24\,D1\,D3^2\,R1 - 16\,c1^2\,D1\,D3^2\,R1 - 24\,c2^2\,D1\,D3^2\,R1 + 32\,c1^2\,c2^2\,D1\,D3^2\,R1 - \\
&8\,c1\,c2\,c3\,D1\,D3^2\,R1 - 8\,c3^2\,D1\,D3^2\,R1 - 24\,D2\,D3^2\,R1 + 56\,c1^2\,D2\,D3^2\,R1 - 32\,c1^4\,D2\,D3^2\,R1 + \\
&16\,c2^2\,D2\,D3^2\,R1 - 40\,c1\,c2\,c3\,D2\,D3^2\,R1 + 32\,c1^3\,c2\,c3\,D2\,D3^2\,R1 + 8\,c3^2\,D2\,D3^2\,R1 - \\
&16\,c1^2\,c3^2\,D2\,D3^2\,R1 - 8\,D3^3\,R1 + 8\,c1^2\,D3^3\,R1 + 8\,c2^2\,D3^3\,R1 - 16\,c1^2\,c2^2\,D3^3\,R1 + \\
&8\,c1\,c2\,c3\,D3^3\,R1 + 24\,D1^2\,R1^2 - 8\,c1^2\,D1^2\,R1^2 - 40\,c2^2\,D1^2\,R1^2 + 16\,c1^2\,c2^2\,D1^2\,R1^2 + \\
&16\,c2^4\,D1^2\,R1^2 + 16\,c1\,c2\,c3\,D1^2\,R1^2 - 32\,c1\,c2^3\,c3\,D1^2\,R1^2 - 40\,c3^2\,D1^2\,R1^2 + 16\,c1^2\,c3^2\,D1^2\,R1^2 + \\
&48\,c2^2\,c3^2\,D1^2\,R1^2 - 32\,c1\,c2\,c3^3\,D1^2\,R1^2 + 16\,c3^4\,D1^2\,R1^2 - 48\,D1\,D2\,R1^2 + 48\,c1^2\,D1\,D2\,R1^2 + \\
&48\,c2^2\,D1\,D2\,R1^2 - 32\,c1\,c2\,c3\,D1\,D2\,R1^2 - 32\,c1^3\,c2\,c3\,D1\,D2\,R1^2 - 32\,c1\,c2^3\,c3\,D1\,D2\,R1^2 + \\
&80\,c3^2\,D1\,D2\,R1^2 - 64\,c1^2\,c3^2\,D1\,D2\,R1^2 - 64\,c2^2\,c3^2\,D1\,D2\,R1^2 + 64\,c1^2\,c2^2\,c3^2\,D1\,D2\,R1^2 + \\
&64\,c1\,c2\,c3^3\,D1\,D2\,R1^2 - 32\,c3^4\,D1\,D2\,R1^2 + 24\,D2^2\,R1^2 - 40\,c1^2\,D2^2\,R1^2 + 16\,c1^4\,D2^2\,R1^2 - \\
&8\,c2^2\,D2^2\,R1^2 + 16\,c1^2\,c2^2\,D2^2\,R1^2 + 16\,c1\,c2\,c3\,D2^2\,R1^2 - 32\,c1^3\,c2\,c3\,D2^2\,R1^2 - \\
&40\,c3^2\,D2^2\,R1^2 + 48\,c1^2\,c3^2\,D2^2\,R1^2 + 16\,c2^2\,c3^2\,D2^2\,R1^2 - 32\,c1\,c2\,c3^3\,D2^2\,R1^2 + \\
&16\,c3^4\,D2^2\,R1^2 - 48\,D1\,D3\,R1^2 + 48\,c1^2\,D1\,D3\,R1^2 + 80\,c2^2\,D1\,D3\,R1^2 - 64\,c1^2\,c2^2\,D1\,D3\,R1^2 - \\
&32\,c2^4\,D1\,D3\,R1^2 - 32\,c1\,c2\,c3\,D1\,D3\,R1^2 - 32\,c1^3\,c2\,c3\,D1\,D3\,R1^2 + 64\,c1\,c2^3\,c3\,D1\,D3\,R1^2 + \\
&48\,c3^2\,D1\,D3\,R1^2 - 64\,c2^2\,c3^2\,D1\,D3\,R1^2 + 64\,c1^2\,c2^2\,c3^2\,D1\,D3\,R1^2 - 32\,c1\,c2\,c3^3\,D1\,D3\,R1^2 + \\
&48\,D2\,D3\,R1^2 - 112\,c1^2\,D2\,D3\,R1^2 + 64\,c1^4\,D2\,D3\,R1^2 - 48\,c2^2\,D2\,D3\,R1^2 + 32\,c1^2\,c2^2\,D2\,D3\,R1^2 + \\
&160\,c1\,c2\,c3\,D2\,D3\,R1^2 - 128\,c1^3\,c2\,c3\,D2\,D3\,R1^2 - 32\,c1\,c2^3\,c3\,D2\,D3\,R1^2 - 48\,c3^2\,D2\,D3\,R1^2 + \\
&32\,c1^2\,c3^2\,D2\,D3\,R1^2 + 64\,c1^2\,c2^2\,c3^2\,D2\,D3\,R1^2 - 32\,c1\,c2\,c3^3\,D2\,D3\,R1^2 + 24\,D3^2\,R1^2 - \\
&40\,c1^2\,D3^2\,R1^2 + 16\,c1^4\,D3^2\,R1^2 - 40\,c2^2\,D3^2\,R1^2 + 48\,c1^2\,c2^2\,D3^2\,R1^2 + 16\,c2^4\,D3^2\,R1^2 + \\
&16\,c1\,c2\,c3\,D3^2\,R1^2 - 32\,c1^3\,c2\,c3\,D3^2\,R1^2 - 32\,c1\,c2^3\,c3\,D3^2\,R1^2 - 8\,c3^2\,D3^2\,R1^2 + \\
&16\,c1^2\,c3^2\,D3^2\,R1^2 + 16\,c2^2\,c3^2\,D3^2\,R1^2 + 32\,D1\,R1^3 - 32\,c1^2\,D1\,R1^3 - 64\,c2^2\,D1\,R1^3 + \\
&32\,c1^2\,c2^2\,D1\,R1^3 + 32\,c2^4\,D1\,R1^3 + 32\,c1\,c2\,c3\,D1\,R1^3 + 32\,c1^3\,c2\,c3\,D1\,R1^3 - 32\,c1\,c2^3\,c3\,D1\,R1^3 - \\
&64\,c3^2\,D1\,R1^3 + 32\,c1^2\,c3^2\,D1\,R1^3 + 128\,c2^2\,c3^2\,D1\,R1^3 - 128\,c1^2\,c2^2\,c3^2\,D1\,R1^3 - 64\,c2^4\,c3^2\,D1\,R1^3 - \\
&32\,c1\,c2\,c3^3\,D1\,R1^3 + 128\,c1\,c2^3\,c3^3\,D1\,R1^3 + 32\,c3^4\,D1\,R1^3 - 64\,c2^2\,c3^4\,D1\,R1^3 - 32\,D2\,R1^3 + \\
&64\,c1^2\,D2\,R1^3 - 32\,c1^4\,D2\,R1^3 + 32\,c2^2\,D2\,R1^3 - 32\,c1^2\,c2^2\,D2\,R1^3 - 96\,c1\,c2\,c3\,D2\,R1^3 + \\
&96\,c1^3\,c2\,c3\,D2\,R1^3 + 32\,c1\,c2^3\,c3\,D2\,R1^3 + 64\,c3^2\,D2\,R1^3 - 64\,c1^2\,c3^2\,D2\,R1^3 - 32\,c2^2\,c3^2\,D2\,R1^3 - \\
&64\,c1^2\,c2^2\,c3^2\,D2\,R1^3 + 96\,c1\,c2\,c3^3\,D2\,R1^3 - 32\,c3^4\,D2\,R1^3 - 32\,D3\,R1^3 + 64\,c1^2\,D3\,R1^3 - \\
&32\,c1^4\,D3\,R1^3 + 64\,c2^2\,D3\,R1^3 - 64\,c1^2\,c2^2\,D3\,R1^3 - 32\,c2^4\,D3\,R1^3 - 96\,c1\,c2\,c3\,D3\,R1^3 + \\
&96\,c1^3\,c2\,c3\,D3\,R1^3 + 96\,c1\,c2^3\,c3\,D3\,R1^3 + 32\,c3^2\,D3\,R1^3 - 32\,c1^2\,c3^2\,D3\,R1^3 - 32\,c2^2\,c3^2\,D3\,R1^3 - \\
&64\,c1^2\,c2^2\,c3^2\,D3\,R1^3 + 32\,c1\,c2\,c3^3\,D3\,R1^3 + 16\,R1^4 - 32\,c1^2\,R1^4 + 16\,c1^4\,R1^4 - 32\,c2^2\,R1^4 + \\
&32\,c1^2\,c2^2\,R1^4 + 16\,c2^4\,R1^4 + 64\,c1\,c2\,c3\,R1^4 - 64\,c1^3\,c2\,c3\,R1^4 - 64\,c1\,c2^3\,c3\,R1^4 - \\
&32\,c3^2\,R1^4 + 32\,c1^2\,c3^2\,R1^4 + 32\,c2^2\,c3^2\,R1^4 + 64\,c1^2\,c2^2\,c3^2\,R1^4 - 64\,c1\,c2\,c3^3\,R1^4 + 16\,c3^4\,R1^4
\end{aligned}
$$

In[351]:=
```
Factor[CoefficientList[p5, {c3, c2, c1}]]
```

Out[351]=
$$\Big\{\Big\{\big\{(D1-D2-D3+2\,R1)^4,\ 0,\ -8\,(D2-R1)\,(D3-R1)\,(D1-D2-D3+2\,R1)^2,\ 0,$$
$$16\,(D2-R1)^2\,(D3-R1)^2\big\},\ \{0,0,0,0,0\},\ \big\{-8\,R1\,(D1-D3+R1)\,(D1-D2-D3+2\,R1)^2,$$
$$0,\ -16\,(D3-R1)\,R1\,\left(D1^2+D2^2-2\,D1\,D3+D3^2+2\,D1\,R1-2\,D2\,R1-2\,D3\,R1+2\,R1^2\right),\ 0,\ 0\big\},$$
$$\{0,0,0,0,0\},\ \big\{16\,R1^2\,(D1-D3+R1)^2,\ 0,\ 0,\ 0,\ 0\big\}\Big\},\ \Big\{\{0,0,0,0,0\},$$
$$\big\{0,\ 8\,R1\,(D1-D2-D3+2\,R1)\,\left(D1^2-D2^2+6\,D2\,D3-D3^2-4\,D2\,R1-4\,D3\,R1+4\,R1^2\right),$$
$$0,\ 32\,(D1+D2+D3-2\,R1)\,(D2-R1)\,(D3-R1)\,R1,\ 0\big\},\ \{0,0,0,0,0\},$$
$$\big\{0,\ -32\,R1^2\,\left(D1^2+D1\,D2-2\,D1\,D3+D2\,D3+D3^2+D1\,R1-D2\,R1-3\,D3\,R1+2\,R1^2\right),\ 0,\ 0,\ 0\big\},$$
$$\{0,0,0,0,0\}\big\},\ \Big\{\big\{-8\,R1\,(D1-D2+R1)\,(D1-D2-D3+2\,R1)^2,\ 0,\ -16\,(D2-R1)\,R1$$
$$\left(D1^2-2\,D1\,D2+D2^2+D3^2+2\,D1\,R1-2\,D2\,R1-2\,D3\,R1+2\,R1^2\right),\ 0,\ 0\big\},\ \{0,0,0,0,0\},$$
$$\big\{16\,R1^2\,\left(3\,D1^2-4\,D1\,D2+D2^2-4\,D1\,D3+D3^2+8\,D1\,R1-2\,D2\,R1-2\,D3\,R1+2\,R1^2\right),$$
$$0,\ 64\,R1^2\,\left(D1\,D2+D1\,D3+D2\,D3-2\,D1\,R1-D2\,R1-D3\,R1+R1^2\right),\ 0,\ 0\big\},$$
$$\{0,0,0,0,0\},\ \big\{-64\,D1\,R1^3,\ 0,\ 0,\ 0,\ 0\big\}\Big\},\ \Big\{\{0,0,0,0,0\},$$
$$\big\{0,\ -32\,R1^2\,\left(D1^2-2\,D1\,D2+D2^2+D1\,D3+D2\,D3+D1\,R1-3\,D2\,R1-D3\,R1+2\,R1^2\right),\ 0,\ 0,\ 0\big\},$$
$$\{0,0,0,0,0\},\ \big\{0,\ 128\,D1\,R1^3,\ 0,\ 0,\ 0\big\},\ \{0,0,0,0,0\}\big\},$$
$$\Big\{\big\{16\,R1^2\,(D1-D2+R1)^2,\ 0,\ 0,\ 0,\ 0\big\},\ \{0,0,0,0,0\},$$
$$\big\{-64\,D1\,R1^3,\ 0,\ 0,\ 0,\ 0\big\},\ \{0,0,0,0,0\},\ \{0,0,0,0,0\}\big\}\Big\}$$

In[352]:=
```
quarticDiscriminant = Factor[Discriminant[p5, R1]]
```

Out[352]=
$$16\,777\,216\,\left(-1+c1^2+c2^2-2\,c1\,c2\,c3+c3^2\right)^2$$
$$\left(c1^2\,D1+c2^2\,D1-2\,c1\,c2\,c3\,D1-c1^2\,D2-c2^2\,D2+2\,c1\,c2\,c3\,D2+c1^2\,D3-c2^2\,D3\right)^2$$
$$\left(c1^2\,D1-2\,c1\,c2\,c3\,D1+c3^2\,D1+c1^2\,D2-c3^2\,D2-c1^2\,D3+2\,c1\,c2\,c3\,D3-c3^2\,D3\right)^2$$
$$\left(-c2^2\,D1+c3^2\,D1-c2^2\,D2+2\,c1\,c2\,c3\,D2-c3^2\,D2+c2^2\,D3-2\,c1\,c2\,c3\,D3+c3^2\,D3\right)^2$$
$$\Big(D1^6-2\,c2^2\,D1^6+c2^4\,D1^6-2\,c3^2\,D1^6+4\,c2^2\,c3^2\,D1^6-2\,c2^4\,c3^2\,D1^6+c3^4\,D1^6-2\,c2^2\,c3^4\,D1^6+$$
$$c2^4\,c3^4\,D1^6-2\,D1^5\,D2+4\,c2^2\,D1^5\,D2-2\,c2^4\,D1^5\,D2+4\,c1\,c2\,c3\,D1^5\,D2-4\,c1\,c2^3\,c3\,D1^5\,D2-$$
$$4\,c2^2\,c3^2\,D1^5\,D2+4\,c2^4\,c3^2\,D1^5\,D2-8\,c1\,c2\,c3^3\,D1^5\,D2+8\,c1\,c2^3\,c3^3\,D1^5\,D2+6\,c3^4\,D1^5\,D2-$$
$$4\,c2^2\,c3^4\,D1^5\,D2-2\,c2^4\,c3^4\,D1^5\,D2+4\,c1\,c2\,c3^5\,D1^5\,D2-4\,c1\,c2^3\,c3^5\,D1^5\,D2-4\,c3^6\,D1^5\,D2+$$
$$4\,c2^2\,c3^6\,D1^5\,D2-D1^4\,D2^2+2\,c1^2\,D1^4\,D2^2+2\,c1^2\,c2^2\,D1^4\,D2^2+c2^4\,D1^4\,D2^2-16\,c1\,c2\,c3\,D1^4\,D2^2+$$
$$8\,c1\,c2^3\,c3\,D1^4\,D2^2+18\,c3^2\,D1^4\,D2^2-8\,c1^2\,c3^2\,D1^4\,D2^2-12\,c2^2\,c3^2\,D1^4\,D2^2-2\,c2^4\,c3^2\,D1^4\,D2^2+$$
$$32\,c1\,c2\,c3^3\,D1^4\,D2^2-16\,c1\,c2^3\,c3^3\,D1^4\,D2^2-33\,c3^4\,D1^4\,D2^2+10\,c1^2\,c3^4\,D1^4\,D2^2+$$
$$24\,c2^2\,c3^4\,D1^4\,D2^2-6\,c1^2\,c2^2\,c3^4\,D1^4\,D2^2+c2^4\,c3^4\,D1^4\,D2^2-16\,c1\,c2\,c3^5\,D1^4\,D2^2+$$
$$8\,c1\,c2^3\,c3^5\,D1^4\,D2^2+16\,c3^6\,D1^4\,D2^2-4\,c1^2\,c3^6\,D1^4\,D2^2-12\,c2^2\,c3^6\,D1^4\,D2^2+$$
$$4\,c1^2\,c2^2\,c3^6\,D1^4\,D2^2+4\,D1^3\,D2^3-4\,c1^2\,D1^3\,D2^3-4\,c2^2\,D1^3\,D2^3-4\,c1^2\,c2^2\,D1^3\,D2^3+$$
$$24\,c1\,c2\,c3\,D1^3\,D2^3-4\,c1^3\,c2\,c3\,D1^3\,D2^3-4\,c1\,c2^3\,c3\,D1^3\,D2^3-32\,c3^2\,D1^3\,D2^3+$$
$$20\,c1^2\,c3^2\,D1^3\,D2^3+20\,c2^2\,c3^2\,D1^3\,D2^3-48\,c1\,c2\,c3^3\,D1^3\,D2^3+8\,c1^3\,c2\,c3^3\,D1^3\,D2^3+$$
$$8\,c1\,c2^3\,c3^3\,D1^3\,D2^3+52\,c3^4\,D1^3\,D2^3-28\,c1^2\,c3^4\,D1^3\,D2^3-28\,c2^2\,c3^4\,D1^3\,D2^3+$$
$$12\,c1^2\,c2^2\,c3^4\,D1^3\,D2^3+24\,c1\,c2\,c3^5\,D1^3\,D2^3-4\,c1^3\,c2\,c3^5\,D1^3\,D2^3-4\,c1\,c2^3\,c3^5\,D1^3\,D2^3-$$
$$24\,c3^6\,D1^3\,D2^3+12\,c1^2\,c3^6\,D1^3\,D2^3+12\,c2^2\,c3^6\,D1^3\,D2^3-8\,c1^2\,c2^2\,c3^6\,D1^3\,D2^3-D1^2\,D2^4+$$
$$c1^4\,D1^2\,D2^4+2\,c2^2\,D1^2\,D2^4+2\,c1^2\,c2^2\,D1^2\,D2^4-16\,c1\,c2\,c3\,D1^2\,D2^4+8\,c1^3\,c2\,c3\,D1^2\,D2^4+$$
$$18\,c3^2\,D1^2\,D2^4-12\,c1^2\,c3^2\,D1^2\,D2^4-2\,c1^4\,c3^2\,D1^2\,D2^4-8\,c2^2\,c3^2\,D1^2\,D2^4+32\,c1\,c2\,c3^3\,D1^2\,D2^4-$$
$$16\,c1^3\,c2\,c3^3\,D1^2\,D2^4-33\,c3^4\,D1^2\,D2^4+24\,c1^2\,c3^4\,D1^2\,D2^4+c1^4\,c3^4\,D1^2\,D2^4+$$

$$
\begin{aligned}
&10\,c2^2\,c3^4\,D1^2\,D2^4 - 6\,c1^2\,c2^2\,c3^4\,D1^2\,D2^4 - 16\,c1\,c2\,c3^5\,D1^2\,D2^4 + 8\,c1^3\,c2\,c3^5\,D1^2\,D2^4 + \\
&16\,c3^6\,D1^2\,D2^4 - 12\,c1^2\,c3^6\,D1^2\,D2^4 - 4\,c2^2\,c3^6\,D1^2\,D2^4 + 4\,c1^2\,c2^2\,c3^6\,D1^2\,D2^4 - 2\,D1\,D2^5 + \\
&4\,c1^2\,D1\,D2^5 - 2\,c1^4\,D1\,D2^5 + 4\,c1\,c2\,c3\,D1\,D2^5 - 4\,c1^3\,c2\,c3\,D1\,D2^5 - 4\,c1^2\,c3^2\,D1\,D2^5 + \\
&4\,c1^4\,c3^2\,D1\,D2^5 - 8\,c1\,c2\,c3^3\,D1\,D2^5 + 8\,c1^3\,c2\,c3^3\,D1\,D2^5 + 6\,c3^4\,D1\,D2^5 - 4\,c1^2\,c3^4\,D1\,D2^5 - \\
&2\,c1^4\,c3^4\,D1\,D2^5 + 4\,c1\,c2\,c3^5\,D1\,D2^5 - 4\,c1^3\,c2\,c3^5\,D1\,D2^5 - 4\,c3^6\,D1\,D2^5 + 4\,c1^2\,c3^6\,D1\,D2^5 + \\
&D2^6 - 2\,c1^2\,D2^6 + c1^4\,D2^6 - 2\,c3^2\,D2^6 + 4\,c1^2\,c3^2\,D2^6 - 2\,c1^4\,c3^2\,D2^6 + c3^4\,D2^6 - \\
&2\,c1^2\,c3^4\,D2^6 + c1^4\,c3^4\,D2^6 - 2\,D1^5\,D3 + 6\,c2^4\,D1^5\,D3 - 4\,c2^6\,D1^5\,D3 + 4\,c1\,c2\,c3\,D1^5\,D3 - \\
&8\,c1\,c2^3\,c3\,D1^5\,D3 + 4\,c1\,c2^5\,c3\,D1^5\,D3 + 4\,c3^2\,D1^5\,D3 - 4\,c2^2\,c3^2\,D1^5\,D3 - 4\,c2^4\,c3^2\,D1^5\,D3 + \\
&4\,c2^6\,c3^2\,D1^5\,D3 - 4\,c1\,c2\,c3^3\,D1^5\,D3 + 8\,c1\,c2^3\,c3^3\,D1^5\,D3 - 4\,c1\,c2^5\,c3^3\,D1^5\,D3 - \\
&2\,c3^4\,D1^5\,D3 + 4\,c2^2\,c3^4\,D1^5\,D3 - 2\,c2^4\,c3^4\,D1^5\,D3 + 6\,D1^4\,D2\,D3 - 8\,c1^2\,D1^4\,D2\,D3 - \\
&12\,c2^2\,D1^4\,D2\,D3 + 12\,c1^2\,c2^2\,D1^4\,D2\,D3 + 6\,c2^4\,D1^4\,D2\,D3 - 4\,c1^2\,c2^4\,D1^4\,D2\,D3 + \\
&8\,c1\,c2\,c3\,D1^4\,D2\,D3 - 28\,c1\,c2^3\,c3\,D1^4\,D2\,D3 + 20\,c1\,c2^5\,c3\,D1^4\,D2\,D3 - 12\,c3^2\,D1^4\,D2\,D3 + \\
&12\,c1^2\,c3^2\,D1^4\,D2\,D3 + 40\,c2^2\,c3^2\,D1^4\,D2\,D3 - 28\,c2^4\,c3^2\,D1^4\,D2\,D3 - 12\,c1^2\,c2^4\,c3^2\,D1^4\,D2\,D3 - \\
&28\,c1\,c2\,c3^3\,D1^4\,D2\,D3 + 48\,c1\,c2^3\,c3^3\,D1^4\,D2\,D3 - 20\,c1\,c2^5\,c3^3\,D1^4\,D2\,D3 + 6\,c3^4\,D1^4\,D2\,D3 - \\
&4\,c1^2\,c3^4\,D1^4\,D2\,D3 - 28\,c2^2\,c3^4\,D1^4\,D2\,D3 - 12\,c1^2\,c2^2\,c3^4\,D1^4\,D2\,D3 + 22\,c2^4\,c3^4\,D1^4\,D2\,D3 + \\
&16\,c1^2\,c2^4\,c3^4\,D1^4\,D2\,D3 + 20\,c1\,c2\,c3^5\,D1^4\,D2\,D3 - 20\,c1\,c2^3\,c3^5\,D1^4\,D2\,D3 - 4\,D1^3\,D2^2\,D3 + \\
&4\,c1^2\,D1^3\,D2^2\,D3 + 16\,c2^2\,D1^3\,D2^2\,D3 - 12\,c1^2\,c2^2\,D1^3\,D2^2\,D3 - 12\,c2^4\,D1^3\,D2^2\,D3 - \\
&8\,c1^2\,c2^4\,D1^3\,D2^2\,D3 - 12\,c1\,c2\,c3\,D1^3\,D2^2\,D3 - 8\,c1^3\,c2\,c3\,D1^3\,D2^2\,D3 + 44\,c1\,c2^3\,c3\,D1^3\,D2^2\,D3 + \\
&24\,c1^3\,c2^3\,c3\,D1^3\,D2^2\,D3 + 8\,c3^2\,D1^3\,D2^2\,D3 + 8\,c1^2\,c3^2\,D1^3\,D2^2\,D3 - 56\,c2^2\,c3^2\,D1^3\,D2^2\,D3 + \\
&32\,c2^4\,c3^2\,D1^3\,D2^2\,D3 - 24\,c1^2\,c2^4\,c3^2\,D1^3\,D2^2\,D3 + 32\,c1\,c2\,c3^3\,D1^3\,D2^2\,D3 - \\
&8\,c1^3\,c2\,c3^3\,D1^3\,D2^2\,D3 - 48\,c1\,c2^3\,c3^3\,D1^3\,D2^2\,D3 - 8\,c1^3\,c2^3\,c3^3\,D1^3\,D2^2\,D3 - \\
&4\,c3^4\,D1^3\,D2^2\,D3 - 12\,c1^2\,c3^4\,D1^3\,D2^2\,D3 + 40\,c2^2\,c3^4\,D1^3\,D2^2\,D3 + 12\,c1^2\,c2^2\,c3^4\,D1^3\,D2^2\,D3 - \\
&20\,c2^4\,c3^4\,D1^3\,D2^2\,D3 + 32\,c1^2\,c2^4\,c3^4\,D1^3\,D2^2\,D3 - 20\,c1\,c2\,c3^5\,D1^3\,D2^2\,D3 + \\
&16\,c1^3\,c2\,c3^5\,D1^3\,D2^2\,D3 + 4\,c1\,c2^3\,c3^5\,D1^3\,D2^2\,D3 - 16\,c1^3\,c2^3\,c3^5\,D1^3\,D2^2\,D3 - 4\,D1^2\,D2^3\,D3 + \\
&16\,c1^2\,D1^2\,D2^3\,D3 - 12\,c1^4\,D1^2\,D2^3\,D3 + 4\,c2^2\,D1^2\,D2^3\,D3 - 12\,c1^2\,c2^2\,D1^2\,D2^3\,D3 - \\
&8\,c1^4\,c2^2\,D1^2\,D2^3\,D3 - 12\,c1\,c2\,c3\,D1^2\,D2^3\,D3 + 44\,c1^3\,c2\,c3\,D1^2\,D2^3\,D3 - 8\,c1\,c2^3\,c3\,D1^2\,D2^3\,D3 + \\
&24\,c1^3\,c2^3\,c3\,D1^2\,D2^3\,D3 + 8\,c3^2\,D1^2\,D2^3\,D3 - 56\,c1^2\,c3^2\,D1^2\,D2^3\,D3 + 32\,c1^4\,c3^2\,D1^2\,D2^3\,D3 + \\
&8\,c2^2\,c3^2\,D1^2\,D2^3\,D3 - 24\,c1^4\,c2^2\,c3^2\,D1^2\,D2^3\,D3 + 32\,c1\,c2\,c3^3\,D1^2\,D2^3\,D3 - \\
&48\,c1^3\,c2\,c3^3\,D1^2\,D2^3\,D3 - 8\,c1\,c2^3\,c3^3\,D1^2\,D2^3\,D3 - 8\,c1^3\,c2^3\,c3^3\,D1^2\,D2^3\,D3 - \\
&4\,c3^4\,D1^2\,D2^3\,D3 + 40\,c1^2\,c3^4\,D1^2\,D2^3\,D3 - 20\,c1^4\,c3^4\,D1^2\,D2^3\,D3 - 12\,c2^2\,c3^4\,D1^2\,D2^3\,D3 + \\
&12\,c1^2\,c2^2\,c3^4\,D1^2\,D2^3\,D3 + 32\,c1^4\,c2^2\,c3^4\,D1^2\,D2^3\,D3 - 20\,c1\,c2\,c3^5\,D1^2\,D2^3\,D3 + \\
&4\,c1^3\,c2\,c3^5\,D1^2\,D2^3\,D3 + 16\,c1\,c2^3\,c3^5\,D1^2\,D2^3\,D3 - 16\,c1^3\,c2^3\,c3^5\,D1^2\,D2^3\,D3 + 6\,D1\,D2^4\,D3 - \\
&12\,c1^2\,D1\,D2^4\,D3 + 6\,c1^4\,D1\,D2^4\,D3 - 8\,c2^2\,D1\,D2^4\,D3 + 12\,c1^2\,c2^2\,D1\,D2^4\,D3 - 4\,c1^4\,c2^2\,D1\,D2^4\,D3 + \\
&8\,c1\,c2\,c3\,D1\,D2^4\,D3 - 28\,c1^3\,c2\,c3\,D1\,D2^4\,D3 + 20\,c1^5\,c2\,c3\,D1\,D2^4\,D3 - 12\,c3^2\,D1\,D2^4\,D3 + \\
&40\,c1^2\,c3^2\,D1\,D2^4\,D3 - 28\,c1^4\,c3^2\,D1\,D2^4\,D3 + 12\,c2^2\,c3^2\,D1\,D2^4\,D3 - 12\,c1^4\,c2^2\,c3^2\,D1\,D2^4\,D3 - \\
&28\,c1\,c2\,c3^3\,D1\,D2^4\,D3 + 48\,c1^3\,c2\,c3^3\,D1\,D2^4\,D3 - 20\,c1^5\,c2\,c3^3\,D1\,D2^4\,D3 + 6\,c3^4\,D1\,D2^4\,D3 - \\
&28\,c1^2\,c3^4\,D1\,D2^4\,D3 + 22\,c1^4\,c3^4\,D1\,D2^4\,D3 - 4\,c2^2\,c3^4\,D1\,D2^4\,D3 - 12\,c1^2\,c2^2\,c3^4\,D1\,D2^4\,D3 + \\
&16\,c1^4\,c2^2\,c3^4\,D1\,D2^4\,D3 + 20\,c1\,c2\,c3^5\,D1\,D2^4\,D3 - 20\,c1^3\,c2\,c3^5\,D1\,D2^4\,D3 - 2\,D2^5\,D3 + \\
&6\,c1^4\,D2^5\,D3 - 4\,c1^6\,D2^5\,D3 + 4\,c1\,c2\,c3\,D2^5\,D3 - 8\,c1^3\,c2\,c3\,D2^5\,D3 + 4\,c1^5\,c2\,c3\,D2^5\,D3 + \\
&4\,c3^2\,D2^5\,D3 - 4\,c1^2\,c3^2\,D2^5\,D3 - 4\,c1^4\,c3^2\,D2^5\,D3 + 4\,c1^6\,c3^2\,D2^5\,D3 - 4\,c1\,c2\,c3^3\,D2^5\,D3 + \\
&8\,c1^3\,c2\,c3^3\,D2^5\,D3 - 4\,c1^5\,c2\,c3^3\,D2^5\,D3 - 2\,c3^4\,D2^5\,D3 + 4\,c1^2\,c3^4\,D2^5\,D3 - 2\,c1^4\,c3^4\,D2^5\,D3 - \\
&D1^4\,D3^2 + 2\,c1^2\,D1^4\,D3^2 + 18\,c2^2\,D1^4\,D3^2 - 8\,c1^2\,c2^2\,D1^4\,D3^2 - 33\,c2^4\,D1^4\,D3^2 + 10\,c1^2\,c2^4\,D1^4\,D3^2 + \\
&16\,c2^6\,D1^4\,D3^2 - 4\,c1^2\,c2^6\,D1^4\,D3^2 - 16\,c1\,c2\,c3\,D1^4\,D3^2 + 32\,c1\,c2^3\,c3\,D1^4\,D3^2 - \\
&16\,c1\,c2^5\,c3\,D1^4\,D3^2 + 2\,c1^2\,c3^2\,D1^4\,D3^2 - 12\,c2^2\,c3^2\,D1^4\,D3^2 + 24\,c2^4\,c3^2\,D1^4\,D3^2 -
\end{aligned}
$$

$$6\,c1^2\,c2^4\,c3^2\,D1^4\,D3^2 - 12\,c2^6\,c3^2\,D1^4\,D3^2 + 4\,c1^2\,c2^6\,c3^2\,D1^4\,D3^2 + 8\,c1\,c2\,c3^3\,D1^4\,D3^2 -$$
$$16\,c1\,c2^3\,c3^3\,D1^4\,D3^2 + 8\,c1\,c2^5\,c3^3\,D1^4\,D3^2 + c3^4\,D1^4\,D3^2 - 2\,c2^2\,c3^4\,D1^4\,D3^2 + c2^4\,c3^4\,D1^4\,D3^2 -$$
$$4\,D1^3\,D2\,D3^2 + 4\,c1^2\,D1^3\,D2\,D3^2 + 8\,c2^2\,D1^3\,D2\,D3^2 + 8\,c1^2\,c2^2\,D1^3\,D2\,D3^2 - 4\,c2^4\,D1^3\,D2\,D3^2 -$$
$$12\,c1^2\,c2^4\,D1^3\,D2\,D3^2 - 12\,c1\,c2\,c3\,D1^3\,D2\,D3^2 - 8\,c1^3\,c2\,c3\,D1^3\,D2\,D3^2 + 32\,c1\,c2^3\,c3\,D1^3\,D2\,D3^2 -$$
$$8\,c1^3\,c2^3\,c3\,D1^3\,D2\,D3^2 - 20\,c1\,c2^5\,c3\,D1^3\,D2\,D3^2 + 16\,c1^3\,c2^5\,c3\,D1^3\,D2\,D3^2 + 16\,c3^2\,D1^3\,D2\,D3^2 -$$
$$12\,c1^2\,c3^2\,D1^3\,D2\,D3^2 - 56\,c2^2\,c3^2\,D1^3\,D2\,D3^2 + 40\,c2^4\,c3^2\,D1^3\,D2\,D3^2 + 12\,c1^2\,c2^4\,c3^2\,D1^3\,D2\,D3^2 +$$
$$44\,c1\,c2\,c3^3\,D1^3\,D2\,D3^2 + 24\,c1^3\,c2\,c3^3\,D1^3\,D2\,D3^2 - 48\,c1\,c2^3\,c3^3\,D1^3\,D2\,D3^2 -$$
$$8\,c1^3\,c2^3\,c3^3\,D1^3\,D2\,D3^2 + 4\,c1\,c2^5\,c3^3\,D1^3\,D2\,D3^2 - 16\,c1^3\,c2^5\,c3^3\,D1^3\,D2\,D3^2 -$$
$$12\,c3^4\,D1^3\,D2\,D3^2 - 8\,c1^2\,c3^4\,D1^3\,D2\,D3^2 + 32\,c2^2\,c3^4\,D1^3\,D2\,D3^2 - 24\,c1^2\,c2^2\,c3^4\,D1^3\,D2\,D3^2 -$$
$$20\,c2^4\,c3^4\,D1^3\,D2\,D3^2 + 32\,c1^2\,c2^4\,c3^4\,D1^3\,D2\,D3^2 + 10\,D1^2\,D2^2\,D3^2 - 32\,c1^2\,D1^2\,D2^2\,D3^2 +$$
$$22\,c1^4\,D1^2\,D2^2\,D3^2 - 32\,c2^2\,D1^2\,D2^2\,D3^2 + 78\,c1^2\,c2^2\,D1^2\,D2^2\,D3^2 - 22\,c1^4\,c2^2\,D1^2\,D2^2\,D3^2 +$$
$$22\,c2^4\,D1^2\,D2^2\,D3^2 - 22\,c1^2\,c2^4\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c2^4\,D1^2\,D2^2\,D3^2 + 56\,c1\,c2\,c3\,D1^2\,D2^2\,D3^2 -$$
$$104\,c1^3\,c2\,c3\,D1^2\,D2^2\,D3^2 - 104\,c1\,c2^3\,c3\,D1^2\,D2^2\,D3^2 + 88\,c1^3\,c2^3\,c3\,D1^2\,D2^2\,D3^2 -$$
$$32\,c3^2\,D1^2\,D2^2\,D3^2 + 78\,c1^2\,c3^2\,D1^2\,D2^2\,D3^2 - 22\,c1^4\,c3^2\,D1^2\,D2^2\,D3^2 + 78\,c2^2\,c3^2\,D1^2\,D2^2\,D3^2 -$$
$$84\,c1^2\,c2^2\,c3^2\,D1^2\,D2^2\,D3^2 + 54\,c1^4\,c2^2\,c3^2\,D1^2\,D2^2\,D3^2 - 22\,c2^4\,c3^2\,D1^2\,D2^2\,D3^2 +$$
$$54\,c1^2\,c2^4\,c3^2\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c2^4\,c3^2\,D1^2\,D2^2\,D3^2 - 104\,c1\,c2\,c3^3\,D1^2\,D2^2\,D3^2 +$$
$$88\,c1^3\,c2\,c3^3\,D1^2\,D2^2\,D3^2 + 88\,c1\,c2^3\,c3^3\,D1^2\,D2^2\,D3^2 - 136\,c1^3\,c2^3\,c3^3\,D1^2\,D2^2\,D3^2 +$$
$$22\,c3^4\,D1^2\,D2^2\,D3^2 - 22\,c1^2\,c3^4\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c3^4\,D1^2\,D2^2\,D3^2 - 22\,c2^2\,c3^4\,D1^2\,D2^2\,D3^2 +$$
$$54\,c1^2\,c2^2\,c3^4\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c2^2\,c3^4\,D1^2\,D2^2\,D3^2 - 8\,c2^4\,c3^4\,D1^2\,D2^2\,D3^2 -$$
$$8\,c1^2\,c2^4\,c3^4\,D1^2\,D2^2\,D3^2 + 16\,c1^4\,c2^4\,c3^4\,D1^2\,D2^2\,D3^2 - 4\,D1\,D2^3\,D3^2 + 8\,c1^2\,D1\,D2^3\,D3^2 -$$
$$4\,c1^4\,D1\,D2^3\,D3^2 + 4\,c2^2\,D1\,D2^3\,D3^2 + 8\,c1^2\,c2^2\,D1\,D2^3\,D3^2 - 12\,c1^4\,c2^2\,D1\,D2^3\,D3^2 -$$
$$12\,c1\,c2\,c3\,D1\,D2^3\,D3^2 + 32\,c1^3\,c2\,c3\,D1\,D2^3\,D3^2 - 20\,c1^5\,c2\,c3\,D1\,D2^3\,D3^2 -$$
$$8\,c1\,c2^3\,c3\,D1\,D2^3\,D3^2 - 8\,c1^3\,c2^3\,c3\,D1\,D2^3\,D3^2 + 16\,c1^5\,c2^3\,c3\,D1\,D2^3\,D3^2 + 16\,c3^2\,D1\,D2^3\,D3^2 -$$
$$56\,c1^2\,c3^2\,D1\,D2^3\,D3^2 + 40\,c1^4\,c3^2\,D1\,D2^3\,D3^2 - 12\,c2^2\,c3^2\,D1\,D2^3\,D3^2 + 12\,c1^4\,c2^2\,c3^2\,D1\,D2^3\,D3^2 +$$
$$44\,c1\,c2\,c3^3\,D1\,D2^3\,D3^2 - 48\,c1^3\,c2\,c3^3\,D1\,D2^3\,D3^2 + 4\,c1^5\,c2\,c3^3\,D1\,D2^3\,D3^2 +$$
$$24\,c1\,c2^3\,c3^3\,D1\,D2^3\,D3^2 - 8\,c1^3\,c2^3\,c3^3\,D1\,D2^3\,D3^2 - 16\,c1^5\,c2^3\,c3^3\,D1\,D2^3\,D3^2 -$$
$$12\,c3^4\,D1\,D2^3\,D3^2 + 32\,c1^2\,c3^4\,D1\,D2^3\,D3^2 - 20\,c1^4\,c3^4\,D1\,D2^3\,D3^2 - 8\,c2^2\,c3^4\,D1\,D2^3\,D3^2 -$$
$$24\,c1^2\,c2^2\,c3^4\,D1\,D2^3\,D3^2 + 32\,c1^4\,c2^2\,c3^4\,D1\,D2^3\,D3^2 - D2^4\,D3^2 + 18\,c1^2\,D2^4\,D3^2 -$$
$$33\,c1^4\,D2^4\,D3^2 + 16\,c1^6\,D2^4\,D3^2 + 2\,c2^2\,D2^4\,D3^2 - 8\,c1^2\,c2^2\,D2^4\,D3^2 + 10\,c1^4\,c2^2\,D2^4\,D3^2 -$$
$$4\,c1^6\,c2^2\,D2^4\,D3^2 - 16\,c1\,c2\,c3\,D2^4\,D3^2 + 32\,c1^3\,c2\,c3\,D2^4\,D3^2 - 16\,c1^5\,c2\,c3\,D2^4\,D3^2 -$$
$$12\,c1^2\,c3^2\,D2^4\,D3^2 + 24\,c1^4\,c3^2\,D2^4\,D3^2 - 12\,c1^6\,c3^2\,D2^4\,D3^2 + 2\,c2^2\,c3^2\,D2^4\,D3^2 -$$
$$6\,c1^4\,c2^2\,c3^2\,D2^4\,D3^2 + 4\,c1^6\,c2^2\,c3^2\,D2^4\,D3^2 + 8\,c1\,c2\,c3^3\,D2^4\,D3^2 - 16\,c1^3\,c2\,c3^3\,D2^4\,D3^2 +$$
$$8\,c1^5\,c2\,c3^3\,D2^4\,D3^2 + c3^4\,D2^4\,D3^2 - 2\,c1^2\,c3^4\,D2^4\,D3^2 + c1^4\,c3^4\,D2^4\,D3^2 + 4\,D1^3\,D3^3 -$$
$$4\,c1^2\,D1^3\,D3^3 - 32\,c2^2\,D1^3\,D3^3 + 20\,c1^2\,c2^2\,D1^3\,D3^3 + 52\,c2^4\,D1^3\,D3^3 - 28\,c1^2\,c2^4\,D1^3\,D3^3 -$$
$$24\,c2^6\,D1^3\,D3^3 + 12\,c1^2\,c2^6\,D1^3\,D3^3 + 24\,c1\,c2\,c3\,D1^3\,D3^3 - 4\,c1^3\,c2\,c3\,D1^3\,D3^3 -$$
$$48\,c1\,c2^3\,c3\,D1^3\,D3^3 + 8\,c1^3\,c2^3\,c3\,D1^3\,D3^3 + 24\,c1\,c2^5\,c3\,D1^3\,D3^3 - 4\,c1^3\,c2^5\,c3\,D1^3\,D3^3 -$$
$$4\,c3^2\,D1^3\,D3^3 - 4\,c1^2\,c3^2\,D1^3\,D3^3 + 20\,c2^2\,c3^2\,D1^3\,D3^3 - 28\,c2^4\,c3^2\,D1^3\,D3^3 +$$
$$12\,c1^2\,c2^4\,c3^2\,D1^3\,D3^3 + 12\,c2^6\,c3^2\,D1^3\,D3^3 - 8\,c1^2\,c2^6\,c3^2\,D1^3\,D3^3 - 4\,c1\,c2\,c3^3\,D1^3\,D3^3 +$$
$$8\,c1\,c2^3\,c3^3\,D1^3\,D3^3 - 4\,c1\,c2^5\,c3^3\,D1^3\,D3^3 - 4\,D1^2\,D2\,D3^3 + 16\,c1^2\,D1^2\,D2\,D3^3 -$$
$$12\,c1^4\,D1^2\,D2\,D3^3 + 8\,c2^2\,D1^2\,D2\,D3^3 - 56\,c1^2\,c2^2\,D1^2\,D2\,D3^3 + 32\,c1^4\,c2^2\,D1^2\,D2\,D3^3 -$$
$$4\,c2^4\,D1^2\,D2\,D3^3 + 40\,c1^2\,c2^4\,D1^2\,D2\,D3^3 - 20\,c1^4\,c2^4\,D1^2\,D2\,D3^3 - 12\,c1\,c2\,c3\,D1^2\,D2\,D3^3 +$$
$$44\,c1^3\,c2\,c3\,D1^2\,D2\,D3^3 + 32\,c1\,c2^3\,c3\,D1^2\,D2\,D3^3 - 48\,c1^3\,c2^3\,c3\,D1^2\,D2\,D3^3 -$$
$$20\,c1\,c2^5\,c3\,D1^2\,D2\,D3^3 + 4\,c1^3\,c2^5\,c3\,D1^2\,D2\,D3^3 + 4\,c3^2\,D1^2\,D2\,D3^3 - 12\,c1^2\,c3^2\,D1^2\,D2\,D3^3 -$$
$$8\,c1^4\,c3^2\,D1^2\,D2\,D3^3 + 8\,c2^2\,c3^2\,D1^2\,D2\,D3^3 - 24\,c1^4\,c2^2\,c3^2\,D1^2\,D2\,D3^3 - 12\,c2^4\,c3^2\,D1^2\,D2\,D3^3 +$$

$$12\,c1^2\,c2^4\,c3^2\,D1^2\,D2\,D3^3 + 32\,c1^4\,c2^4\,c3^2\,D1^2\,D2\,D3^3 - 8\,c1\,c2\,c3^3\,D1^2\,D2\,D3^3 +$$
$$24\,c1^3\,c2\,c3^3\,D1^2\,D2\,D3^3 - 8\,c1\,c2^3\,c3^3\,D1^2\,D2\,D3^3 - 8\,c1^3\,c2^3\,c3^3\,D1^2\,D2\,D3^3 +$$
$$16\,c1\,c2^5\,c3^3\,D1^2\,D2\,D3^3 - 16\,c1^3\,c2^5\,c3^3\,D1^2\,D2\,D3^3 - 4\,D1\,D2^2\,D3^3 + 8\,c1^2\,D1\,D2^2\,D3^3 -$$
$$4\,c1^4\,D1\,D2^2\,D3^3 + 16\,c2^2\,D1\,D2^2\,D3^3 - 56\,c1^2\,c2^2\,D1\,D2^2\,D3^3 + 40\,c1^4\,c2^2\,D1\,D2^2\,D3^3 -$$
$$12\,c2^4\,D1\,D2^2\,D3^3 + 32\,c1^2\,c2^4\,D1\,D2^2\,D3^3 - 20\,c1^4\,c2^4\,D1\,D2^2\,D3^3 - 12\,c1\,c2\,c3\,D1\,D2^2\,D3^3 +$$
$$32\,c1^3\,c2\,c3\,D1\,D2^2\,D3^3 - 20\,c1^5\,c2\,c3\,D1\,D2^2\,D3^3 + 44\,c1\,c2^3\,c3\,D1\,D2^2\,D3^3 -$$
$$48\,c1^3\,c2^3\,c3\,D1\,D2^2\,D3^3 + 4\,c1^5\,c2^3\,c3\,D1\,D2^2\,D3^3 + 4\,c3^2\,D1\,D2^2\,D3^3 + 8\,c1^2\,c3^2\,D1\,D2^2\,D3^3 -$$
$$12\,c1^4\,c3^2\,D1\,D2^2\,D3^3 - 12\,c2^2\,c3^2\,D1\,D2^2\,D3^3 + 12\,c1^4\,c2^2\,c3^2\,D1\,D2^2\,D3^3 - 8\,c2^4\,c3^2\,D1\,D2^2\,D3^3 -$$
$$24\,c1^2\,c2^4\,c3^2\,D1\,D2^2\,D3^3 + 32\,c1^4\,c2^4\,c3^2\,D1\,D2^2\,D3^3 - 8\,c1\,c2\,c3^3\,D1\,D2^2\,D3^3 -$$
$$8\,c1^3\,c2\,c3^3\,D1\,D2^2\,D3^3 + 16\,c1^5\,c2\,c3^3\,D1\,D2^2\,D3^3 + 24\,c1\,c2^3\,c3^3\,D1\,D2^2\,D3^3 -$$
$$8\,c1^3\,c2^3\,c3^3\,D1\,D2^2\,D3^3 - 16\,c1^5\,c2^3\,c3^3\,D1\,D2^2\,D3^3 + 4\,D2^3\,D3^3 - 32\,c1^2\,D2^3\,D3^3 + 52\,c1^4\,D2^3\,D3^3 -$$
$$24\,c1^6\,D2^3\,D3^3 - 4\,c2^2\,D2^3\,D3^3 + 20\,c1^2\,c2^2\,D2^3\,D3^3 - 28\,c1^4\,c2^2\,D2^3\,D3^3 + 12\,c1^6\,c2^2\,D2^3\,D3^3 +$$
$$24\,c1\,c2\,c3\,D2^3\,D3^3 - 48\,c1^3\,c2\,c3\,D2^3\,D3^3 + 24\,c1^5\,c2\,c3\,D2^3\,D3^3 - 4\,c1\,c2^3\,c3\,D2^3\,D3^3 +$$
$$8\,c1^3\,c2^3\,c3\,D2^3\,D3^3 - 4\,c1^5\,c2^3\,c3\,D2^3\,D3^3 - 4\,c3^2\,D2^3\,D3^3 + 20\,c1^2\,c3^2\,D2^3\,D3^3 -$$
$$28\,c1^4\,c3^2\,D2^3\,D3^3 + 12\,c1^6\,c3^2\,D2^3\,D3^3 - 4\,c2^2\,c3^2\,D2^3\,D3^3 + 12\,c1^4\,c2^2\,c3^2\,D2^3\,D3^3 -$$
$$8\,c1^6\,c2^2\,c3^2\,D2^3\,D3^3 - 4\,c1\,c2\,c3^3\,D2^3\,D3^3 + 8\,c1^3\,c2\,c3^3\,D2^3\,D3^3 - 4\,c1^5\,c2\,c3^3\,D2^3\,D3^3 -$$
$$D1^2\,D3^4 + c1^4\,D1^2\,D3^4 + 18\,c2^2\,D1^2\,D3^4 - 12\,c1^2\,c2^2\,D1^2\,D3^4 - 2\,c1^4\,c2^2\,D1^2\,D3^4 -$$
$$33\,c2^4\,D1^2\,D3^4 + 24\,c1^2\,c2^4\,D1^2\,D3^4 + c1^4\,c2^4\,D1^2\,D3^4 + 16\,c2^6\,D1^2\,D3^4 - 12\,c1^2\,c2^6\,D1^2\,D3^4 -$$
$$16\,c1\,c2\,c3\,D1^2\,D3^4 + 8\,c1^3\,c2\,c3\,D1^2\,D3^4 + 32\,c1\,c2^3\,c3\,D1^2\,D3^4 - 16\,c1^3\,c2^3\,c3\,D1^2\,D3^4 -$$
$$16\,c1\,c2^5\,c3\,D1^2\,D3^4 + 8\,c1^3\,c2^5\,c3\,D1^2\,D3^4 + 2\,c3^2\,D1^2\,D3^4 + 2\,c1^2\,c3^2\,D1^2\,D3^4 -$$
$$8\,c2^2\,c3^2\,D1^2\,D3^4 + 10\,c2^4\,c3^2\,D1^2\,D3^4 - 6\,c1^2\,c2^4\,c3^2\,D1^2\,D3^4 - 4\,c2^6\,c3^2\,D1^2\,D3^4 +$$
$$4\,c1^2\,c2^6\,c3^2\,D1^2\,D3^4 + 6\,D1\,D2\,D3^4 - 12\,c1^2\,D1\,D2\,D3^4 + 6\,c1^4\,D1\,D2\,D3^4 - 12\,c2^2\,D1\,D2\,D3^4 +$$
$$40\,c1^2\,c2^2\,D1\,D2\,D3^4 - 28\,c1^4\,c2^2\,D1\,D2\,D3^4 + 6\,c2^4\,D1\,D2\,D3^4 - 28\,c1^2\,c2^4\,D1\,D2\,D3^4 +$$
$$22\,c1^4\,c2^4\,D1\,D2\,D3^4 + 8\,c1\,c2\,c3\,D1\,D2\,D3^4 - 28\,c1^3\,c2\,c3\,D1\,D2\,D3^4 + 20\,c1^5\,c2\,c3\,D1\,D2\,D3^4 -$$
$$28\,c1\,c2^3\,c3\,D1\,D2\,D3^4 + 48\,c1^3\,c2^3\,c3\,D1\,D2\,D3^4 - 20\,c1^5\,c2^3\,c3\,D1\,D2\,D3^4 +$$
$$20\,c1\,c2^5\,c3\,D1\,D2\,D3^4 - 20\,c1^3\,c2^5\,c3\,D1\,D2\,D3^4 - 8\,c3^2\,D1\,D2\,D3^4 + 12\,c1^2\,c3^2\,D1\,D2\,D3^4 -$$
$$4\,c1^4\,c3^2\,D1\,D2\,D3^4 + 12\,c2^2\,c3^2\,D1\,D2\,D3^4 - 12\,c1^4\,c2^2\,c3^2\,D1\,D2\,D3^4 - 4\,c2^4\,c3^2\,D1\,D2\,D3^4 -$$
$$12\,c1^2\,c2^4\,c3^2\,D1\,D2\,D3^4 + 16\,c1^4\,c2^4\,c3^2\,D1\,D2\,D3^4 - D2^2\,D3^4 + 18\,c1^2\,D2^2\,D3^4 - 33\,c1^4\,D2^2\,D3^4 +$$
$$16\,c1^6\,D2^2\,D3^4 - 12\,c1^2\,c2^2\,D2^2\,D3^4 + 24\,c1^4\,c2^2\,D2^2\,D3^4 - 12\,c1^6\,c2^2\,D2^2\,D3^4 + c2^4\,D2^2\,D3^4 -$$
$$2\,c1^2\,c2^4\,D2^2\,D3^4 + c1^4\,c2^4\,D2^2\,D3^4 - 16\,c1\,c2\,c3\,D2^2\,D3^4 + 32\,c1^3\,c2\,c3\,D2^2\,D3^4 -$$
$$16\,c1^5\,c2\,c3\,D2^2\,D3^4 + 8\,c1\,c2^3\,c3\,D2^2\,D3^4 - 16\,c1^3\,c2^3\,c3\,D2^2\,D3^4 + 8\,c1^5\,c2^3\,c3\,D2^2\,D3^4 +$$
$$2\,c3^2\,D2^2\,D3^4 - 8\,c1^2\,c3^2\,D2^2\,D3^4 + 10\,c1^4\,c3^2\,D2^2\,D3^4 - 4\,c1^6\,c3^2\,D2^2\,D3^4 + 2\,c2^2\,c3^2\,D2^2\,D3^4 -$$
$$6\,c1^4\,c2^2\,c3^2\,D2^2\,D3^4 + 4\,c1^6\,c2^2\,c3^2\,D2^2\,D3^4 - 2\,D1\,D3^5 + 4\,c1^2\,D1\,D3^5 - 2\,c1^4\,D1\,D3^5 -$$
$$4\,c1^2\,c2^2\,D1\,D3^5 + 4\,c1^4\,c2^2\,D1\,D3^5 + 6\,c2^4\,D1\,D3^5 - 4\,c1^2\,c2^4\,D1\,D3^5 - 2\,c1^4\,c2^4\,D1\,D3^5 -$$
$$4\,c2^6\,D1\,D3^5 + 4\,c1^2\,c2^6\,D1\,D3^5 + 4\,c1\,c2\,c3\,D1\,D3^5 - 4\,c1^3\,c2\,c3\,D1\,D3^5 - 8\,c1\,c2^3\,c3\,D1\,D3^5 +$$
$$8\,c1^3\,c2^3\,c3\,D1\,D3^5 + 4\,c1\,c2^5\,c3\,D1\,D3^5 - 4\,c1^3\,c2^5\,c3\,D1\,D3^5 - 2\,D2\,D3^5 + 6\,c1^4\,D2\,D3^5 -$$
$$4\,c1^6\,D2\,D3^5 + 4\,c2^2\,D2\,D3^5 - 4\,c1^2\,c2^2\,D2\,D3^5 - 4\,c1^4\,c2^2\,D2\,D3^5 + 4\,c1^6\,c2^2\,D2\,D3^5 - 2\,c2^4\,D2\,D3^5 +$$
$$4\,c1^2\,c2^4\,D2\,D3^5 - 2\,c1^4\,c2^4\,D2\,D3^5 + 4\,c1\,c2\,c3\,D2\,D3^5 - 8\,c1^3\,c2\,c3\,D2\,D3^5 + 4\,c1^5\,c2\,c3\,D2\,D3^5 -$$
$$4\,c1\,c2^3\,c3\,D2\,D3^5 + 8\,c1^3\,c2^3\,c3\,D2\,D3^5 - 4\,c1^5\,c2^3\,c3\,D2\,D3^5 + D3^6 - 2\,c1^2\,D3^6 +$$
$$c1^4\,D3^6 - 2\,c2^2\,D3^6 + 4\,c1^2\,c2^2\,D3^6 - 2\,c1^4\,c2^2\,D3^6 + c2^4\,D3^6 - 2\,c1^2\,c2^4\,D3^6 + c1^4\,c2^4\,D3^6\Big)$$

In[353]:=
```
quarticDiscriminant0 =
 Factor[quarticDiscriminant / (16 777 216 (-1 + c1^2 + c2^2 - 2 c1 c2 c3 + c3^2)^2
     (c1^2 D1 + c2^2 D1 - 2 c1 c2 c3 D1 - c1^2 D2 - c2^2 D2 + 2 c1 c2 c3 D2 + c1^2 D3 - c2^2 D3)^2
     (c1^2 D1 - 2 c1 c2 c3 D1 + c3^2 D1 + c1^2 D2 - c3^2 D2 - c1^2 D3 + 2 c1 c2 c3 D3 - c3^2 D3)^2
     (-c2^2 D1 + c3^2 D1 - c2^2 D2 + 2 c1 c2 c3 D2 - c3^2 D2 + c2^2 D3 - 2 c1 c2 c3 D3 + c3^2 D3)^2))]
```

Out[353]=
```
D1^6 - 2 c2^2 D1^6 + c2^4 D1^6 - 2 c3^2 D1^6 + 4 c2^2 c3^2 D1^6 - 2 c2^4 c3^2 D1^6 + c3^4 D1^6 - 2 c2^2 c3^4 D1^6 +
 c2^4 c3^4 D1^6 - 2 D1^5 D2 + 4 c2^2 D1^5 D2 - 2 c2^4 D1^5 D2 + 4 c1 c2 c3 D1^5 D2 - 4 c1 c2^3 c3 D1^5 D2 -
 4 c2^2 c3^2 D1^5 D2 + 4 c2^4 c3^2 D1^5 D2 - 8 c1 c2 c3^3 D1^5 D2 + 8 c1 c2^3 c3^3 D1^5 D2 + 6 c3^4 D1^5 D2 -
 4 c2^2 c3^4 D1^5 D2 - 2 c2^4 c3^4 D1^5 D2 + 4 c1 c2 c3^5 D1^5 D2 - 4 c1 c2^3 c3^5 D1^5 D2 - 4 c3^6 D1^5 D2 +
 4 c2^2 c3^6 D1^5 D2 - D1^4 D2^2 + 2 c1^2 D1^4 D2^2 + 2 c1^2 c2^2 D1^4 D2^2 + c2^4 D1^4 D2^2 - 16 c1 c2 c3 D1^4 D2^2 +
 8 c1 c2^3 c3 D1^4 D2^2 + 18 c3^2 D1^4 D2^2 - 8 c1^2 c3^2 D1^4 D2^2 - 12 c2^2 c3^2 D1^4 D2^2 - 2 c2^4 c3^2 D1^4 D2^2 +
 32 c1 c2 c3^3 D1^4 D2^2 - 16 c1 c2^3 c3^3 D1^4 D2^2 - 33 c3^4 D1^4 D2^2 + 10 c1^2 c3^4 D1^4 D2^2 +
 24 c2^2 c3^4 D1^4 D2^2 - 6 c1^2 c2^2 c3^4 D1^4 D2^2 + c2^4 c3^4 D1^4 D2^2 - 16 c1 c2 c3^5 D1^4 D2^2 +
 8 c1 c2^3 c3^5 D1^4 D2^2 + 16 c3^6 D1^4 D2^2 - 4 c1^2 c3^6 D1^4 D2^2 - 12 c2^2 c3^6 D1^4 D2^2 +
 4 c1^2 c2^2 c3^6 D1^4 D2^2 + 4 D1^3 D2^3 - 4 c1^2 D1^3 D2^3 - 4 c2^2 D1^3 D2^3 - 4 c1^2 c2^2 D1^3 D2^3 +
 24 c1 c2 c3 D1^3 D2^3 - 4 c1^3 c2 c3 D1^3 D2^3 - 4 c1 c2^3 c3 D1^3 D2^3 - 32 c3^2 D1^3 D2^3 +
 20 c1^2 c3^2 D1^3 D2^3 + 20 c2^2 c3^2 D1^3 D2^3 - 48 c1 c2 c3^3 D1^3 D2^3 + 8 c1^3 c2 c3^3 D1^3 D2^3 +
 8 c1 c2^3 c3^3 D1^3 D2^3 + 52 c3^4 D1^3 D2^3 - 28 c1^2 c3^4 D1^3 D2^3 - 28 c2^2 c3^4 D1^3 D2^3 +
 12 c1^2 c2^2 c3^4 D1^3 D2^3 + 24 c1 c2 c3^5 D1^3 D2^3 - 4 c1^3 c2 c3^5 D1^3 D2^3 - 4 c1 c2^3 c3^5 D1^3 D2^3 -
 24 c3^6 D1^3 D2^3 + 12 c1^2 c3^6 D1^3 D2^3 + 12 c2^2 c3^6 D1^3 D2^3 - 8 c1^2 c2^2 c3^6 D1^3 D2^3 - D1^2 D2^4 +
 c1^4 D1^2 D2^4 + 2 c2^2 D1^2 D2^4 + 2 c1^2 c2^2 D1^2 D2^4 - 16 c1 c2 c3 D1^2 D2^4 + 8 c1^3 c2 c3 D1^2 D2^4 +
 18 c3^2 D1^2 D2^4 - 12 c1^2 c3^2 D1^2 D2^4 - 2 c1^4 c3^2 D1^2 D2^4 - 8 c2^2 c3^2 D1^2 D2^4 + 32 c1 c2 c3^3 D1^2 D2^4 -
 16 c1^3 c2 c3^3 D1^2 D2^4 - 33 c3^4 D1^2 D2^4 + 24 c1^2 c3^4 D1^2 D2^4 + c1^4 c3^4 D1^2 D2^4 +
 10 c2^2 c3^4 D1^2 D2^4 - 6 c1^2 c2^2 c3^4 D1^2 D2^4 - 16 c1 c2 c3^5 D1^2 D2^4 + 8 c1^3 c2 c3^5 D1^2 D2^4 +
 16 c3^6 D1^2 D2^4 - 12 c1^2 c3^6 D1^2 D2^4 - 4 c2^2 c3^6 D1^2 D2^4 + 4 c1^2 c2^2 c3^6 D1^2 D2^4 - 2 D1 D2^5 +
 4 c1^2 D1 D2^5 - 2 c1^4 D1 D2^5 + 4 c1 c2 c3 D1 D2^5 - 4 c1^3 c2 c3 D1 D2^5 - 4 c1^2 c3^2 D1 D2^5 +
 4 c1^4 c3^2 D1 D2^5 - 8 c1 c2 c3^3 D1 D2^5 + 8 c1^3 c2 c3^3 D1 D2^5 + 6 c3^4 D1 D2^5 - 4 c1^2 c3^4 D1 D2^5 -
 2 c1^4 c3^4 D1 D2^5 + 4 c1 c2 c3^5 D1 D2^5 - 4 c1^3 c2 c3^5 D1 D2^5 - 4 c3^6 D1 D2^5 + 4 c1^2 c3^6 D1 D2^5 +
 D2^6 - 2 c1^2 D2^6 + c1^4 D2^6 - 2 c3^2 D2^6 + 4 c1^2 c3^2 D2^6 - 2 c1^4 c3^2 D2^6 + c3^4 D2^6 - 2 c1^2 c3^4 D2^6 +
 c1^4 c3^4 D2^6 - 2 D1^5 D3 + 6 c2^4 D1^5 D3 - 4 c2^6 D1^5 D3 + 4 c1 c2 c3 D1^5 D3 - 8 c1 c2^3 c3 D1^5 D3 +
 4 c1 c2^5 c3 D1^5 D3 + 4 c3^2 D1^5 D3 - 4 c2^2 c3^2 D1^5 D3 - 4 c2^4 c3^2 D1^5 D3 + 4 c2^6 c3^2 D1^5 D3 -
 4 c1 c2 c3^3 D1^5 D3 + 8 c1 c2^3 c3^3 D1^5 D3 - 4 c1 c2^5 c3^3 D1^5 D3 - 2 c3^4 D1^5 D3 + 4 c2^2 c3^4 D1^5 D3 -
 2 c2^4 c3^4 D1^5 D3 + 6 D1^4 D2 D3 - 8 c1^2 D1^4 D2 D3 - 12 c2^2 D1^4 D2 D3 + 12 c1^2 c2^2 D1^4 D2 D3 +
 6 c2^4 D1^4 D2 D3 - 4 c1^2 c2^4 D1^4 D2 D3 + 8 c1 c2 c3 D1^4 D2 D3 - 28 c1 c2^3 c3 D1^4 D2 D3 +
 20 c1 c2^5 c3 D1^4 D2 D3 - 12 c3^2 D1^4 D2 D3 + 12 c1^2 c3^2 D1^4 D2 D3 + 40 c2^2 c3^2 D1^4 D2 D3 -
 28 c2^4 c3^2 D1^4 D2 D3 - 12 c1^2 c2^4 c3^2 D1^4 D2 D3 - 28 c1 c2 c3^3 D1^4 D2 D3 + 48 c1 c2^3 c3^3 D1^4 D2 D3 -
 20 c1 c2^5 c3^3 D1^4 D2 D3 + 6 c3^4 D1^4 D2 D3 - 4 c1^2 c3^4 D1^4 D2 D3 - 28 c2^2 c3^4 D1^4 D2 D3 -
 12 c1^2 c2^2 c3^4 D1^4 D2 D3 + 22 c2^4 c3^4 D1^4 D2 D3 + 16 c1^2 c2^4 c3^4 D1^4 D2 D3 + 20 c1 c2 c3^5 D1^4 D2 D3 -
 20 c1 c2^3 c3^5 D1^4 D2 D3 - 4 D1^3 D2^2 D3 + 4 c1^2 D1^3 D2^2 D3 + 16 c2^2 D1^3 D2^2 D3 -
 12 c1^2 c2^2 D1^3 D2^2 D3 - 12 c2^4 D1^3 D2^2 D3 - 8 c1^2 c2^4 D1^3 D2^2 D3 - 12 c1 c2 c3 D1^3 D2^2 D3 -
 8 c1^3 c2 c3 D1^3 D2^2 D3 + 44 c1 c2^3 c3 D1^3 D2^2 D3 + 24 c1^3 c2^3 c3 D1^3 D2^2 D3 + 8 c3^2 D1^3 D2^2 D3 +
 8 c1^2 c3^2 D1^3 D2^2 D3 - 56 c2^2 c3^2 D1^3 D2^2 D3 + 32 c2^4 c3^2 D1^3 D2^2 D3 - 24 c1^2 c2^4 c3^2 D1^3 D2^2 D3 +
```

$$
\begin{aligned}
&32\,c1\,c2\,c3^3\,D1^3\,D2^2\,D3 - 8\,c1^3\,c2\,c3^3\,D1^3\,D2^2\,D3 - 48\,c1\,c2^3\,c3^3\,D1^3\,D2^2\,D3 - \\
&8\,c1^3\,c2^3\,c3^3\,D1^3\,D2^2\,D3 - 4\,c3^4\,D1^3\,D2^2\,D3 - 12\,c1^2\,c3^4\,D1^3\,D2^2\,D3 + 40\,c2^2\,c3^4\,D1^3\,D2^2\,D3 + \\
&12\,c1^2\,c2^2\,c3^4\,D1^3\,D2^2\,D3 - 20\,c2^4\,c3^4\,D1^3\,D2^2\,D3 + 32\,c1^2\,c2^4\,c3^4\,D1^3\,D2^2\,D3 - \\
&20\,c1\,c2\,c3^5\,D1^3\,D2^2\,D3 + 16\,c1^3\,c2\,c3^5\,D1^3\,D2^2\,D3 + 4\,c1\,c2^3\,c3^5\,D1^3\,D2^2\,D3 - \\
&16\,c1^3\,c2^3\,c3^5\,D1^3\,D2^2\,D3 - 4\,D1^2\,D2^3\,D3 + 16\,c1^2\,D1^2\,D2^3\,D3 - 12\,c1^4\,D1^2\,D2^3\,D3 + 4\,c2^2\,D1^2\,D2^3\,D3 - \\
&12\,c1^2\,c2^2\,D1^2\,D2^3\,D3 - 8\,c1^4\,c2^2\,D1^2\,D2^3\,D3 - 12\,c1\,c2\,c3\,D1^2\,D2^3\,D3 + 44\,c1^3\,c2\,c3\,D1^2\,D2^3\,D3 - \\
&8\,c1\,c2^3\,c3\,D1^2\,D2^3\,D3 + 24\,c1^3\,c2^3\,c3\,D1^2\,D2^3\,D3 + 8\,c3^2\,D1^2\,D2^3\,D3 - 56\,c1^2\,c3^2\,D1^2\,D2^3\,D3 + \\
&32\,c1^4\,c3^2\,D1^2\,D2^3\,D3 + 8\,c2^2\,c3^2\,D1^2\,D2^3\,D3 - 24\,c1^4\,c2^2\,c3^2\,D1^2\,D2^3\,D3 + 32\,c1\,c2\,c3^3\,D1^2\,D2^3\,D3 - \\
&48\,c1^3\,c2\,c3^3\,D1^2\,D2^3\,D3 - 8\,c1\,c2^3\,c3^3\,D1^2\,D2^3\,D3 - 8\,c1^3\,c2^3\,c3^3\,D1^2\,D2^3\,D3 - \\
&4\,c3^4\,D1^2\,D2^3\,D3 + 40\,c1^2\,c3^4\,D1^2\,D2^3\,D3 - 20\,c1^4\,c3^4\,D1^2\,D2^3\,D3 - 12\,c2^2\,c3^4\,D1^2\,D2^3\,D3 + \\
&12\,c1^2\,c2^2\,c3^4\,D1^2\,D2^3\,D3 + 32\,c1^4\,c2^2\,c3^4\,D1^2\,D2^3\,D3 - 20\,c1\,c2\,c3^5\,D1^2\,D2^3\,D3 + \\
&4\,c1^3\,c2\,c3^5\,D1^2\,D2^3\,D3 + 16\,c1\,c2^3\,c3^5\,D1^2\,D2^3\,D3 - 16\,c1^3\,c2^3\,c3^5\,D1^2\,D2^3\,D3 + 6\,D1\,D2^4\,D3 - \\
&12\,c1^2\,D1\,D2^4\,D3 + 6\,c1^4\,D1\,D2^4\,D3 - 8\,c2^2\,D1\,D2^4\,D3 + 12\,c1^2\,c2^2\,D1\,D2^4\,D3 - 4\,c1^4\,c2^2\,D1\,D2^4\,D3 + \\
&8\,c1\,c2\,c3\,D1\,D2^4\,D3 - 28\,c1^3\,c2\,c3\,D1\,D2^4\,D3 + 20\,c1^5\,c2\,c3\,D1\,D2^4\,D3 - 12\,c3^2\,D1\,D2^4\,D3 + \\
&40\,c1^2\,c3^2\,D1\,D2^4\,D3 - 28\,c1^4\,c3^2\,D1\,D2^4\,D3 + 12\,c2^2\,c3^2\,D1\,D2^4\,D3 - 12\,c1^4\,c2^2\,c3^2\,D1\,D2^4\,D3 - \\
&28\,c1\,c2\,c3^3\,D1\,D2^4\,D3 + 48\,c1^3\,c2\,c3^3\,D1\,D2^4\,D3 - 20\,c1^5\,c2\,c3^3\,D1\,D2^4\,D3 + 6\,c3^4\,D1\,D2^4\,D3 - \\
&28\,c1^2\,c3^4\,D1\,D2^4\,D3 + 22\,c1^4\,c3^4\,D1\,D2^4\,D3 - 4\,c2^2\,c3^4\,D1\,D2^4\,D3 - 12\,c1^2\,c2^2\,c3^4\,D1\,D2^4\,D3 + \\
&16\,c1^4\,c2^2\,c3^4\,D1\,D2^4\,D3 + 20\,c1\,c2\,c3^5\,D1\,D2^4\,D3 - 20\,c1^3\,c2\,c3^5\,D1\,D2^4\,D3 - 2\,D2^5\,D3 + \\
&6\,c1^4\,D2^5\,D3 - 4\,c1^6\,D2^5\,D3 + 4\,c1\,c2\,c3\,D2^5\,D3 - 8\,c1^3\,c2\,c3\,D2^5\,D3 + 4\,c1^5\,c2\,c3\,D2^5\,D3 + \\
&4\,c3^2\,D2^5\,D3 - 4\,c1^2\,c3^2\,D2^5\,D3 - 4\,c1^4\,c3^2\,D2^5\,D3 + 4\,c1^6\,c3^2\,D2^5\,D3 - 4\,c1\,c2\,c3^3\,D2^5\,D3 + \\
&8\,c1^3\,c2\,c3^3\,D2^5\,D3 - 4\,c1^5\,c2\,c3^3\,D2^5\,D3 - 2\,c3^4\,D2^5\,D3 + 4\,c1^2\,c3^4\,D2^5\,D3 - 2\,c1^4\,c3^4\,D2^5\,D3 - \\
&D1^4\,D3^2 + 2\,c1^2\,D1^4\,D3^2 + 18\,c2^2\,D1^4\,D3^2 - 8\,c1^2\,c2^2\,D1^4\,D3^2 - 33\,c2^4\,D1^4\,D3^2 + 10\,c1^2\,c2^4\,D1^4\,D3^2 + \\
&16\,c2^6\,D1^4\,D3^2 - 4\,c1^2\,c2^6\,D1^4\,D3^2 - 16\,c1\,c2\,c3\,D1^4\,D3^2 + 32\,c1\,c2^3\,c3\,D1^4\,D3^2 - \\
&16\,c1\,c2^5\,c3\,D1^4\,D3^2 + 2\,c1^2\,c3^2\,D1^4\,D3^2 - 12\,c2^2\,c3^2\,D1^4\,D3^2 + 24\,c2^4\,c3^2\,D1^4\,D3^2 - \\
&6\,c1^2\,c2^4\,c3^2\,D1^4\,D3^2 - 12\,c2^6\,c3^2\,D1^4\,D3^2 + 4\,c1^2\,c2^6\,c3^2\,D1^4\,D3^2 + 8\,c1\,c2\,c3^3\,D1^4\,D3^2 - \\
&16\,c1\,c2^3\,c3^3\,D1^4\,D3^2 + 8\,c1\,c2^5\,c3^3\,D1^4\,D3^2 + c3^4\,D1^4\,D3^2 - 2\,c2^2\,c3^4\,D1^4\,D3^2 + c2^4\,c3^4\,D1^4\,D3^2 - \\
&4\,D1^3\,D2\,D3^2 + 4\,c1^2\,D1^3\,D2\,D3^2 + 8\,c2^2\,D1^3\,D2\,D3^2 + 8\,c1^2\,c2^2\,D1^3\,D2\,D3^2 - 4\,c2^4\,D1^3\,D2\,D3^2 - \\
&12\,c1^2\,c2^4\,D1^3\,D2\,D3^2 - 12\,c1\,c2\,c3\,D1^3\,D2\,D3^2 - 8\,c1^3\,c2\,c3\,D1^3\,D2\,D3^2 + 32\,c1\,c2^3\,c3\,D1^3\,D2\,D3^2 - \\
&8\,c1^3\,c2^3\,c3\,D1^3\,D2\,D3^2 - 20\,c1\,c2^5\,c3\,D1^3\,D2\,D3^2 + 16\,c1^3\,c2^5\,c3\,D1^3\,D2\,D3^2 + 16\,c3^2\,D1^3\,D2\,D3^2 - \\
&12\,c1^2\,c3^2\,D1^3\,D2\,D3^2 - 56\,c2^2\,c3^2\,D1^3\,D2\,D3^2 + 40\,c2^4\,c3^2\,D1^3\,D2\,D3^2 + 12\,c1^2\,c2^4\,c3^2\,D1^3\,D2\,D3^2 + \\
&44\,c1\,c2\,c3^3\,D1^3\,D2\,D3^2 + 24\,c1^3\,c2\,c3^3\,D1^3\,D2\,D3^2 - 48\,c1\,c2^3\,c3^3\,D1^3\,D2\,D3^2 - \\
&8\,c1^3\,c2^3\,c3^3\,D1^3\,D2\,D3^2 + 4\,c1\,c2^5\,c3^3\,D1^3\,D2\,D3^2 - 16\,c1^3\,c2^5\,c3^3\,D1^3\,D2\,D3^2 - \\
&12\,c3^4\,D1^3\,D2\,D3^2 - 8\,c1^2\,c3^4\,D1^3\,D2\,D3^2 + 32\,c2^2\,c3^4\,D1^3\,D2\,D3^2 - 24\,c1^2\,c2^2\,c3^4\,D1^3\,D2\,D3^2 - \\
&20\,c2^4\,c3^4\,D1^3\,D2\,D3^2 + 32\,c1^2\,c2^4\,c3^4\,D1^3\,D2\,D3^2 + 10\,D1^2\,D2^2\,D3^2 - 32\,c1^2\,D1^2\,D2^2\,D3^2 + \\
&22\,c1^4\,D1^2\,D2^2\,D3^2 - 32\,c2^2\,D1^2\,D2^2\,D3^2 + 78\,c1^2\,c2^2\,D1^2\,D2^2\,D3^2 - 22\,c1^4\,c2^2\,D1^2\,D2^2\,D3^2 + \\
&22\,c2^4\,D1^2\,D2^2\,D3^2 - 22\,c1^2\,c2^4\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c2^4\,D1^2\,D2^2\,D3^2 + 56\,c1\,c2\,c3\,D1^2\,D2^2\,D3^2 - \\
&104\,c1^3\,c2\,c3\,D1^2\,D2^2\,D3^2 - 104\,c1\,c2^3\,c3\,D1^2\,D2^2\,D3^2 + 88\,c1^3\,c2^3\,c3\,D1^2\,D2^2\,D3^2 - \\
&32\,c3^2\,D1^2\,D2^2\,D3^2 + 78\,c1^2\,c3^2\,D1^2\,D2^2\,D3^2 - 22\,c1^4\,c3^2\,D1^2\,D2^2\,D3^2 + 78\,c2^2\,c3^2\,D1^2\,D2^2\,D3^2 - \\
&84\,c1^2\,c2^2\,c3^2\,D1^2\,D2^2\,D3^2 + 54\,c1^4\,c2^2\,c3^2\,D1^2\,D2^2\,D3^2 - 22\,c2^4\,c3^2\,D1^2\,D2^2\,D3^2 + \\
&54\,c1^2\,c2^4\,c3^2\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c2^4\,c3^2\,D1^2\,D2^2\,D3^2 - 104\,c1\,c2\,c3^3\,D1^2\,D2^2\,D3^2 + \\
&88\,c1^3\,c2\,c3^3\,D1^2\,D2^2\,D3^2 + 88\,c1\,c2^3\,c3^3\,D1^2\,D2^2\,D3^2 - 136\,c1^3\,c2^3\,c3^3\,D1^2\,D2^2\,D3^2 + \\
&22\,c3^4\,D1^2\,D2^2\,D3^2 - 22\,c1^2\,c3^4\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c3^4\,D1^2\,D2^2\,D3^2 - 22\,c2^2\,c3^4\,D1^2\,D2^2\,D3^2 + \\
&54\,c1^2\,c2^2\,c3^4\,D1^2\,D2^2\,D3^2 - 8\,c1^4\,c2^2\,c3^4\,D1^2\,D2^2\,D3^2 - 8\,c2^4\,c3^4\,D1^2\,D2^2\,D3^2 - \\
&8\,c1^2\,c2^4\,c3^4\,D1^2\,D2^2\,D3^2 + 16\,c1^4\,c2^4\,c3^4\,D1^2\,D2^2\,D3^2 - 4\,D1\,D2^3\,D3^2 + 8\,c1^2\,D1\,D2^3\,D3^2 -
\end{aligned}
$$

$$4\,c1^4\,D1\,D2^3\,D3^2 + 4\,c2^2\,D1\,D2^3\,D3^2 + 8\,c1^2\,c2^2\,D1\,D2^3\,D3^2 - 12\,c1^4\,c2^2\,D1\,D2^3\,D3^2 -$$
$$12\,c1\,c2\,c3\,D1\,D2^3\,D3^2 + 32\,c1^3\,c2\,c3\,D1\,D2^3\,D3^2 - 20\,c1^5\,c2\,c3\,D1\,D2^3\,D3^2 -$$
$$8\,c1\,c2^3\,c3\,D1\,D2^3\,D3^2 - 8\,c1^3\,c2^3\,c3\,D1\,D2^3\,D3^2 + 16\,c1^5\,c2^3\,c3\,D1\,D2^3\,D3^2 + 16\,c3^2\,D1\,D2^3\,D3^2 -$$
$$56\,c1^2\,c3^2\,D1\,D2^3\,D3^2 + 40\,c1^4\,c3^2\,D1\,D2^3\,D3^2 - 12\,c2^2\,c3^2\,D1\,D2^3\,D3^2 + 12\,c1^4\,c2^2\,c3^2\,D1\,D2^3\,D3^2 +$$
$$44\,c1\,c2\,c3^3\,D1\,D2^3\,D3^2 - 48\,c1^3\,c2\,c3^3\,D1\,D2^3\,D3^2 + 4\,c1^5\,c2\,c3^3\,D1\,D2^3\,D3^2 +$$
$$24\,c1\,c2^3\,c3^3\,D1\,D2^3\,D3^2 - 8\,c1^3\,c2^3\,c3^3\,D1\,D2^3\,D3^2 - 16\,c1^5\,c2^3\,c3^3\,D1\,D2^3\,D3^2 -$$
$$12\,c3^4\,D1\,D2^3\,D3^2 + 32\,c1^2\,c3^4\,D1\,D2^3\,D3^2 - 20\,c1^4\,c3^4\,D1\,D2^3\,D3^2 - 8\,c2^2\,c3^4\,D1\,D2^3\,D3^2 -$$
$$24\,c1^2\,c2^2\,c3^4\,D1\,D2^3\,D3^2 + 32\,c1^4\,c2^2\,c3^4\,D1\,D2^3\,D3^2 - D2^4\,D3^2 + 18\,c1^2\,D2^4\,D3^2 - 33\,c1^4\,D2^4\,D3^2 +$$
$$16\,c1^6\,D2^4\,D3^2 + 2\,c2^2\,D2^4\,D3^2 - 8\,c1^2\,c2^2\,D2^4\,D3^2 + 10\,c1^4\,c2^2\,D2^4\,D3^2 - 4\,c1^6\,c2^2\,D2^4\,D3^2 -$$
$$16\,c1\,c2\,c3\,D2^4\,D3^2 + 32\,c1^3\,c2\,c3\,D2^4\,D3^2 - 16\,c1^5\,c2\,c3\,D2^4\,D3^2 - 12\,c1^2\,c3^2\,D2^4\,D3^2 +$$
$$24\,c1^4\,c3^2\,D2^4\,D3^2 - 12\,c1^6\,c3^2\,D2^4\,D3^2 + 2\,c2^2\,c3^2\,D2^4\,D3^2 - 6\,c1^4\,c2^2\,c3^2\,D2^4\,D3^2 +$$
$$4\,c1^6\,c2^2\,c3^2\,D2^4\,D3^2 + 8\,c1\,c2\,c3^3\,D2^4\,D3^2 - 16\,c1^3\,c2\,c3^3\,D2^4\,D3^2 + 8\,c1^5\,c2\,c3^3\,D2^4\,D3^2 +$$
$$c3^4\,D2^4\,D3^2 - 2\,c1^2\,c3^4\,D2^4\,D3^2 + c1^4\,c3^4\,D2^4\,D3^2 + 4\,D1^3\,D3^3 - 4\,c1^2\,D1^3\,D3^3 - 32\,c2^2\,D1^3\,D3^3 +$$
$$20\,c1^2\,c2^2\,D1^3\,D3^3 + 52\,c2^4\,D1^3\,D3^3 - 28\,c1^2\,c2^4\,D1^3\,D3^3 - 24\,c2^6\,D1^3\,D3^3 + 12\,c1^2\,c2^6\,D1^3\,D3^3 +$$
$$24\,c1\,c2\,c3\,D1^3\,D3^3 - 4\,c1^3\,c2\,c3\,D1^3\,D3^3 - 48\,c1\,c2^3\,c3\,D1^3\,D3^3 + 8\,c1^3\,c2^3\,c3\,D1^3\,D3^3 +$$
$$24\,c1\,c2^5\,c3\,D1^3\,D3^3 - 4\,c1^3\,c2^5\,c3\,D1^3\,D3^3 - 4\,c3^2\,D1^3\,D3^3 - 4\,c1^2\,c3^2\,D1^3\,D3^3 + 20\,c2^2\,c3^2\,D1^3\,D3^3 -$$
$$28\,c2^4\,c3^2\,D1^3\,D3^3 + 12\,c1^2\,c2^4\,c3^2\,D1^3\,D3^3 + 12\,c2^6\,c3^2\,D1^3\,D3^3 - 8\,c1^2\,c2^6\,c3^2\,D1^3\,D3^3 -$$
$$4\,c1\,c2\,c3^3\,D1^3\,D3^3 + 8\,c1\,c2^3\,c3^3\,D1^3\,D3^3 - 4\,c1\,c2^5\,c3^3\,D1^3\,D3^3 - 4\,D1^2\,D2\,D3^3 + 16\,c1^2\,D1^2\,D2\,D3^3 -$$
$$12\,c1^4\,D1^2\,D2\,D3^3 + 8\,c2^2\,D1^2\,D2\,D3^3 - 56\,c1^2\,c2^2\,D1^2\,D2\,D3^3 + 32\,c1^4\,c2^2\,D1^2\,D2\,D3^3 -$$
$$4\,c2^4\,D1^2\,D2\,D3^3 + 40\,c1^2\,c2^4\,D1^2\,D2\,D3^3 - 20\,c1^4\,c2^4\,D1^2\,D2\,D3^3 - 12\,c1\,c2\,c3\,D1^2\,D2\,D3^3 +$$
$$44\,c1^3\,c2\,c3\,D1^2\,D2\,D3^3 + 32\,c1\,c2^3\,c3\,D1^2\,D2\,D3^3 - 48\,c1^3\,c2^3\,c3\,D1^2\,D2\,D3^3 -$$
$$20\,c1\,c2^5\,c3\,D1^2\,D2\,D3^3 + 4\,c1^3\,c2^5\,c3\,D1^2\,D2\,D3^3 + 4\,c3^2\,D1^2\,D2\,D3^3 - 12\,c1^2\,c3^2\,D1^2\,D2\,D3^3 -$$
$$8\,c1^4\,c3^2\,D1^2\,D2\,D3^3 + 8\,c2^2\,c3^2\,D1^2\,D2\,D3^3 - 24\,c1^4\,c2^2\,c3^2\,D1^2\,D2\,D3^3 - 12\,c2^4\,c3^2\,D1^2\,D2\,D3^3 +$$
$$12\,c1^2\,c2^4\,c3^2\,D1^2\,D2\,D3^3 + 32\,c1^4\,c2^4\,c3^2\,D1^2\,D2\,D3^3 - 8\,c1\,c2\,c3^3\,D1^2\,D2\,D3^3 +$$
$$24\,c1^3\,c2\,c3^3\,D1^2\,D2\,D3^3 - 8\,c1\,c2^3\,c3^3\,D1^2\,D2\,D3^3 - 8\,c1^3\,c2^3\,c3^3\,D1^2\,D2\,D3^3 +$$
$$16\,c1\,c2^5\,c3^3\,D1^2\,D2\,D3^3 - 16\,c1^3\,c2^5\,c3^3\,D1^2\,D2\,D3^3 - 4\,D1\,D2^2\,D3^3 + 8\,c1^2\,D1\,D2^2\,D3^3 -$$
$$4\,c1^4\,D1\,D2^2\,D3^3 + 16\,c2^2\,D1\,D2^2\,D3^3 - 56\,c1^2\,c2^2\,D1\,D2^2\,D3^3 + 40\,c1^4\,c2^2\,D1\,D2^2\,D3^3 -$$
$$12\,c2^4\,D1\,D2^2\,D3^3 + 32\,c1^2\,c2^4\,D1\,D2^2\,D3^3 - 20\,c1^4\,c2^4\,D1\,D2^2\,D3^3 - 12\,c1\,c2\,c3\,D1\,D2^2\,D3^3 +$$
$$32\,c1^3\,c2\,c3\,D1\,D2^2\,D3^3 - 20\,c1^5\,c2\,c3\,D1\,D2^2\,D3^3 + 44\,c1\,c2^3\,c3\,D1\,D2^2\,D3^3 -$$
$$48\,c1^3\,c2^3\,c3\,D1\,D2^2\,D3^3 + 4\,c1^5\,c2^3\,c3\,D1\,D2^2\,D3^3 + 4\,c3^2\,D1\,D2^2\,D3^3 + 8\,c1^2\,c3^2\,D1\,D2^2\,D3^3 -$$
$$12\,c1^4\,c3^2\,D1\,D2^2\,D3^3 - 12\,c2^2\,c3^2\,D1\,D2^2\,D3^3 + 12\,c1^4\,c2^2\,c3^2\,D1\,D2^2\,D3^3 - 8\,c2^4\,c3^2\,D1\,D2^2\,D3^3 -$$
$$24\,c1^2\,c2^4\,c3^2\,D1\,D2^2\,D3^3 + 32\,c1^4\,c2^4\,c3^2\,D1\,D2^2\,D3^3 - 8\,c1\,c2\,c3^3\,D1\,D2^2\,D3^3 -$$
$$8\,c1^3\,c2\,c3^3\,D1\,D2^2\,D3^3 + 16\,c1^5\,c2\,c3^3\,D1\,D2^2\,D3^3 + 24\,c1\,c2^3\,c3^3\,D1\,D2^2\,D3^3 -$$
$$8\,c1^3\,c2^3\,c3^3\,D1\,D2^2\,D3^3 - 16\,c1^5\,c2^3\,c3^3\,D1\,D2^2\,D3^3 + 4\,D2^3\,D3^3 - 32\,c1^2\,D2^3\,D3^3 + 52\,c1^4\,D2^3\,D3^3 -$$
$$24\,c1^6\,D2^3\,D3^3 - 4\,c2^2\,D2^3\,D3^3 + 20\,c1^2\,c2^2\,D2^3\,D3^3 - 28\,c1^4\,c2^2\,D2^3\,D3^3 + 12\,c1^6\,c2^2\,D2^3\,D3^3 +$$
$$24\,c1\,c2\,c3\,D2^3\,D3^3 - 48\,c1^3\,c2\,c3\,D2^3\,D3^3 + 24\,c1^5\,c2\,c3\,D2^3\,D3^3 - 4\,c1\,c2^3\,c3\,D2^3\,D3^3 +$$
$$8\,c1^3\,c2^3\,c3\,D2^3\,D3^3 - 4\,c1^5\,c2^3\,c3\,D2^3\,D3^3 - 4\,c3^2\,D2^3\,D3^3 + 20\,c1^2\,c3^2\,D2^3\,D3^3 - 28\,c1^4\,c3^2\,D2^3\,D3^3 +$$
$$12\,c1^6\,c3^2\,D2^3\,D3^3 - 4\,c2^2\,c3^2\,D2^3\,D3^3 + 12\,c1^4\,c2^2\,c3^2\,D2^3\,D3^3 - 8\,c1^6\,c2^2\,c3^2\,D2^3\,D3^3 -$$
$$4\,c1\,c2\,c3^3\,D2^3\,D3^3 + 8\,c1^3\,c2\,c3^3\,D2^3\,D3^3 - 4\,c1^5\,c2\,c3^3\,D2^3\,D3^3 - D1^2\,D3^4 + c1^4\,D1^2\,D3^4 +$$
$$18\,c2^2\,D1^2\,D3^4 - 12\,c1^2\,c2^2\,D1^2\,D3^4 - 2\,c1^4\,c2^2\,D1^2\,D3^4 - 33\,c2^4\,D1^2\,D3^4 + 24\,c1^2\,c2^4\,D1^2\,D3^4 +$$
$$c1^4\,c2^4\,D1^2\,D3^4 + 16\,c2^6\,D1^2\,D3^4 - 12\,c1^2\,c2^6\,D1^2\,D3^4 - 16\,c1\,c2\,c3\,D1^2\,D3^4 + 8\,c1^3\,c2\,c3\,D1^2\,D3^4 +$$
$$32\,c1\,c2^3\,c3\,D1^2\,D3^4 - 16\,c1^3\,c2^3\,c3\,D1^2\,D3^4 - 16\,c1\,c2^5\,c3\,D1^2\,D3^4 + 8\,c1^3\,c2^5\,c3\,D1^2\,D3^4 +$$
$$2\,c3^2\,D1^2\,D3^4 + 2\,c1^2\,c3^2\,D1^2\,D3^4 - 8\,c2^2\,c3^2\,D1^2\,D3^4 + 10\,c2^4\,c3^2\,D1^2\,D3^4 - 6\,c1^2\,c2^4\,c3^2\,D1^2\,D3^4 -$$
$$4\,c2^6\,c3^2\,D1^2\,D3^4 + 4\,c1^2\,c2^6\,c3^2\,D1^2\,D3^4 + 6\,D1\,D2\,D3^4 - 12\,c1^2\,D1\,D2\,D3^4 + 6\,c1^4\,D1\,D2\,D3^4 -$$

$$12\,c2^2\,D1\,D2\,D3^4 + 40\,c1^2\,c2^2\,D1\,D2\,D3^4 - 28\,c1^4\,c2^2\,D1\,D2\,D3^4 + 6\,c2^4\,D1\,D2\,D3^4 - 28\,c1^2\,c2^4\,D1\,D2\,D3^4 + 22\,c1^4\,c2^4\,D1\,D2\,D3^4 + 8\,c1\,c2\,c3\,D1\,D2\,D3^4 - 28\,c1^3\,c2\,c3\,D1\,D2\,D3^4 + 20\,c1^5\,c2\,c3\,D1\,D2\,D3^4 - 28\,c1\,c2^3\,c3\,D1\,D2\,D3^4 + 48\,c1^3\,c2^3\,c3\,D1\,D2\,D3^4 - 20\,c1^5\,c2^3\,c3\,D1\,D2\,D3^4 + 20\,c1\,c2^5\,c3\,D1\,D2\,D3^4 - 20\,c1^3\,c2^5\,c3\,D1\,D2\,D3^4 - 8\,c3^2\,D1\,D2\,D3^4 + 12\,c1^2\,c3^2\,D1\,D2\,D3^4 - 4\,c1^4\,c3^2\,D1\,D2\,D3^4 + 12\,c2^2\,c3^2\,D1\,D2\,D3^4 - 12\,c1^4\,c2^2\,c3^2\,D1\,D2\,D3^4 - 4\,c2^4\,c3^2\,D1\,D2\,D3^4 - 12\,c1^2\,c2^4\,c3^2\,D1\,D2\,D3^4 + 16\,c1^4\,c2^4\,c3^2\,D1\,D2\,D3^4 - D2^2\,D3^4 + 18\,c1^2\,D2^2\,D3^4 - 33\,c1^4\,D2^2\,D3^4 + 16\,c1^6\,D2^2\,D3^4 - 12\,c1^2\,c2^2\,D2^2\,D3^4 + 24\,c1^4\,c2^2\,D2^2\,D3^4 - 12\,c1^6\,c2^2\,D2^2\,D3^4 + c2^4\,D2^2\,D3^4 - 2\,c1^2\,c2^4\,D2^2\,D3^4 + c1^4\,c2^4\,D2^2\,D3^4 - 16\,c1\,c2\,c3\,D2^2\,D3^4 + 32\,c1^3\,c2\,c3\,D2^2\,D3^4 - 16\,c1^5\,c2\,c3\,D2^2\,D3^4 + 8\,c1\,c2^3\,c3\,D2^2\,D3^4 - 16\,c1^3\,c2^3\,c3\,D2^2\,D3^4 + 8\,c1^5\,c2^3\,c3\,D2^2\,D3^4 + 2\,c3^2\,D2^2\,D3^4 - 8\,c1^2\,c3^2\,D2^2\,D3^4 + 10\,c1^4\,c3^2\,D2^2\,D3^4 - 4\,c1^6\,c3^2\,D2^2\,D3^4 + 2\,c2^2\,c3^2\,D2^2\,D3^4 - 6\,c1^4\,c2^2\,c3^2\,D2^2\,D3^4 + 4\,c1^6\,c2^2\,c3^2\,D2^2\,D3^4 - 2\,D1\,D3^5 + 4\,c1^2\,D1\,D3^5 - 2\,c1^4\,D1\,D3^5 - 4\,c1^2\,c2^2\,D1\,D3^5 + 4\,c1^4\,c2^2\,D1\,D3^5 + 6\,c2^4\,D1\,D3^5 - 4\,c1^2\,c2^4\,D1\,D3^5 - 2\,c1^4\,c2^4\,D1\,D3^5 - 4\,c2^6\,D1\,D3^5 + 4\,c1^2\,c2^6\,D1\,D3^5 + 4\,c1\,c2\,c3\,D1\,D3^5 - 4\,c1^3\,c2\,c3\,D1\,D3^5 - 8\,c1\,c2^3\,c3\,D1\,D3^5 + 8\,c1^3\,c2^3\,c3\,D1\,D3^5 + 4\,c1\,c2^5\,c3\,D1\,D3^5 - 4\,c1^3\,c2^5\,c3\,D1\,D3^5 - 2\,D2\,D3^5 + 6\,c1^4\,D2\,D3^5 - 4\,c1^6\,D2\,D3^5 + 4\,c2^2\,D2\,D3^5 - 4\,c1^2\,c2^2\,D2\,D3^5 - 4\,c1^4\,c2^2\,D2\,D3^5 + 4\,c1^6\,c2^2\,D2\,D3^5 - 2\,c2^4\,D2\,D3^5 + 4\,c1^2\,c2^4\,D2\,D3^5 - 2\,c1^4\,c2^4\,D2\,D3^5 + 4\,c1\,c2\,c3\,D2\,D3^5 - 8\,c1^3\,c2\,c3\,D2\,D3^5 + 4\,c1^5\,c2\,c3\,D2\,D3^5 - 4\,c1\,c2^3\,c3\,D2\,D3^5 + 8\,c1^3\,c2^3\,c3\,D2\,D3^5 - 4\,c1^5\,c2^3\,c3\,D2\,D3^5 + D3^6 - 2\,c1^2\,D3^6 + c1^4\,D3^6 - 2\,c2^2\,D3^6 + 4\,c1^2\,c2^2\,D3^6 - 2\,c1^4\,c2^2\,D3^6 + c2^4\,D3^6 - 2\,c1^2\,c2^4\,D3^6 + c1^4\,c2^4\,D3^6$$

In[354]:=
```
Factor[CoefficientList[quarticDiscriminant0, {c3, c2, c1}]]
```

Out[354]=
$$\Big\{\Big\{\Big\{(D1-D2-D3)^2\,(D1+D2-D3)^2\,(D1-D2+D3)^2,\ 0,\ 2\,(D1+D2-D3)\,(D1-D2+D3)\,\big(D1^2\,D2^2-2\,D1\,D2^3+D2^4-4\,D1^2\,D2\,D3+2\,D1\,D2^2\,D3+2\,D2^3\,D3+D1^2\,D3^2+2\,D1\,D2\,D3^2-6\,D2^2\,D3^2-2\,D1\,D3^3+2\,D2\,D3^3+D3^4\big),\ 0,\ (D2-D3)^2\,\big(D1^2\,D2^2-2\,D1\,D2^3+D2^4-10\,D1^2\,D2\,D3+2\,D1\,D2^2\,D3+8\,D2^3\,D3+D1^2\,D3^2+2\,D1\,D2\,D3^2-18\,D2^2\,D3^2-2\,D1\,D3^3+8\,D2\,D3^3+D3^4\big),\ 0,\ -4\,D2\,(D2-D3)^4\,D3\Big\},\ \{0,0,0,0,0,0,0\},\ \Big\{-2\,(D1-D2-D3)\,(D1+D2-D3)\,\big(D1^4-2\,D1^3\,D2+D1^2\,D2^2+2\,D1^3\,D3+2\,D1^2\,D2\,D3-4\,D1\,D2^2\,D3-6\,D1^2\,D3^2+2\,D1\,D2\,D3^2+D2^2\,D3^2+2\,D1\,D3^3-2\,D2\,D3^3+D3^4\big),\ 0,\ 2\,\big(D1^4\,D2^2-2\,D1^3\,D2^3+D1^2\,D2^4+6\,D1^4\,D2\,D3-6\,D1^3\,D2^2\,D3-6\,D1^2\,D2^3\,D3+6\,D1\,D2^4\,D3-4\,D1^4\,D3^2+4\,D1^3\,D2\,D3^2+39\,D1^2\,D2^2\,D3^2+4\,D1\,D2^3\,D3^2-4\,D2^4\,D3^2+10\,D1^3\,D3^3-28\,D1^2\,D2\,D3^3-28\,D1\,D2^2\,D3^3+10\,D2^3\,D3^3-6\,D1^2\,D3^4+20\,D1\,D2\,D3^4-6\,D2^2\,D3^4-2\,D1\,D3^5-2\,D2\,D3^5+2\,D3^6\big),\ 0,\ -2\,(D2-D3)\,D3\,\big(4\,D1^2\,D2^2+2\,D1\,D2^3+15\,D1^2\,D2\,D3+8\,D1\,D2^2\,D3-5\,D2^3\,D3-D1^2\,D3^2-12\,D1\,D2\,D3^2+9\,D2^2\,D3^2+2\,D1\,D3^3-3\,D2\,D3^3-D3^4\big),\ 0,\ -4\,D2\,(D2-D3)^3\,D3^2\Big\},\ \{0,0,0,0,0,0,0\},\ \Big\{(D1-D3)^2\,\big(D1^4-2\,D1^3\,D2+D1^2\,D2^2+8\,D1^3\,D3+2\,D1^2\,D2\,D3-10\,D1\,D2^2\,D3-18\,D1^2\,D3^2+2\,D1\,D2\,D3^2+D2^2\,D3^2+8\,D1\,D3^3-2\,D2\,D3^3+D3^4\big),\ 0,\ -2\,(D1-D3)\,D3\,\big(2\,D1^3\,D2+4\,D1^2\,D2^2-5\,D1^3\,D3+8\,D1^2\,D2\,D3+15\,D1\,D2^2\,D3+9\,D1^2\,D3^2-12\,D1\,D2\,D3^2-D2^2\,D3^2-3\,D1\,D3^3+2\,D2\,D3^3-D3^4\big),\ 0,\ -D3^2\,\big(8\,D1^2\,D2^2+20\,D1^2\,D2\,D3+20\,D1\,D2^2\,D3-D1^2\,D3^2-22\,D1\,D2\,D3^2-D2^2\,D3^2+2\,D1\,D3^3+2\,D2\,D3^3-D3^4\big),\ 0,\ 0\Big\},\ \{0,0,0,0,0,0,0\},\ \Big\{-4\,D1\,(D1-D3)^4\,D3,\ 0,\ -4\,D1\,(D1-D3)^3\,D3^2,\ 0,\ 0,\ 0,\ 0\Big\}\Big\},\ \Big\{\{0,0,0,0,0,0,0\},\ \Big\{0,\ 4\,\big(D1^5\,D2-4\,D1^4\,D2^2+6\,D1^3\,D2^3-4\,D1^2\,D2^4+D1\,D2^5+D1^5\,D3+2\,D1^4\,D2\,D3-$$

```
      3 D1^3 D2^2 D3 - 3 D1^2 D2^3 D3 + 2 D1 D2^4 D3 + D2^5 D3 - 4 D1^4 D3^2 - 3 D1^3 D2 D3^2 +
      14 D1^2 D2^2 D3^2 - 3 D1 D2^3 D3^2 - 4 D2^4 D3^2 + 6 D1^3 D3^3 - 3 D1^2 D2 D3^3 - 3 D1 D2^2 D3^3 +
      6 D2^3 D3^3 - 4 D1^2 D3^4 + 2 D1 D2 D3^4 - 4 D2^2 D3^4 + D1 D3^5 + D2 D3^5), 0,
   -4 (D1^3 D2^3 - 2 D1^2 D2^4 + D1 D2^5 + 2 D1^3 D2^2 D3 - 11 D1^2 D2^3 D3 + 7 D1 D2^4 D3 + 2 D2^5 D3 +
      2 D1^3 D2 D3^2 + 26 D1^2 D2^2 D3^2 - 8 D1 D2^3 D3^2 - 8 D2^4 D3^2 + D1^3 D3^3 - 11 D1^2 D2 D3^3 -
      8 D1 D2^2 D3^3 + 12 D2^3 D3^3 - 2 D1^2 D3^4 + 7 D1 D2 D3^4 - 8 D2^2 D3^4 + D1 D3^5 + 2 D2 D3^5), 0,
   4 D2 (D2 - D3)^2 D3 (5 D1 D2 + D2^2 + 5 D1 D3 - 2 D2 D3 + D3^2), 0}, {0, 0, 0, 0, 0, 0, 0},
  {0, -4 (D1^5 D2 - 2 D1^4 D2^2 + D1^3 D2^3 + 2 D1^5 D3 + 7 D1^4 D2 D3 - 11 D1^3 D2^2 D3 + 2 D1^2 D2^3 D3 -
      8 D1^4 D3^2 - 8 D1^3 D2 D3^2 + 26 D1^2 D2^2 D3^2 + 2 D1 D2^3 D3^2 + 12 D1^3 D3^3 - 8 D1^2 D2 D3^3 -
      11 D1 D2^2 D3^3 + D2^3 D3^3 - 8 D1^2 D3^4 + 7 D1 D2 D3^4 - 2 D2^2 D3^4 + 2 D1 D3^5 + D2 D3^5),
   0, 8 D3 (3 D1^3 D2^2 + 3 D1^2 D2^3 - D1^3 D2 D3 + 11 D1^2 D2^2 D3 - D1 D2^3 D3 + D1^3 D3^2 -
      6 D1^2 D2 D3^2 - 6 D1 D2^2 D3^2 + D2^3 D3^2 - 2 D1^2 D3^3 + 6 D1 D2 D3^3 - 2 D2^2 D3^3 + D1 D3^4 + D2 D3^4),
   0, 4 D2 (D2 - D3) D3^2 (4 D1 D2 + 5 D1 D3 - D2 D3 + D3^2), 0}, {0, 0, 0, 0, 0, 0, 0},
  {0, 4 D1 (D1 - D3)^2 D3 (D1^2 + 5 D1 D2 - 2 D1 D3 + 5 D2 D3 + D3^2), 0,
   4 D1 (D1 - D3) D3^2 (4 D1 D2 - D1 D3 + 5 D2 D3 + D3^2),
   0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}},
 {{-2 (D1 - D2 - D3) (D1 - D2 + D3) (D1^4 + 2 D1^3 D2 - 6 D1^2 D2^2 + 2 D1 D2^3 + D2^4 - 2 D1^3 D3 +
      2 D1^2 D2 D3 + 2 D1 D2^2 D3 - 2 D2^3 D3 + D1^2 D3^2 - 4 D1 D2 D3^2 + D2^2 D3^2), 0,
   -2 (4 D1^4 D2^2 - 10 D1^3 D2^3 + 6 D1^2 D2^4 + 2 D1 D2^5 - 2 D2^6 - 6 D1^4 D2 D3 - 4 D1^3 D2^2 D3 +
      28 D1^2 D2^3 D3 - 20 D1 D2^4 D3 + 2 D2^5 D3 - D1^4 D3^2 + 6 D1^3 D2 D3^2 -
      39 D1^2 D2^2 D3^2 + 28 D1 D2^3 D3^2 + 6 D2^4 D3^2 + 2 D1^3 D3^3 + 6 D1^2 D2 D3^3 -
      4 D1 D2^2 D3^3 - 10 D2^3 D3^3 - D1^2 D3^4 - 6 D1 D2 D3^4 + 4 D2^2 D3^4), 0,
   -2 D2 (D2 - D3) (D1^2 D2^2 - 2 D1 D2^3 + D2^4 - 15 D1^2 D2 D3 + 12 D1 D2^2 D3 + 3 D2^3 D3 -
      4 D1^2 D3^2 - 8 D1 D2 D3^2 - 9 D2^2 D3^2 - 2 D1 D3^3 + 5 D2 D3^3), 0, 4 D2^2 (D2 - D3)^3 D3},
  {0, 0, 0, 0, 0, 0, 0}, {2 (2 D1^6 - 2 D1^5 D2 - 6 D1^4 D2^2 + 10 D1^3 D2^3 - 4 D1^2 D2^4 -
      2 D1^5 D3 + 20 D1^4 D2 D3 - 28 D1^3 D2^2 D3 + 4 D1^2 D2^3 D3 + 6 D1 D2^4 D3 - 6 D1^4 D3^2 -
      28 D1^3 D2 D3^2 + 39 D1^2 D2^2 D3^2 - 6 D1 D2^3 D3^2 + D2^4 D3^2 + 10 D1^3 D3^3 + 4 D1^2 D2 D3^3 -
      6 D1 D2^2 D3^3 - 2 D2^3 D3^3 - 4 D1^2 D3^4 + 6 D1 D2 D3^4 + D2^2 D3^4), 0, -84 D1^2 D2^2 D3^2,
   0, -6 D2 D3 (4 D1^2 D2^2 + 2 D1 D2^3 - 9 D1^2 D2 D3 - 2 D1 D2^2 D3 + D2^3 D3 + 4 D1^2 D3^2 -
      2 D1 D2 D3^2 - 2 D2^2 D3^2 + 2 D1 D3^3 + D2 D3^3), 0, 4 D2^2 (D2 - D3)^2 D3^2},
  {0, 0, 0, 0, 0, 0, 0}, {-2 D1 (D1 - D3) (D1^4 - 2 D1^3 D2 + D1^2 D2^2 + 3 D1^3 D3 +
      12 D1^2 D2 D3 - 15 D1 D2^2 D3 - 9 D1^2 D3^2 - 8 D1 D2 D3^2 - 4 D2^2 D3^2 + 5 D1 D3^3 - 2 D2 D3^3),
   0, -6 D1 D3 (2 D1^3 D2 + 4 D1^2 D2^2 + D1^3 D3 - 2 D1^2 D2 D3 - 9 D1 D2^2 D3 -
      2 D1^2 D3^2 - 2 D1 D2 D3^2 + 4 D2^2 D3^2 + D1 D3^3 + 2 D2 D3^3), 0,
   -8 D1 D2 D3^2 (D1 D2 - 4 D1 D3 - 4 D2 D3 - 2 D3^2), 0, 0}, {0, 0, 0, 0, 0, 0, 0},
  {4 D1^2 (D1 - D3)^3 D3, 0, 4 D1^2 (D1 - D3)^2 D3^2, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0},
  {0, -4 (2 D1^5 D2 - 8 D1^4 D2^2 + 12 D1^3 D2^3 - 8 D1^2 D2^4 + 2 D1 D2^5 + D1^5 D3 + 7 D1^4 D2 D3 -
      8 D1^3 D2^2 D3 - 8 D1^2 D2^3 D3 + 7 D1 D2^4 D3 + D2^5 D3 - 2 D1^4 D3^2 - 11 D1^3 D2 D3^2 +
      26 D1^2 D2^2 D3^2 - 11 D1 D2^3 D3^2 - 2 D2^4 D3^2 + D1^3 D3^3 + 2 D1^2 D2 D3^3 + 2 D1 D2^2 D3^3 + D2^3 D3^3),
```

```
    0, 8 D2 (D1^3 D2^2 - 2 D1^2 D2^3 + D1 D2^4 - D1^3 D2 D3 - 6 D1^2 D2^2 D3 + 6 D1 D2^3 D3 + D2^4 D3 +
        3 D1^3 D3^2 + 11 D1^2 D2 D3^2 - 6 D1 D2^2 D3^2 - 2 D2^3 D3^2 + 3 D1^2 D3^3 - D1 D2 D3^3 + D2^2 D3^3),
    0, -4 D2^2 (D2 - D3) D3 (5 D1 D2 + D2^2 + 4 D1 D3 - D2 D3), 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 8 D1 (D1^4 D2 - 2 D1^3 D2^2 + D1^2 D2^3 + D1^4 D3 + 6 D1^3 D2 D3 - 6 D1^2 D2^2 D3 - D1 D2^3 D3 -
        2 D1^3 D3^2 - 6 D1^2 D2 D3^2 + 11 D1 D2^2 D3^2 + 3 D2^3 D3^2 + D1^2 D3^3 - D1 D2 D3^3 + 3 D2^2 D3^3),
    0, -8 D1 D2 D3 (D1^2 D2 + D1 D2^2 + D1^2 D3 + 17 D1 D2 D3 + D2^2 D3 + D1 D3^2 + D2 D3^2),
    0, -16 D1 D2^2 D3^2 (D2 + D3), 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, -4 D1^2 (D1 - D3) D3 (D1^2 + 5 D1 D2 - D1 D3 + 4 D2 D3), 0,
    -16 D1^2 D2 D3^2 (D1 + D3), 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}},
  {{(D1 - D2)^2 (D1^4 + 8 D1^3 D2 - 18 D1^2 D2^2 + 8 D1 D2^3 + D2^4 - 2 D1^3 D3 +
        2 D1^2 D2 D3 + 2 D1 D2^2 D3 - 2 D2^3 D3 + D1^2 D3^2 - 10 D1 D2 D3^2 + D2^2 D3^2),
    0, 2 (D1 - D2) D2 (5 D1^3 D2 - 9 D1^2 D2^2 + 3 D1 D2^3 + D2^4 - 2 D1^3 D3 - 8 D1^2 D2 D3 +
        12 D1 D2^2 D3 - 2 D2^3 D3 - 4 D1^2 D3^2 - 15 D1 D2 D3^2 + D2^2 D3^2),
    0, D2^2 (D1^2 D2^2 - 2 D1 D2^3 + D2^4 - 20 D1^2 D2 D3 + 22 D1 D2^2 D3 - 2 D2^3 D3 -
        8 D1^2 D3^2 - 20 D1 D2 D3^2 + D2^2 D3^2), 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {-2 D1 (D1 - D2) (D1^4 + 3 D1^3 D2 - 9 D1^2 D2^2 + 5 D1 D2^3 - 2 D1^3 D3 + 12 D1^2 D2 D3 -
        8 D1 D2^2 D3 - 2 D2^3 D3 + D1^2 D3^2 - 15 D1 D2 D3^2 - 4 D2^2 D3^2), 0,
    -6 D1 D2 (D1^3 D2 - 2 D1^2 D2^2 + D1 D2^3 + 2 D1^3 D3 - 2 D1^2 D2 D3 - 2 D1 D2^2 D3 +
        2 D2^3 D3 + 4 D1^2 D3^2 - 9 D1 D2 D3^2 + 4 D2^2 D3^2), 0,
    8 D1 D2^2 D3 (4 D1 D2 + 2 D2^2 - D1 D3 + 4 D2 D3), 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {D1^2 (D1^4 - 2 D1^3 D2 + D1^2 D2^2 - 2 D1^3 D3 + 22 D1^2 D2 D3 -
        20 D1 D2^2 D3 + D1^2 D3^2 - 20 D1 D2 D3^2 - 8 D2^2 D3^2), 0,
    8 D1^2 D2 D3 (2 D1^2 + 4 D1 D2 + 4 D1 D3 - D2 D3), 0, 16 D1^2 D2^2 D3^2, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0}, {0, 4 D1 (D1 - D2)^2 D2 (D1^2 - 2 D1 D2 + D2^2 + 5 D1 D3 + 5 D2 D3),
    0, -4 D1 (D1 - D2) D2^2 (D1 D2 - D2^2 - 4 D1 D3 - 5 D2 D3), 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, -4 D1^2 (D1 - D2) D2 (D1^2 - D1 D2 + 5 D1 D3 + 4 D2 D3),
    0, -16 D1^2 D2^2 (D1 + D2) D3, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}},
  {{-4 D1 (D1 - D2)^4 D2, 0, -4 D1 (D1 - D2)^3 D2^2, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {4 D1^2 (D1 - D2)^3 D2, 0, 4 D1^2 (D1 - D2)^2 D2^2, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}}}
```

In[355]:= **Factor[Factor[% /. DRSubs] //. tSubsSp]**

Out[355]=

{ ··· 1 ··· }

large output | show less | show more | show all | set size limit...

In[356]:=
```
quarticDiscriminant00 = Factor[UnCoefficientList[ % , {c3, c2, c1}]]
```

Out[356]=

$$-\frac{1}{(1+t2^2)^{12}\,(1+t3^2)^{12}}$$
$$16\,384\,(t2-t3)^2\,(\cdots 1 \cdots)^2\,(\cdots 1 \cdots)^2\,(-64\,t2^6+208\,c1^2\,t2^6-225\,c1^4\,t2^6+$$
$$81\,c1^6\,t2^6+\cdots 11\,163 \cdots+6\,c1^3\,c2\,c3^5\,t2^{16}\,t3^{16}+c3^6\,t2^{16}\,t3^{16}-c1^2\,c3^6\,t2^{16}\,t3^{16})$$

large output | show less | show more | show all | set size limit...

```
(* Now compute formula in theorem, and compare *)
```

In[357]:=
```
deltoid
```

Out[357]=
$$-27+8\,(1+R)\,\left(-3\,L^2+(1+R)^2\right)+18\,\left(L^2+(1+R)^2\right)+\left(L^2+(1+R)^2\right)^2$$

In[358]:=
```
Factor[Factor[Factor[ deltoid /. {L → LL, R → RR}] //. ySubs] //. ySubs]
```

Out[358]=

$$-\frac{1}{(-1+c1^2+c2^2-2\,c1\,c2\,c3+c3^2)^4}$$
$$16\,(-27+54\,c1^2-27\,c1^4-4\,c1^6+\cdots 7191 \cdots+32\,c2^2\,c3^4\,x2\,x3^4\,y2\,y3-16\,c1\,c2\,c3^5\,x2\,x3^4$$
$$y2\,y3+16\,c1\,c2^3\,c3^5\,x2\,x3^4\,y2\,y3+16\,c3^6\,x2\,x3^4\,y2\,y3-16\,c2^2\,c3^6\,x2\,x3^4\,y2\,y3)$$

large output | show less | show more | show all | set size limit...

In[359]:=
```
Factor[
 Factor[Factor[CoefficientList[Numerator[%], {c3, c2, c1}]] //. ySubs] //. ySubs]
```

Out[359]=

{{ ··· 1 ··· }, {{0, 0, 0, 0, 0, 0, 0, 0}, ··· 6 ···,
{0, −32 (−1 + x2) (1 + 2 x2)^3 (−x1 + x1 x2 − x3 + x2 x3 − y1 y2 − y2 y3), 0, 0, 0, 0, 0, 0}},
{ ··· 1 ··· }, { ··· 1 ··· }, { ··· 1 ··· }, { ··· 1 ··· }, { ··· 1 ··· }, {{0, 0, 0, 0, 0, 0, 0, 0},
{0, −32 (−1 + x3) (1 + 2 x3)^3 (−x1 − x2 + x1 x3 + x2 x3 − y1 y3 − y2 y3), 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}

large output | show less | show more | show all | set size limit...

In[360]:=
```
Factor[% //. tSubsSp]
```

Out[360]=
$$\left\{\left\{\left\{-\frac{4096\,(t2-t3)^2\,(1+t2\,t3)^2\,\left(-1+t2^2+2\,t2\,t3\right)^2\,\left(-2\,t2-t3+t2^2\,t3\right)^2\,\left(-1+2\,t2\,t3+t3^2\right)^2\,\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^8\,\left(1+t3^2\right)^8},\right.\right.\right.$$
$$0,\ \frac{2048\,(t2-t3)^2\,\langle\langle 3\rangle\rangle\,\left(\langle\langle 1\rangle\rangle\right)\,\left(\langle\langle 54\rangle\rangle+13\,t2^6\,t3^8\right)}{\left(1+t2^2\right)^8\,\left(1+t3^2\right)^8},\ 0,$$
$$-\frac{256\,\langle\langle 3\rangle\rangle\,\left(\langle\langle 1\rangle\rangle\right)}{\langle\langle 1\rangle\rangle^8\,\langle\langle 1\rangle\rangle^8},\ 0,\ \left.\left.\frac{256\,(t2-t3)^2\,(t2+t3)^4\,\left(3-t2^2-8\,t2\,t3-t3^2+3\,t2^2\,t3^2\right)^4}{\left(1+t2^2\right)^7\,\left(1+t3^2\right)^7},\ 0\right\},\right.$$
$$\left.\langle\langle 6\rangle\rangle,\ \{0,0,0,0,0,0,0,0\}\right\},\ \langle\langle 6\rangle\rangle,\ \left\{\langle\langle 1\rangle\rangle\right\}\right\}$$

large output | show less | show more | show all | set size limit...

In[361]:=
```
discriminantGrunert = Factor[UnCoefficientList[ % , {c3, c2, c1}]]
```

Out[361]=
$$\frac{256\left(-64\,t2^6+208\,c1^2\,t2^6-225\,c1^4\,t2^6+81\,c1^6\,t2^6+\langle\langle 11\,163\rangle\rangle+6\,c1^3\,c2\,c3^5\,t2^{16}\,t3^{16}+c3^6\,t2^{16}\,t3^{16}-c1^2\,c3^6\,t2^{16}\,t3^{16}\right)}{\left(1+t2^2\right)^8\,\left(1+t3^2\right)^8}$$

large output | show less | show more | show all | set size limit...

In[362]:=
```
quarticDiscriminant00
```

Out[362]=
$$-\frac{1}{\left(1+t2^2\right)^{12}\,\left(1+t3^2\right)^{12}}$$
$$16\,384\,(t2-t3)^2\,\left(\langle\langle 1\rangle\rangle\right)^2\,\left(\langle\langle 1\rangle\rangle\right)^2\,\left(-64\,t2^6+208\,c1^2\,t2^6-225\,c1^4\,t2^6+\right.$$
$$\left.81\,c1^6\,t2^6+\langle\langle 11\,163\rangle\rangle+6\,c1^3\,c2\,c3^5\,t2^{16}\,t3^{16}+c3^6\,t2^{16}\,t3^{16}-c1^2\,c3^6\,t2^{16}\,t3^{16}\right)$$

large output | show less | show more | show all | set size limit...

In[363]:=
```
Factor[%% / %]
```

Out[363]=
$$-\frac{\left(1+t2^2\right)^4\,\left(1+t3^2\right)^4}{64\,(t2-t3)^2\,\left(-2\,t2-t3+t2^2\,t3\right)^2\,\left(-t2-2\,t3+t2\,t3^2\right)^2}$$

```
(*
    Up to a scalar multiple, the quantity in Theorem 2 ("discriminantGrunert")
    equals the "big factor" in the
 discriminant of the quartic polynomial (in R1).
    The other factors vanish in the "flat case" (z = 0) and in the "altitude
    plane case," and these do NOT correspond to repeated solutions to the
    Grunert system, only to repeated roots of the quartic polynomial.
    Now, what about the degree of the discriminant to Gruenert's system?
*)
```

In[364]:=
```
{num, den} = {Numerator[discriminantGrunert], Denominator[discriminantGrunert]}
```

Out[364]=
```
{256 (-64 t2^6 + 208 c1^2 t2^6 - 225 c1^4 t2^6 + 81 c1^6 t2^6 + 208 c2^2 t2^6 -
      558 c1^2 c2^2 t2^6 + 459 c1^4 c2^2 t2^6 - 108 c1^6 c2^2 t2^6 - 225 c2^4 t2^6 + ··· 11 147 ··· +
      4 c1^3 c2^3 c3^3 t2^16 t3^16 + 8 c1^5 c2^3 c3^3 t2^16 t3^16 + 3 c2^2 c3^4 t2^16 t3^16 +
      9 c1^2 c2^2 c3^4 t2^16 t3^16 - 12 c1^4 c2^2 c3^4 t2^16 t3^16 - 6 c1 c2 c3^5 t2^16 t3^16 +
      6 c1^3 c2 c3^5 t2^16 t3^16 + c3^6 t2^16 t3^16 - c1^2 c3^6 t2^16 t3^16), (1 + ··· 1 ···)^8 (··· 1 ···)^8}
```
large output | show less | show more | show all | set size limit...

In[365]:=
```
coeffs = Factor[CoefficientList[num, {c3, c2, c1}]]
```

Out[365]=
```
{··· 1 ···}
```
large output | show less | show more | show all | set size limit...

In[366]:=
```
Length[coeffs]
```

Out[366]= 7

In[367]:=
```
Factor[coeffs[[5]][[5]][[5]] / den]
```

Out[367]=
$$-\frac{1024\,(t2-t3)^2\,\left(-2\,t2-t3+t2^2\,t3\right)^2\,\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\,\left(1+t3^2\right)^4}$$

In[368]:=
```
Factor[Factor[Δ^2 /. unXYΔSubs] //. tSubsSp]
```

Out[368]=
$$\frac{16\,(t2-t3)^2\,\left(-2\,t2-t3+t2^2\,t3\right)^2\,\left(-t2-2\,t3+t2\,t3^2\right)^2}{\left(1+t2^2\right)^4\,\left(1+t3^2\right)^4}$$

In[369]:=
```
Factor[%% / %]
```

Out[369]= -64

```
(*
    coeffs[[5]][[5]][[5]] is the coefficient of c1^4 c2^4 c3^4,
    which is a degree-12 monomial. No higher degree monomials are
    possible since discriminant is a fourth degree polynomial in
    polynomials that are linear in c1^2, c2^2, c3^2 and c1 c2 c3.
*)


(*  Lemma 15 Check.  *)
```

In[370]:= $\mu\nu\xi$Subs

Out[370]= $\left\{x \to \frac{\mu + XH\,\xi}{1+\xi},\ y \to \frac{\nu + YH\,\xi}{1+\xi},\ z \to \sqrt{Z},\ Z \to -\frac{\left(1 - x^2 - y^2\right)\,(1-\xi)}{2\,\xi}\right\}$

In[371]:= un$\mu\nu\xi$Subs

Out[371]= $\left\{\mu \to \frac{-2\,x + 2\,x^3 + XH - x^2\,XH + 2\,x\,y^2 - XH\,y^2 + 2\,x\,Z}{-1 + x^2 + y^2 + 2\,Z},\right.$

$\left.\nu \to \frac{-2\,y + 2\,x^2\,y + 2\,y^3 + YH - x^2\,YH - y^2\,YH + 2\,y\,Z}{-1 + x^2 + y^2 + 2\,Z},\ \xi \to \frac{-1 + x^2 + y^2}{-1 + x^2 + y^2 + 2\,Z}\right\}$

In[372]:= Factor[{x, y, Z} /. $\mu\nu\xi$Subs]

Out[372]= $\left\{\frac{\mu + XH\,\xi}{1+\xi},\ \frac{\nu + YH\,\xi}{1+\xi},\ -\frac{\left(-1 + x^2 + y^2\right)\,(-1+\xi)}{2\,\xi}\right\}$

In[373]:= Factor[{x, y, Z} //. $\mu\nu\xi$Subs]

Out[373]= $\left\{\frac{\mu + XH\,\xi}{1+\xi},\ \frac{\nu + YH\,\xi}{1+\xi},\ -\frac{(-1+\xi)\,\left(-1 + \mu^2 + \nu^2 - 2\,\xi + 2\,XH\,\mu\,\xi + 2\,YH\,\nu\,\xi - \xi^2 + XH^2\,\xi^2 + YH^2\,\xi^2\right)}{2\,\xi\,(1+\xi)^2}\right\}$

In[374]:= Factor[% /. un$\mu\nu\xi$Subs]

Out[374]= {x, y, Z}

In[375]:= Factor[{μ, ν, ξ} //. un$\mu\nu\xi$Subs]

Out[375]= $\left\{\frac{-2\,x + 2\,x^3 + XH - x^2\,XH + 2\,x\,y^2 - XH\,y^2 + 2\,x\,Z}{-1 + x^2 + y^2 + 2\,Z},\right.$

$\left.\frac{-2\,y + 2\,x^2\,y + 2\,y^3 + YH - x^2\,YH - y^2\,YH + 2\,y\,Z}{-1 + x^2 + y^2 + 2\,Z},\ \frac{-1 + x^2 + y^2}{-1 + x^2 + y^2 + 2\,Z}\right\}$

In[376]:= Factor[% //. $\mu\nu\xi$Subs]

Out[376]= {μ, ν, ξ}

(*

Corollary 2 Check.

*)

In[377]:= {LLL, RRR}

Out[377]= $\left\{\frac{\left(-1 + x^2 + y^2\right)\,(y + 2\,x\,y - XH\,y - YH - x\,YH) + 2\,(1+x)\,y\,z^2}{z^2},\right.$

$\left.\frac{-\left(-1 + x^2 + y^2\right)\,\left(-x + x^2 + XH - x\,XH - y^2 + y\,YH\right) - (-1 + x - y)\,(-1 + x + y)\,z^2}{z^2}\right\}$

In[378]:=
```
{LLLμνξ, RRRμνξ} = Factor[ % //. μνξSubs]
```

Out[378]=
$$\left\{\frac{2\left(-\nu-\mu\,\nu+\text{YH}\,\xi^2+\text{XH}\,\text{YH}\,\xi^2\right)}{(-1+\xi)\,(1+\xi)},\ -\frac{-1+2\,\mu-\mu^2+\nu^2+\xi^2-2\,\text{XH}\,\xi^2+\text{XH}^2\,\xi^2-\text{YH}^2\,\xi^2}{(-1+\xi)\,(1+\xi)}\right\}$$

```
(*
    Lemma 16 Check.
    TYPO IN PAPER - forgot to square denominator in proof in paper
*)
```

In[379]:=
```
Factor[ μ^2 + ν^2 - 1 //. unμνξSubs ]
```

Out[379]=
$$\frac{1}{\left(-1+x^2+y^2+2\,Z\right)^2}\left(-1+x^2+y^2\right)$$
$$\left(1-5\,x^2+4\,x^4+4\,x\,\text{XH}-4\,x^3\,\text{XH}-\text{XH}^2+x^2\,\text{XH}^2-5\,y^2+8\,x^2\,y^2-4\,x\,\text{XH}\,y^2+\text{XH}^2\,y^2+4\,y^4+4\,y\,\text{YH}-\right.$$
$$\left.4\,x^2\,y\,\text{YH}-4\,y^3\,\text{YH}-\text{YH}^2+x^2\,\text{YH}^2+y^2\,\text{YH}^2-4\,Z+8\,x^2\,Z-4\,x\,\text{XH}\,Z+8\,y^2\,Z-4\,y\,\text{YH}\,Z+4\,Z^2\right)$$

In[380]:=
```
Factor[
  (x^2 + y^2 - 1) (4 (x^2 + y^2 + Z)^2 + 4 (1 + XH x + YH y) (1 - x^2 - y^2 - Z) +
       (1 - XH^2 - YH^2) (1 - x^2 - y^2) - 4 ) / (x^2 + y^2 + 2 Z - 1)^2
]
```

Out[380]=
$$\frac{1}{\left(-1+x^2+y^2+2\,Z\right)^2}\left(-1+x^2+y^2\right)$$
$$\left(1-5\,x^2+4\,x^4+4\,x\,\text{XH}-4\,x^3\,\text{XH}-\text{XH}^2+x^2\,\text{XH}^2-5\,y^2+8\,x^2\,y^2-4\,x\,\text{XH}\,y^2+\text{XH}^2\,y^2+4\,y^4+4\,y\,\text{YH}-\right.$$
$$\left.4\,x^2\,y\,\text{YH}-4\,y^3\,\text{YH}-\text{YH}^2+x^2\,\text{YH}^2+y^2\,\text{YH}^2-4\,Z+8\,x^2\,Z-4\,x\,\text{XH}\,Z+8\,y^2\,Z-4\,y\,\text{YH}\,Z+4\,Z^2\right)$$

In[381]:=
```
Factor[%% - %]
```

Out[381]=
```
0
```

In[382]:=
```
long = (1 - 5 x^2 + 4 x^4 + 4 x XH - 4 x^3 XH - XH^2 + x^2 XH^2 - 5 y^2 + 8 x^2 y^2 - 4 x XH y^2 + XH^2 y^2 + 4 y^4 +
    4 y YH - 4 x^2 y YH - 4 y^3 YH - YH^2 + x^2 YH^2 + y^2 YH^2 - 4 Z + 8 x^2 Z - 4 x XH Z + 8 y^2 Z - 4 y YH Z + 4 Z^2)
```

Out[382]=
$$1-5\,x^2+4\,x^4+4\,x\,\text{XH}-4\,x^3\,\text{XH}-\text{XH}^2+x^2\,\text{XH}^2-5\,y^2+8\,x^2\,y^2-4\,x\,\text{XH}\,y^2+\text{XH}^2\,y^2+4\,y^4+4\,y\,\text{YH}-$$
$$4\,x^2\,y\,\text{YH}-4\,y^3\,\text{YH}-\text{YH}^2+x^2\,\text{YH}^2+y^2\,\text{YH}^2-4\,Z+8\,x^2\,Z-4\,x\,\text{XH}\,Z+8\,y^2\,Z-4\,y\,\text{YH}\,Z+4\,Z^2$$

In[383]:=
```
Factor[D[ long, Z]]
```

Out[383]=
$$4\left(-1+2\,x^2-x\,\text{XH}+2\,y^2-y\,\text{YH}+2\,Z\right)$$

In[384]:=
```
Factor[8 Z + 8 (x - XH / 4)^2 + 8 (y - YH / 4)^2 - (XH^2 + YH^2 + 8) / 2]
```

Out[384]=
$$4\left(-1+2\,x^2-x\,\text{XH}+2\,y^2-y\,\text{YH}+2\,Z\right)$$

In[385]:=
```
Factor[%% - %]
```

Out[385]=
```
0
```

In[386]:=
```
Factor[long /. Z → 4]
```

Out[386]=
$$49 + 27\,x^2 + 4\,x^4 - 12\,x\,XH - 4\,x^3\,XH - XH^2 + x^2\,XH^2 + 27\,y^2 + 8\,x^2\,y^2 - 4\,x\,XH\,y^2 + XH^2\,y^2 + 4\,y^4 - 12\,y\,YH - 4\,x^2\,y\,YH - 4\,y^3\,YH - YH^2 + x^2\,YH^2 + y^2\,YH^2$$

In[387]:=
```
Factor[Φ^2 - Ψ^2 - 4 * (XH^2 + YH^2 - 1) /. {
    Φ -> 2 * ((x - XH / 4) ^2 + (y - YH / 4) ^2) + (XH^2 + YH^2 + 54) / 8,
  Ψ -> XH * x + YH * y - (XH^2 + YH^2 + 3) / 4
  }]
```

Out[387]=
$$49 + 27\,x^2 + 4\,x^4 - 12\,x\,XH - 4\,x^3\,XH - XH^2 + x^2\,XH^2 + 27\,y^2 + 8\,x^2\,y^2 - 4\,x\,XH\,y^2 + XH^2\,y^2 + 4\,y^4 - 12\,y\,YH - 4\,x^2\,y\,YH - 4\,y^3\,YH - YH^2 + x^2\,YH^2 + y^2\,YH^2$$

In[388]:=
```
Factor[%% - %]
```

Out[388]=
$$0$$

```
(*
      Lemma 17 Check.
*)
```

In[389]:=
```
trigSubs = {
   Cos[4 ϕ] → Cos[2 ϕ] ^2 - Sin[2 ϕ] ^2,
   Sin[4 ϕ] → 2 Cos[2 ϕ] Sin[2 ϕ],
   Cos[2 ϕ] → Cos[ϕ] ^2 - Sin[ϕ] ^2,
   Sin[2 ϕ] → 2 Cos[ϕ] Sin[ϕ],
   Sin[ϕ]^2 → 1 - Cos[ϕ]^2,
   Sin[ϕ]^3 → Sin[ϕ] (1 - Cos[ϕ]^2),
   Sin[ϕ]^4 → (1 - Cos[ϕ]^2) ^2,
   Sin[ϕ]^5 → Sin[ϕ] (1 - Cos[ϕ]^2) ^2,
   Sin[ϕ]^6 → (1 - Cos[ϕ]^2) ^3
  }
```

Out[389]=
$$\{\text{Cos}[4\,\phi] \to \text{Cos}[2\,\phi]^2 - \text{Sin}[2\,\phi]^2,\ \text{Sin}[4\,\phi] \to 2\,\text{Cos}[2\,\phi]\,\text{Sin}[2\,\phi],\ \text{Cos}[2\,\phi] \to \text{Cos}[\phi]^2 - \text{Sin}[\phi]^2,\ \text{Sin}[2\,\phi] \to 2\,\text{Cos}[\phi]\,\text{Sin}[\phi],\ \text{Sin}[\phi]^2 \to 1 - \text{Cos}[\phi]^2,\ \text{Sin}[\phi]^3 \to (1 - \text{Cos}[\phi]^2)\,\text{Sin}[\phi],\ \text{Sin}[\phi]^4 \to (1 - \text{Cos}[\phi]^2)^2,\ \text{Sin}[\phi]^5 \to (1 - \text{Cos}[\phi]^2)^2\,\text{Sin}[\phi],\ \text{Sin}[\phi]^6 \to (1 - \text{Cos}[\phi]^2)^3\}$$

In[390]:=
```
Factor[
 Factor[

   ( Cos[2 ϕ] (2 (1 + μ) ν) + Sin[2 ϕ] (ν^2 - (μ - 1)^2) ) -

     ( 2 (Cos[ϕ] (μ + Cos[4 ϕ]) + Sin[ϕ] (ν + Sin[4 ϕ])) *
        (-Sin[ϕ] (μ + Cos[4 ϕ]) + Cos[ϕ] (ν + Sin[4 ϕ])) - 4 Cos[2 ϕ]^2 Sin[2 ϕ] )

     //. trigSubs] //. trigSubs]
```

Out[390]= 0

```
(*
      Lemma 18 Check.
*)
```

In[391]:=
```
LLL
```

Out[391]=
$$\frac{\left(-1+x^2+y^2\right)\ (y+2\,x\,y-XH\,y-YH-x\,YH)+2\ (1+x)\ y\ z^2}{z^2}$$

In[392]:=
```
RRR
```

Out[392]=
$$\frac{-\left(-1+x^2+y^2\right)\ \left(-x+x^2+XH-x\,XH-y^2+y\,YH\right)-(-1+x-y)\ (-1+x+y)\ z^2}{z^2}$$

In[393]:=
```
Factor[
 Factor[

   ( Cos[2 ϕ] LLLμνξ + Sin[2 ϕ] RRRμνξ ) -

     ( ( ( Cos[ϕ] (μ + Cos[4 ϕ]) + Sin[ϕ] (ν + Sin[4 ϕ]))
           (-Sin[ϕ] (μ + Cos[4 ϕ]) + Cos[ϕ] (ν + Sin[4 ϕ])) -
         ( Cos[ϕ] (XH + Cos[4 ϕ]) + Sin[ϕ] (YH + Sin[4 ϕ])) (-Sin[ϕ] (XH + Cos[4 ϕ]) +
           Cos[ϕ] (YH + Sin[4 ϕ])) ξ^2 ) (2 / (1 - ξ^2)) - 4 Cos[2 ϕ]^2 Sin[2 ϕ] )

     //. trigSubs] //. trigSubs]
```

Out[393]= 0

```
(*
      Lemma 22 Check.
*)
```

In[394]:=
```
μt = ( 2 μ0 + 2 (- Sin[2 ϕ] + μ0 (1 - Sin[2 ϕ]) + ν0 Cos[2 ϕ]) t +
     (- Cos[4 ϕ] - Sin[2 ϕ] + μ0 (1 - Sin[2 ϕ]) + ν0 Cos[2 ϕ]) t^2 ) / (2 (1 + t))
```

Out[394]=
$$\frac{1}{2\,(1+t)}\Big(2\,\mu 0+2\,t\,(\nu 0\,\text{Cos}[2\,\phi]+\mu 0\,(1-\text{Sin}[2\,\phi])-\text{Sin}[2\,\phi])+ t^2\,(\nu 0\,\text{Cos}[2\,\phi]-\text{Cos}[4\,\phi]+\mu 0\,(1-\text{Sin}[2\,\phi])-\text{Sin}[2\,\phi])\Big)$$

In[395]:=
```
νt = ( 2 ν0 + 2 (- Cos[2 ϕ] + μ0 Cos[2 ϕ] + ν0 (1 + Sin[2 ϕ])) t +
     (- Sin[4 ϕ] - Cos[2 ϕ] + μ0 Cos[2 ϕ] + ν0 (1 + Sin[2 ϕ])) t^2 ) / (2 (1 + t))
```

Out[395]=
$$\frac{1}{2\,(1+t)}\Big(2\,\nu 0+2\,t\,(-\text{Cos}[2\,\phi]+\mu 0\,\text{Cos}[2\,\phi]+\nu 0\,(1+\text{Sin}[2\,\phi]))+ t^2\,(-\text{Cos}[2\,\phi]+\mu 0\,\text{Cos}[2\,\phi]+\nu 0\,(1+\text{Sin}[2\,\phi])-\text{Sin}[4\,\phi])\Big)$$

In[396]:=
```
Ξt = ( 4 (μ0^2 + ν0^2 - 1) t +

     2 (-3 - 2 μ0 Sin[2 ϕ] + μ0^2 (3 - 2 Sin[2 ϕ]) -
        2 ν0 Cos[2 ϕ] + 4 μ0 ν0 Cos[2 ϕ] + ν0^2 (3 + 2 Sin[2 ϕ])) t^2 +

     4 ( μ0 (Sin[2 ϕ] - 1) (2 Sin[2 ϕ] + 1) + μ0^2 (1 - Sin[2 ϕ]) -
        ν0 Cos[2 ϕ] (2 Sin[2 ϕ] + 1) + 2 μ0 ν0 Cos[2 ϕ] + ν0^2 (1 + Sin[2 ϕ])) t^3 +

     ((1 - Sin[2 ϕ]) (1 + 2 Sin[2 ϕ])^2 -
        2 μ0 (1 - Sin[2 ϕ]) (1 + 2 Sin[2 ϕ]) + μ0^2 (1 - Sin[2 ϕ]) -
        2 ν0 Cos[2 ϕ] (1 + 2 Sin[2 ϕ]) + 2 μ0 ν0 Cos[2 ϕ] + ν0^2 (1 + Sin[2 ϕ])) t^4 ) /

   (2 κ (1 + t)^2)
```

Out[396]=
$$\frac{1}{2\,(1+t)^2\,\kappa}\Big(4\,t\,\left(-1+\mu 0^2+\nu 0^2\right)+4\,t^3\,\Big(2\,\mu 0\,\nu 0\,\text{Cos}[2\,\phi]+\mu 0^2\,(1-\text{Sin}[2\,\phi])+\nu 0^2\,(1+\text{Sin}[2\,\phi])- \nu 0\,\text{Cos}[2\,\phi]\,(1+2\,\text{Sin}[2\,\phi])+\mu 0\,(-1+\text{Sin}[2\,\phi])\,(1+2\,\text{Sin}[2\,\phi])\Big)+ t^4\,\Big(2\,\mu 0\,\nu 0\,\text{Cos}[2\,\phi]+\mu 0^2\,(1-\text{Sin}[2\,\phi])+\nu 0^2\,(1+\text{Sin}[2\,\phi])-2\,\nu 0\,\text{Cos}[2\,\phi]\,(1+2\,\text{Sin}[2\,\phi])- 2\,\mu 0\,(1-\text{Sin}[2\,\phi])\,(1+2\,\text{Sin}[2\,\phi])+(1-\text{Sin}[2\,\phi])\,(1+2\,\text{Sin}[2\,\phi])^2\Big)+ 2\,t^2\,\Big(-3-2\,\nu 0\,\text{Cos}[2\,\phi]+4\,\mu 0\,\nu 0\,\text{Cos}[2\,\phi]+\mu 0^2\,(3-2\,\text{Sin}[2\,\phi])- 2\,\mu 0\,\text{Sin}[2\,\phi]+\nu 0^2\,(3+2\,\text{Sin}[2\,\phi])\Big)\Big)$$

In[397]:=
```
Factor[Factor[
   2 (μt + 1) νt - (2 (μ0 + 1) ν0 + κ Cos[2 ϕ] Ξt)
     //. trigSubs] //. trigSubs]
```

Out[397]= 0

In[398]:=
```
Factor[Factor[
    νt^2 - (μt - 1)^2 - (ν0^2 - (μ0 - 1)^2 + κ Sin[2 ϕ] Ξt)
      //. trigSubs] //. trigSubs]
```

Out[398]= 0

```
(*
     Lemma 23 Check.
*)
```

In[399]:=
```
MatrixForm[ M = {{ν, -μ - 1}, {μ - 1, ν}} ]
```

Out[399]//MatrixForm=

$$\begin{pmatrix} \nu & -1-\mu \\ -1+\mu & \nu \end{pmatrix}$$

In[400]:=
```
MatrixForm[ v = {{ 2 (XH + 1) YH - 2 (μ + 1) ν }, {YH^2 - (XH - 1)^2 - ν^2 + (μ - 1)^2}}]
```

Out[400]//MatrixForm=

$$\begin{pmatrix} 2\,(1+XH)\,YH - 2\,(1+\mu)\,\nu \\ -(-1+XH)^2 + YH^2 + (-1+\mu)^2 - \nu^2 \end{pmatrix}$$

In[401]:=
```
M.v
```

Out[401]=
$$\left\{\left\{\nu\,(2\,(1+XH)\,YH - 2\,(1+\mu)\,\nu) + (-1-\mu)\,\left(-(-1+XH)^2 + YH^2 + (-1+\mu)^2 - \nu^2\right)\right\},\right.$$
$$\left.\left\{(-1+\mu)\,(2\,(1+XH)\,YH - 2\,(1+\mu)\,\nu) + \nu\,\left(-(-1+XH)^2 + YH^2 + (-1+\mu)^2 - \nu^2\right)\right\}\right\}$$

```
(*
    That equals  2(μ^2+ν^2-1)(1-Ξ)( μ̇ / Ξ̇ , ν̇ / Ξ̇ )^T  =
    2(μ^2+ν^2-1)(1-ξ^2)( μ̇ / 2ξξ̇ , ν̇ / 2ξξ̇ )^T ,
    so the following is ( μ̇ / ξ̇ , ν̇ / ξ̇ , 1 )^T .
*)
```

In[402]:=
```
MatrixForm[Append[ ξ M.v / ((μ^2 + ν^2 - 1) (1 - ξ^2)), {1} ]]
```

Out[402]//MatrixForm=

$$\begin{pmatrix} \frac{\left(\nu\,(2\,(1+XH)\,YH - 2\,(1+\mu)\,\nu) + (-1-\mu)\,\left(-(-1+XH)^2 + YH^2 + (-1+\mu)^2 - \nu^2\right)\right)\,\xi}{\left(-1+\mu^2+\nu^2\right)\,\left(1-\xi^2\right)} \\ \frac{\left((-1+\mu)\,(2\,(1+XH)\,YH - 2\,(1+\mu)\,\nu) + \nu\,\left(-(-1+XH)^2 + YH^2 + (-1+\mu)^2 - \nu^2\right)\right)\,\xi}{\left(-1+\mu^2+\nu^2\right)\,\left(1-\xi^2\right)} \\ 1 \end{pmatrix}$$

```
(*
    which is proportional to ( μ̇ , ν̇ , ξ̇ )^T and also to the following:
*)
```

In[403]:=
```
MatrixForm[ V = Factor[Append[ ξ M.v ,  {((μ^2 + ν^2 - 1) (1 - ξ^2))}]]]
```

Out[403]//MatrixForm=

$$\begin{pmatrix} -\left(2\,XH - XH^2 + YH^2 - 2\,\mu + 2\,XH\,\mu - XH^2\,\mu + YH^2\,\mu - \mu^2 + \mu^3 - 2\,YH\,\nu - 2\,XH\,YH\,\nu + \nu^2 + \mu\,\nu^2\right)\,\xi \\ -\left(2\,YH + 2\,XH\,YH - 2\,YH\,\mu - 2\,XH\,YH\,\mu - 2\,\nu - 2\,XH\,\nu + XH^2\,\nu - YH^2\,\nu + 2\,\mu\,\nu + \mu^2\,\nu + \nu^3\right)\,\xi \\ -\left(-1 + \mu^2 + \nu^2\right)\,(-1 + \xi)\,(1 + \xi) \end{pmatrix}$$

```
(* From paper: *)
```

In[404]:=
```
MatrixForm[
 { { ( ( (XH - 1) ^2 - YH^2) (μ + 1) + 2 (XH + 1) YH ν - (μ + 1) ((μ - 1) ^2 + ν^2) ) ξ } ,
   { ( 2 (XH + 1) YH (μ - 1) + (YH^2 - (XH - 1) ^2) ν  - ν ( ν^2 + (μ - 1) (μ + 3)) ) ξ } ,
  {(μ^2 + ν^2 - 1) (1 - ξ^2) } } ]
```

Out[404]//MatrixForm=

$$\begin{pmatrix} \left(\left((-1 + XH)^2 - YH^2\right)\,(1 + \mu) + 2\,(1 + XH)\,YH\,\nu - (1 + \mu)\,\left((-1 + \mu)^2 + \nu^2\right)\right)\,\xi \\ \left(2\,(1 + XH)\,YH\,(-1 + \mu) + \left(-(-1 + XH)^2 + YH^2\right)\,\nu - \nu\,\left((-1 + \mu)\,(3 + \mu) + \nu^2\right)\right)\,\xi \\ \left(-1 + \mu^2 + \nu^2\right)\,\left(1 - \xi^2\right) \end{pmatrix}$$

In[405]:=
```
Factor[V - %]
```

Out[405]= {{0}, {0}, {0}}

```
(*
      Lemma 24 Check.
*)
```

In[406]:=
```
Factor[{x, y, Z} //. μνξSubs]
```

Out[406]= $$\left\{\frac{\mu + XH\,\xi}{1 + \xi},\ \frac{\nu + YH\,\xi}{1 + \xi},\ -\frac{(-1 + \xi)\,\left(-1 + \mu^2 + \nu^2 - 2\,\xi + 2\,XH\,\mu\,\xi + 2\,YH\,\nu\,\xi - \xi^2 + XH^2\,\xi^2 + YH^2\,\xi^2\right)}{2\,\xi\,(1 + \xi)^2}\right\}$$

In[407]:=
```
J = Factor[Transpose[ { D[%, μ], D[%, ν], D[%, ξ] } ]]
```

Out[407]= $$\left\{\left\{\frac{1}{1 + \xi},\ 0,\ \frac{XH - \mu}{(1 + \xi)^2}\right\},\ \left\{0,\ \frac{1}{1 + \xi},\ \frac{YH - \nu}{(1 + \xi)^2}\right\},\ \left\{-\frac{(-1 + \xi)\,(\mu + XH\,\xi)}{\xi\,(1 + \xi)^2},\ -\frac{(-1 + \xi)\,(\nu + YH\,\xi)}{\xi\,(1 + \xi)^2},\right.\right.$$
$$-\frac{1}{2\,\xi^2\,(1 + \xi)^3}\left(-1 + \mu^2 + \nu^2 - 3\,\xi + 3\,\mu^2\,\xi + 3\,\nu^2\,\xi - 3\,\xi^2 - XH^2\,\xi^2 - YH^2\,\xi^2 + 6\,XH\,\mu\,\xi^2 -\right.$$
$$\left.\left.\left.2\,\mu^2\,\xi^2 + 6\,YH\,\nu\,\xi^2 - 2\,\nu^2\,\xi^2 - \xi^3 + 3\,XH^2\,\xi^3 + 3\,YH^2\,\xi^3 - 2\,XH\,\mu\,\xi^3 - 2\,YH\,\nu\,\xi^3\right)\right\}\right\}$$

In[408]:=
```
V
```

Out[408]= $$\left\{\left\{-\left(2\,XH - XH^2 + YH^2 - 2\,\mu + 2\,XH\,\mu - XH^2\,\mu + YH^2\,\mu - \mu^2 + \mu^3 - 2\,YH\,\nu - 2\,XH\,YH\,\nu + \nu^2 + \mu\,\nu^2\right)\,\xi\right\},\right.$$
$$\left\{-\left(2\,YH + 2\,XH\,YH - 2\,YH\,\mu - 2\,XH\,YH\,\mu - 2\,\nu - 2\,XH\,\nu + XH^2\,\nu - YH^2\,\nu + 2\,\mu\,\nu + \mu^2\,\nu + \nu^3\right)\,\xi\right\},$$
$$\left.\left\{-\left(-1 + \mu^2 + \nu^2\right)\,(-1 + \xi)\,(1 + \xi)\right\}\right\}$$

In[409]:=
```
Factor[ J.V ]
```

Out[409]=
$$\left\{\left\{\frac{1}{1+\xi}\left(-\mathrm{XH}+\mu+\mathrm{XH}\,\mu^2-\mu^3+\mathrm{XH}\,\nu^2-\mu\,\nu^2-\mathrm{XH}\,\xi+\mathrm{XH}^2\,\xi-\mathrm{YH}^2\,\xi+\mu\,\xi-\right.\right.\right.$$
$$\left.\left.\left.2\,\mathrm{XH}\,\mu\,\xi+\mathrm{XH}^2\,\mu\,\xi-\mathrm{YH}^2\,\mu\,\xi+\mu^2\,\xi-\mathrm{XH}\,\mu^2\,\xi+2\,\mathrm{YH}\,\nu\,\xi+2\,\mathrm{XH}\,\mathrm{YH}\,\nu\,\xi-\nu^2\,\xi-\mathrm{XH}\,\nu^2\,\xi\right)\right\},$$
$$\left\{\frac{1}{1+\xi}\left(-\mathrm{YH}+\mathrm{YH}\,\mu^2+\nu-\mu^2\,\nu+\mathrm{YH}\,\nu^2-\nu^3-\mathrm{YH}\,\xi-2\,\mathrm{XH}\,\mathrm{YH}\,\xi+2\,\mathrm{YH}\,\mu\,\xi+2\,\mathrm{XH}\,\mathrm{YH}\,\mu\,\xi-\right.\right.$$
$$\left.\left.\mathrm{YH}\,\mu^2\,\xi+\nu\,\xi+2\,\mathrm{XH}\,\nu\,\xi-\mathrm{XH}^2\,\nu\,\xi+\mathrm{YH}^2\,\nu\,\xi-2\,\mu\,\nu\,\xi-\mathrm{YH}\,\nu^2\,\xi\right)\right\},$$
$$\left\{-\frac{1}{2\,\xi^2\,(1+\xi)^2}(-1+\xi)\,\left(-1+2\,\mu^2-\mu^4+2\,\nu^2-2\,\mu^2\,\nu^2-\nu^4-3\,\xi+6\,\mu^2\,\xi-3\,\mu^4\,\xi+6\,\nu^2\,\xi-6\,\mu^2\,\nu^2\,\xi-\right.\right.$$
$$3\,\nu^4\,\xi-3\,\xi^2-\mathrm{XH}^2\,\xi^2-\mathrm{YH}^2\,\xi^2+2\,\mathrm{XH}\,\mu\,\xi^2+2\,\mathrm{XH}^2\,\mu\,\xi^2-2\,\mathrm{YH}^2\,\mu\,\xi^2+5\,\mu^2\,\xi^2-4\,\mathrm{XH}\,\mu^2\,\xi^2+3\,\mathrm{XH}^2$$
$$\mu^2\,\xi^2-\mathrm{YH}^2\,\mu^2\,\xi^2+2\,\mu^3\,\xi^2-6\,\mathrm{XH}\,\mu^3\,\xi^2+2\,\mathrm{YH}\,\nu\,\xi^2-4\,\mathrm{XH}\,\mathrm{YH}\,\nu\,\xi^2+8\,\mathrm{YH}\,\mu\,\nu\,\xi^2+8\,\mathrm{XH}\,\mathrm{YH}\,\mu\,\nu\,\xi^2-$$
$$6\,\mathrm{YH}\,\mu^2\,\nu\,\xi^2+5\,\nu^2\,\xi^2+4\,\mathrm{XH}\,\nu^2\,\xi^2-\mathrm{XH}^2\,\nu^2\,\xi^2+3\,\mathrm{YH}^2\,\nu^2\,\xi^2-6\,\mu\,\nu^2\,\xi^2-6\,\mathrm{XH}\,\mu\,\nu^2\,\xi^2-6\,\mathrm{YH}\,\nu^3\,\xi^2-$$
$$\xi^3-\mathrm{XH}^2\,\xi^3+2\,\mathrm{XH}^3\,\xi^3-\mathrm{YH}^2\,\xi^3-6\,\mathrm{XH}\,\mathrm{YH}^2\,\xi^3+2\,\mathrm{XH}\,\mu\,\xi^3-4\,\mathrm{XH}^2\,\mu\,\xi^3+2\,\mathrm{XH}^3\,\mu\,\xi^3+4\,\mathrm{YH}^2\,\mu\,\xi^3+$$
$$2\,\mathrm{XH}\,\mathrm{YH}^2\,\mu\,\xi^3+\mu^2\,\xi^3+2\,\mathrm{XH}\,\mu^2\,\xi^3-3\,\mathrm{XH}^2\,\mu^2\,\xi^3-3\,\mathrm{YH}^2\,\mu^2\,\xi^3+2\,\mathrm{YH}\,\nu\,\xi^3+8\,\mathrm{XH}\,\mathrm{YH}\,\nu\,\xi^3+$$
$$\left.\left.2\,\mathrm{XH}^2\,\mathrm{YH}\,\nu\,\xi^3+2\,\mathrm{YH}^3\,\nu\,\xi^3-4\,\mathrm{YH}\,\mu\,\nu\,\xi^3+\nu^2\,\xi^3-2\,\mathrm{XH}\,\nu^2\,\xi^3-3\,\mathrm{XH}^2\,\nu^2\,\xi^3-3\,\mathrm{YH}^2\,\nu^2\,\xi^3\right)\right\}\right\}$$

In[410]:=
```
Factor[% /. unμνξSubs]
```

Out[410]=
$$\left\{\left\{-\frac{1}{\left(-1+x^2+y^2+2\,Z\right)^2}\right.\right.$$
$$2\left(-1+x^2+y^2\right)\left(-1+x^2+y^2+Z\right)\left(-x-x^2+2\,x^3+\mathrm{XH}+2\,x\,\mathrm{XH}-3\,x^2\,\mathrm{XH}-\mathrm{XH}^2+x\,\mathrm{XH}^2+\right.$$
$$\left.\left.y^2+2\,x\,y^2-\mathrm{XH}\,y^2-2\,y\,\mathrm{YH}-2\,x\,y\,\mathrm{YH}+\mathrm{YH}^2+x\,\mathrm{YH}^2+2\,x\,Z-2\,\mathrm{XH}\,Z\right)\right\},$$
$$\left\{-\frac{1}{\left(-1+x^2+y^2+2\,Z\right)^2}2\left(-1+x^2+y^2\right)\left(-1+x^2+y^2+Z\right)\left(-y+2\,x\,y+2\,x^2\,y-2\,\mathrm{XH}\,y-\right.\right.$$
$$\left.\left.2\,x\,\mathrm{XH}\,y+\mathrm{XH}^2\,y+2\,y^3+\mathrm{YH}-2\,x\,\mathrm{YH}-x^2\,\mathrm{YH}+2\,\mathrm{XH}\,\mathrm{YH}-3\,y^2\,\mathrm{YH}+y\,\mathrm{YH}^2+2\,y\,Z-2\,\mathrm{YH}\,Z\right)\right\},$$
$$\left\{-\frac{1}{\left(-1+x^2+y^2+2\,Z\right)^2}2\,Z\left(-1+x^2+y^2+Z\right)\left(1-7\,x^2-2\,x^3+8\,x^4+6\,x\,\mathrm{XH}+4\,x^2\,\mathrm{XH}-\right.\right.$$
$$10\,x^3\,\mathrm{XH}-\mathrm{XH}^2-2\,x\,\mathrm{XH}^2+3\,x^2\,\mathrm{XH}^2-7\,y^2+6\,x\,y^2+16\,x^2\,y^2-4\,\mathrm{XH}\,y^2-10\,x\,\mathrm{XH}\,y^2+$$
$$3\,\mathrm{XH}^2\,y^2+8\,y^4+6\,y\,\mathrm{YH}-8\,x\,y\,\mathrm{YH}-10\,x^2\,y\,\mathrm{YH}+4\,\mathrm{XH}\,y\,\mathrm{YH}-10\,y^3\,\mathrm{YH}-\mathrm{YH}^2+$$
$$\left.\left.2\,x\,\mathrm{YH}^2+3\,x^2\,\mathrm{YH}^2+3\,y^2\,\mathrm{YH}^2-4\,Z+12\,x^2\,Z-8\,x\,\mathrm{XH}\,Z+12\,y^2\,Z-8\,y\,\mathrm{YH}\,Z+4\,Z^2\right)\right\}\right\}$$

In[411]:=
```
MatrixForm[ W = Factor[ % (-1 + x^2 + y^2 + 2 Z)^2 / (2 (-1 + x^2 + y^2 + Z)) ]]
```

Out[411]//MatrixForm=
$$\begin{pmatrix} -\left(-1+x^2+y^2\right)\left(-x-x^2+2\,x^3 \\ -\left(-1+x^2+y^2\right)\left(-y+2\,x\,y+ \\ -Z\left(1-7\,x^2-2\,x^3+8\,x^4+6\,x\,\mathrm{XH}+4\,x^2\,\mathrm{XH}-10\,x^3\,\mathrm{XH}-\mathrm{XH}^2-2\,x\,\mathrm{XH}^2+3\,x^2\,\mathrm{XH}^2-7\,y^2+6\,x\,y^2+16\,x^2\,y^2\right. \end{pmatrix}$$

```
(* That is proportional to ( ẋ, ẏ, Ż ) *)
```

In[412]:=
```
poly = Factor[
   (1 - 7 x^2 - 2 x^3 + 8 x^4 + 6 x XH + 4 x^2 XH - 10 x^3 XH - XH^2 - 2 x XH^2 + 3 x^2 XH^2 - 7 y^2 + 6 x y^2 + 16 x^2 y^2 -
      4 XH y^2 - 10 x XH y^2 + 3 XH^2 y^2 + 8 y^4 + 6 y YH - 8 x y YH - 10 x^2 y YH + 4 XH y YH - 10 y^3 YH - YH^2 +
      2 x YH^2 + 3 x^2 YH^2 + 3 y^2 YH^2 - 4 Z + 12 x^2 Z - 8 x XH Z + 12 y^2 Z - 8 y YH Z + 4 Z^2) /. Z → z^2]
```

Out[412]=
$$1 - 7 x^2 - 2 x^3 + 8 x^4 + 6 x XH + 4 x^2 XH - 10 x^3 XH - XH^2 - 2 x XH^2 + 3 x^2 XH^2 - 7 y^2 + 6 x y^2 + 16 x^2 y^2 - 4 XH y^2 - 10 x XH y^2 + 3 XH^2 y^2 + 8 y^4 + 6 y YH - 8 x y YH - 10 x^2 y YH + 4 XH y YH - 10 y^3 YH - YH^2 + 2 x YH^2 + 3 x^2 YH^2 + 3 y^2 YH^2 - 4 z^2 + 12 x^2 z^2 - 8 x XH z^2 + 12 y^2 z^2 - 8 y YH z^2 + 4 z^4$$

```
(* From paper (CORRECTED): *)
```

In[413]:=
```
p = Factor[
  1 - XH^2 - YH^2 + 2 (YH^2 - XH^2 + 3 XH) x + (3 XH^2 + 3 YH^2 + 4 XH - 7) x^2 -
   2 (1 + 5 XH) x^3 + 8 x^4 + 2 (3 + 2 XH) YH y - 8 YH x y - 10 YH x^2 y +
   (3 XH^2 + 3 YH^2 - 4 XH - 7) y^2 + 2 (3 - 5 XH) x y^2 + 16 x^2 y^2 - 10 YH y^3 +
   8 y^4 - 4 z^2 - 8 XH x z^2 + 12 x^2 z^2 - 8 YH y z^2 + 12 y^2 z^2 + 4 z^4
 ]
```

Out[413]=
$$1 - 7 x^2 - 2 x^3 + 8 x^4 + 6 x XH + 4 x^2 XH - 10 x^3 XH - XH^2 - 2 x XH^2 + 3 x^2 XH^2 - 7 y^2 + 6 x y^2 + 16 x^2 y^2 - 4 XH y^2 - 10 x XH y^2 + 3 XH^2 y^2 + 8 y^4 + 6 y YH - 8 x y YH - 10 x^2 y YH + 4 XH y YH - 10 y^3 YH - YH^2 + 2 x YH^2 + 3 x^2 YH^2 + 3 y^2 YH^2 - 4 z^2 + 12 x^2 z^2 - 8 x XH z^2 + 12 y^2 z^2 - 8 y YH z^2 + 4 z^4$$

In[414]:=
```
Factor[poly - p]
```

Out[414]= 0

In[415]:=
```
Clear[p]
```

In[416]:=
```
p[XH_, YH_, x_, y_, Z_] :=
  1 - 7 x^2 - 2 x^3 + 8 x^4 + 6 x XH + 4 x^2 XH - 10 x^3 XH - XH^2 - 2 x XH^2 + 3 x^2 XH^2 - 7 y^2 + 6 x y^2 +
   16 x^2 y^2 - 4 XH y^2 - 10 x XH y^2 + 3 XH^2 y^2 + 8 y^4 + 6 y YH - 8 x y YH - 10 x^2 y YH + 4 XH y YH -
   10 y^3 YH - YH^2 + 2 x YH^2 + 3 x^2 YH^2 + 3 y^2 YH^2 - 4 Z + 12 x^2 Z - 8 x XH Z + 12 y^2 Z - 8 y YH Z + 4 Z^2
```

In[417]:=
```
Factor[D[p[XH, YH, x, y, Z], Z]]
```

Out[417]= $4 \left(-1 + 3 x^2 - 2 x XH + 3 y^2 - 2 y YH + 2 Z\right)$

In[418]:=
```
Factor[ 8 Z + 12 ((x - XH / 3) ^2 + (y - YH / 3) ^2 - (3 + XH^2 + YH^2) / 9)]
```

Out[418]= $4 \left(-1 + 3 x^2 - 2 x XH + 3 y^2 - 2 y YH + 2 Z\right)$

In[419]:=
```
Factor[%% - %]
```

Out[419]= 0

In[420]:=
```
Factor[8 Z - 12 (3 + 9) / 9]
```

Out[420]= $8 (-2 + Z)$

In[421]:=
```
Q[u_, v_] := 8 ((u^2 + v^2) ^2 + u v^2)
```

In[422]:=
```
q[XH_, YH_, x_, y_] := Q[x - (1 + 5 XH) / 16, y - 5 YH / 16]
```

In[423]:=
```
Factor[p[XH, YH, x, y, Z] - q[XH, YH, x, y]]
```

Out[423]=
$$\frac{1}{8192}\left(8191 + 64\,x - 58\,880\,x^2 - 20\,XH + 50\,112\,x\,XH + 17\,408\,x^2\,XH - 8342\,XH^2 - 11\,584\,x\,XH^2 - 13\,824\,x^2\,XH^2 - 500\,XH^3 + 8000\,x\,XH^3 - 625\,XH^4 - 53\,760\,y^2 - 17\,408\,XH\,y^2 + 11\,776\,XH^2\,y^2 + 46\,912\,y\,YH - 34\,816\,x\,y\,YH + 23\,168\,XH\,y\,YH - 51\,200\,x\,XH\,y\,YH + 8000\,XH^2\,y\,YH - 7842\,YH^2 + 11\,584\,x\,YH^2 + 11\,776\,x^2\,YH^2 + 1500\,XH\,YH^2 + 8000\,x\,XH\,YH^2 - 1250\,XH^2\,YH^2 - 13\,824\,y^2\,YH^2 + 8000\,y\,YH^3 - 625\,YH^4 - 32\,768\,Z + 98\,304\,x^2\,Z - 65\,536\,x\,XH\,Z + 98\,304\,y^2\,Z - 65\,536\,y\,YH\,Z + 32\,768\,Z^2\right)$$

In[424]:=
```
r[XH_, YH_, x_, y_, Z_] := 4 Z^2 + 12 x^2 Z + 12 y^2 Z - 8 XH x Z - 8 YH y Z - 4 Z
```

In[425]:=
```
Factor[ p[XH, YH, x, y, Z] - q[XH, YH, x, y] - r[XH, YH, x, y, Z] ]
```

Out[425]=
$$\frac{1}{8192}\left(8191 + 64\,x - 58\,880\,x^2 - 20\,XH + 50\,112\,x\,XH + 17\,408\,x^2\,XH - 8342\,XH^2 - 11\,584\,x\,XH^2 - 13\,824\,x^2\,XH^2 - 500\,XH^3 + 8000\,x\,XH^3 - 625\,XH^4 - 53\,760\,y^2 - 17\,408\,XH\,y^2 + 11\,776\,XH^2\,y^2 + 46\,912\,y\,YH - 34\,816\,x\,y\,YH + 23\,168\,XH\,y\,YH - 51\,200\,x\,XH\,y\,YH + 8000\,XH^2\,y\,YH - 7842\,YH^2 + 11\,584\,x\,YH^2 + 11\,776\,x^2\,YH^2 + 1500\,XH\,YH^2 + 8000\,x\,XH\,YH^2 - 1250\,XH^2\,YH^2 - 13\,824\,y^2\,YH^2 + 8000\,y\,YH^3 - 625\,YH^4\right)$$

In[426]:=
```
s[XH_, YH_, x_, y_] :=
 1/8192 (8191 + 64 x - 58 880 x^2 - 20 XH + 50 112 x XH + 17 408 x^2 XH - 8342 XH^2 -
    11 584 x XH^2 - 13 824 x^2 XH^2 - 500 XH^3 + 8000 x XH^3 - 625 XH^4 - 53 760 y^2 -
    17 408 XH y^2 + 11 776 XH^2 y^2 + 46 912 y YH - 34 816 x y YH + 23 168 XH y YH -
    51 200 x XH y YH + 8000 XH^2 y YH - 7842 YH^2 + 11 584 x YH^2 + 11 776 x^2 YH^2 +
    1500 XH YH^2 + 8000 x XH YH^2 - 1250 XH^2 YH^2 - 13 824 y^2 YH^2 + 8000 y YH^3 - 625 YH^4)
```

In[427]:=
```
r[XH_, YH_, x_, y_, Z_] :=
 4 Z^2 + 12 x^2 Z + 12 y^2 Z - 8 XH x Z - 8 YH y Z - 4 Z  + s[XH, YH, x, y]
```

In[428]:=
```
r[XH, YH, x, y, Z]
```

Out[428]=
$$\frac{1}{8192}\left(8191 + 64\,x - 58\,880\,x^2 - 20\,XH + 50\,112\,x\,XH + 17\,408\,x^2\,XH - 8342\,XH^2 - 11\,584\,x\,XH^2 - 13\,824\,x^2\,XH^2 - 500\,XH^3 + 8000\,x\,XH^3 - 625\,XH^4 - 53\,760\,y^2 - 17\,408\,XH\,y^2 + 11\,776\,XH^2\,y^2 + 46\,912\,y\,YH - 34\,816\,x\,y\,YH + 23\,168\,XH\,y\,YH - 51\,200\,x\,XH\,y\,YH + 8000\,XH^2\,y\,YH - 7842\,YH^2 + 11\,584\,x\,YH^2 + 11\,776\,x^2\,YH^2 + 1500\,XH\,YH^2 + 8000\,x\,XH\,YH^2 - 1250\,XH^2\,YH^2 - 13\,824\,y^2\,YH^2 + 8000\,y\,YH^3 - 625\,YH^4\right) - 4\,Z + 12\,x^2\,Z - 8\,x\,XH\,Z + 12\,y^2\,Z - 8\,y\,YH\,Z + 4\,Z^2$$

In[429]:=
```
Clear[u, v]
```

In[430]:=
```
Q[u, v]
```

Out[430]=
$$8\left(u\,v^2 + \left(u^2 + v^2\right)^2\right)$$

In[431]:=
```
first = Factor[D[Q[u, v], u]]
```
Out[431]= $8\left(4u^3+v^2+4uv^2\right)$

In[432]:=
```
second = Factor[D[Q[u, v], v]]
```
Out[432]= $16v\left(u+2u^2+2v^2\right)$

In[433]:=
```
Factor[Resultant[first, second, v]]
```
Out[433]= $524\,288u^5(1+6u)^2$

In[434]:=
```
Factor[{first, second} /. u → -1 / 6]
```
Out[434]= $\left\{\frac{4}{27}\left(-1+18v^2\right),\ \frac{16}{9}v\left(-1+18v^2\right)\right\}$

In[435]:=
```
Q[-1 / 6, 1 / Sqrt[18]]
```
Out[435]= $-\frac{1}{54}$

```
(*
    That is the least value of Q[u,v] and so least value of q[XH,YH,x,y] too.
    Now consider r[XH,YH,x,y,Z] which needs to be > 1/54 when Z ≥ 9.
*)
```

In[436]:=
```
Factor[r[XH, YH, x, y, 9]]
```
Out[436]=
$$\frac{1}{8192}\left(2\,367\,487+64x+825\,856x^2-20\,XH-539\,712x\,XH+17\,408x^2\,XH-\right.$$
$$8342\,XH^2-11\,584x\,XH^2-13\,824x^2\,XH^2-500\,XH^3+8000x\,XH^3-625\,XH^4+830\,976y^2-$$
$$17\,408\,XH\,y^2+11\,776\,XH^2\,y^2-542\,912y\,YH-34\,816x\,y\,YH+23\,168\,XH\,y\,YH-$$
$$51\,200x\,XH\,y\,YH+8000\,XH^2\,y\,YH-7842\,YH^2+11\,584x\,YH^2+11\,776x^2\,YH^2+$$
$$\left.1500\,XH\,YH^2+8000x\,XH\,YH^2-1250\,XH^2\,YH^2-13\,824y^2\,YH^2+8000y\,YH^3-625\,YH^4\right)$$

In[437]:=
```
first = Factor[D[r[XH, YH, x, y, 9], x]]
```
Out[437]=
$$\frac{1}{128}\left(1+25\,808x-8433\,XH+544x\,XH-181\,XH^2-432x\,XH^2+\right.$$
$$\left.125\,XH^3-544y\,YH-800\,XH\,y\,YH+181\,YH^2+368x\,YH^2+125\,XH\,YH^2\right)$$

In[438]:=
```
second = Factor[D[r[XH, YH, x, y, 9], y]]
```
Out[438]=
$$\frac{1}{128}\left(25\,968y-544\,XH\,y+368\,XH^2\,y-8483\,YH-\right.$$
$$\left.544x\,YH+362\,XH\,YH-800x\,XH\,YH+125\,XH^2\,YH-432y\,YH^2+125\,YH^3\right)$$

In[439]:=
```
Factor[Resultant[first, second, y]]
```

Out[439]=
$$\frac{1}{1024}\left(-1623-41\,886\,384\,x+13\,686\,793\,XH-5440\,x\,XH+7018\,XH^2+126\,048\,x\,XH^2-15\,070\,XH^3-27\,200\,x\,XH^3+8413\,XH^4+9936\,x\,XH^4-2875\,XH^5-5314\,YH^2+118\,048\,x\,YH^2-12\,570\,XH\,YH^2+81\,600\,x\,XH\,YH^2-27\,150\,XH^2\,YH^2+19\,872\,x\,XH^2\,YH^2-5750\,XH^3\,YH^2+637\,YH^4+9936\,x\,YH^4-2875\,XH\,YH^4\right)$$

In[440]:=
```
x0 = Factor[ x /. Solve[Numerator[%] == 0, x][[1]]]
```

Out[440]=
$$\left(1623-13\,686\,793\,XH-7018\,XH^2+15\,070\,XH^3-8413\,XH^4+2875\,XH^5+5314\,YH^2+12\,570\,XH\,YH^2+27\,150\,XH^2\,YH^2+5750\,XH^3\,YH^2-637\,YH^4+2875\,XH\,YH^4\right)\Big/\left(16\left(-2\,617\,899-340\,XH+7878\,XH^2-1700\,XH^3+621\,XH^4+7378\,YH^2+5100\,XH\,YH^2+1242\,XH^2\,YH^2+621\,YH^4\right)\right)$$

In[441]:=
```
Factor[{first, second} /. x → x0]
```

Out[441]=
$$\left\{-\left(\left((17+25\,XH)\,YH\left(-41\,886\,384\,y-5440\,XH\,y+126\,048\,XH^2\,y-27\,200\,XH^3\,y+9936\,XH^4\,y+13\,683\,045\,YH-8812\,XH\,YH-15\,170\,XH^2\,YH+10\,324\,XH^3\,YH-2875\,XH^4\,YH+118\,048\,y\,YH^2+81\,600\,XH\,y\,YH^2+19\,872\,XH^2\,y\,YH^2-12\,670\,YH^3-25\,876\,XH\,YH^3-5750\,XH^2\,YH^3+9936\,y\,YH^4-2875\,YH^5\right)\right)\Big/\left(64\left(-2\,617\,899-340\,XH+7878\,XH^2-1700\,XH^3+621\,XH^4+7378\,YH^2+5100\,XH\,YH^2+1242\,XH^2\,YH^2+621\,YH^4\right)\right)\right),\left(\left(1623-34\,XH+23\,XH^2-27\,YH^2\right)\left(-41\,886\,384\,y-5440\,XH\,y+126\,048\,XH^2\,y-27\,200\,XH^3\,y+9936\,XH^4\,y+13\,683\,045\,YH-8812\,XH\,YH-15\,170\,XH^2\,YH+10\,324\,XH^3\,YH-2875\,XH^4\,YH+118\,048\,y\,YH^2+81\,600\,XH\,y\,YH^2+19\,872\,XH^2\,y\,YH^2-12\,670\,YH^3-25\,876\,XH\,YH^3-5750\,XH^2\,YH^3+9936\,y\,YH^4-2875\,YH^5\right)\right)\Big/\left(128\left(-2\,617\,899-340\,XH+7878\,XH^2-1700\,XH^3+621\,XH^4+7378\,YH^2+5100\,XH\,YH^2+1242\,XH^2\,YH^2+621\,YH^4\right)\right)\right\}$$

In[442]:=
```
y0 = Factor[ y /.
   Solve[(-41 886 384 y - 5440 XH y + 126 048 XH^2 y - 27 200 XH^3 y + 9936 XH^4 y + 13 683 045 YH -
      8812 XH YH - 15 170 XH^2 YH + 10 324 XH^3 YH - 2875 XH^4 YH +
      118 048 y YH^2 + 81 600 XH y YH^2 + 19 872 XH^2 y YH^2 - 12 670 YH^3 -
      25 876 XH YH^3 - 5750 XH^2 YH^3 + 9936 y YH^4 - 2875 YH^5) == 0, y][[1]]]
```

Out[442]=
$$\left(YH\left(-13\,683\,045+8812\,XH+15\,170\,XH^2-10\,324\,XH^3+2875\,XH^4+12\,670\,YH^2+25\,876\,XH\,YH^2+5750\,XH^2\,YH^2+2875\,YH^4\right)\right)\Big/\left(16\left(-2\,617\,899-340\,XH+7878\,XH^2-1700\,XH^3+621\,XH^4+7378\,YH^2+5100\,XH\,YH^2+1242\,XH^2\,YH^2+621\,YH^4\right)\right)$$

In[443]:=
```
Factor[r[XH, YH, x0, y0, 9]]
```

Out[443]=
$$\left(-6\,197\,841\,846\,567 - 807\,334\,704\,XH + 271\,330\,432\,020\,XH^2 + 2\,360\,320\,864\,XH^3 - 632\,463\,154\,XH^4 + 145\,132\,048\,XH^5 - 50\,930\,876\,XH^6 - 2\,392\,000\,XH^7 + 330\,625\,XH^8 + 270\,144\,022\,668\,YH^2 - 6\,949\,989\,152\,XH\,YH^2 - 1\,221\,015\,284\,XH^2\,YH^2 - 289\,762\,976\,XH^3\,YH^2 - 190\,192\,028\,XH^4\,YH^2 + 2\,392\,000\,XH^5\,YH^2 + 1\,322\,500\,XH^6\,YH^2 - 585\,260\,050\,YH^4 - 439\,735\,024\,XH\,YH^4 - 128\,565\,028\,XH^2\,YH^4 + 11\,960\,000\,XH^3\,YH^4 + 1\,983\,750\,XH^4\,YH^4 - 55\,321\,476\,YH^6 + 7\,176\,000\,XH\,YH^6 + 1\,322\,500\,XH^2\,YH^6 + 330\,625\,YH^8\right) \Big/ \left(8192\left(-2\,617\,899 - 340\,XH + 7878\,XH^2 - 1700\,XH^3 + 621\,XH^4 + 7378\,YH^2 + 5100\,XH\,YH^2 + 1242\,XH^2\,YH^2 + 621\,YH^4\right)\right)$$

```
(*
    That is the minimum value of r[XH,YH,x,y,9] for fixed XY and YH.
*)
```

In[444]:=
```
{num, den} = {Numerator[%], Denominator[%]}
```

Out[444]=
$$\left\{-6\,197\,841\,846\,567 - 807\,334\,704\,XH + 271\,330\,432\,020\,XH^2 + 2\,360\,320\,864\,XH^3 - 632\,463\,154\,XH^4 + 145\,132\,048\,XH^5 - 50\,930\,876\,XH^6 - 2\,392\,000\,XH^7 + 330\,625\,XH^8 + 270\,144\,022\,668\,YH^2 - 6\,949\,989\,152\,XH\,YH^2 - 1\,221\,015\,284\,XH^2\,YH^2 - 289\,762\,976\,XH^3\,YH^2 - 190\,192\,028\,XH^4\,YH^2 + 2\,392\,000\,XH^5\,YH^2 + 1\,322\,500\,XH^6\,YH^2 - 585\,260\,050\,YH^4 - 439\,735\,024\,XH\,YH^4 - 128\,565\,028\,XH^2\,YH^4 + 11\,960\,000\,XH^3\,YH^4 + 1\,983\,750\,XH^4\,YH^4 - 55\,321\,476\,YH^6 + 7\,176\,000\,XH\,YH^6 + 1\,322\,500\,XH^2\,YH^6 + 330\,625\,YH^8,\ 8192\left(-2\,617\,899 - 340\,XH + 7878\,XH^2 - 1700\,XH^3 + 621\,XH^4 + 7378\,YH^2 + 5100\,XH\,YH^2 + 1242\,XH^2\,YH^2 + 621\,YH^4\right)\right\}$$

In[445]:=
```
polarSubs = {
  XH → ρ Cos[ω], YH → ρ Sin[ω],
  Sin[ω]^2 → 1 - Cos[ω]^2,
  Sin[ω]^3 → Sin[ω] (1 - Cos[ω]^2),
  Sin[ω]^4 → (1 - Cos[ω]^2)^2,
  Sin[ω]^5 → Sin[ω] (1 - Cos[ω]^2)^2,
  Sin[ω]^6 → (1 - Cos[ω]^2)^3,
  Sin[ω]^7 → Sin[ω] (1 - Cos[ω]^2)^3,
  Sin[ω]^8 → (1 - Cos[ω]^2)^4
 }
```

Out[445]=
$$\left\{XH \to \rho\,\text{Cos}[\omega],\ YH \to \rho\,\text{Sin}[\omega],\ \text{Sin}[\omega]^2 \to 1 - \text{Cos}[\omega]^2,\ \text{Sin}[\omega]^3 \to \left(1 - \text{Cos}[\omega]^2\right)\text{Sin}[\omega],\ \text{Sin}[\omega]^4 \to \left(1 - \text{Cos}[\omega]^2\right)^2,\ \text{Sin}[\omega]^5 \to \left(1 - \text{Cos}[\omega]^2\right)^2\text{Sin}[\omega],\ \text{Sin}[\omega]^6 \to \left(1 - \text{Cos}[\omega]^2\right)^3,\ \text{Sin}[\omega]^7 \to \left(1 - \text{Cos}[\omega]^2\right)^3\text{Sin}[\omega],\ \text{Sin}[\omega]^8 \to \left(1 - \text{Cos}[\omega]^2\right)^4\right\}$$

In[446]:=
```
Factor[ {num, den} //. polarSubs ]
```

Out[446]=
$$\{-6\,197\,841\,846\,567 + 270\,144\,022\,668\,\rho^2 - 585\,260\,050\,\rho^4 - 55\,321\,476\,\rho^6 + 330\,625\,\rho^8 - 807\,334\,704\,\rho\,\text{Cos}[\omega] - 6\,949\,989\,152\,\rho^3\,\text{Cos}[\omega] - 439\,735\,024\,\rho^5\,\text{Cos}[\omega] + 7\,176\,000\,\rho^7\,\text{Cos}[\omega] + 1\,186\,409\,352\,\rho^2\,\text{Cos}[\omega]^2 - 50\,495\,184\,\rho^4\,\text{Cos}[\omega]^2 + 37\,399\,400\,\rho^6\,\text{Cos}[\omega]^2 + 9\,310\,310\,016\,\rho^3\,\text{Cos}[\omega]^3 + 589\,707\,072\,\rho^5\,\text{Cos}[\omega]^3 - 9\,568\,000\,\rho^7\,\text{Cos}[\omega]^3 + 3\,292\,080\,\rho^4\,\text{Cos}[\omega]^4 - 99\,026\,400\,\rho^6\,\text{Cos}[\omega]^4 - 4\,840\,000\,\rho^5\,\text{Cos}[\omega]^5 + 66\,017\,600\,\rho^6\,\text{Cos}[\omega]^6,$$
$$8192\,\left(-2\,617\,899 + 7378\,\rho^2 + 621\,\rho^4 - 340\,\rho\,\text{Cos}[\omega] + 5100\,\rho^3\,\text{Cos}[\omega] + 500\,\rho^2\,\text{Cos}[\omega]^2 - 6800\,\rho^3\,\text{Cos}[\omega]^3\right)\}$$

In[447]:=
```
{numPolar, denPolar} = Factor[ - % /. Cos[ω] → χ ]
```

Out[447]=
$$\{6\,197\,841\,846\,567 - 270\,144\,022\,668\,\rho^2 + 585\,260\,050\,\rho^4 + 55\,321\,476\,\rho^6 - 330\,625\,\rho^8 + 807\,334\,704\,\rho\,\chi + 6\,949\,989\,152\,\rho^3\,\chi + 439\,735\,024\,\rho^5\,\chi - 7\,176\,000\,\rho^7\,\chi - 1\,186\,409\,352\,\rho^2\,\chi^2 + 50\,495\,184\,\rho^4\,\chi^2 - 37\,399\,400\,\rho^6\,\chi^2 - 9\,310\,310\,016\,\rho^3\,\chi^3 - 589\,707\,072\,\rho^5\,\chi^3 + 9\,568\,000\,\rho^7\,\chi^3 - 3\,292\,080\,\rho^4\,\chi^4 + 99\,026\,400\,\rho^6\,\chi^4 + 4\,840\,000\,\rho^5\,\chi^5 - 66\,017\,600\,\rho^6\,\chi^6,$$
$$-8192\,\left(-2\,617\,899 + 7378\,\rho^2 + 621\,\rho^4 - 340\,\rho\,\chi + 5100\,\rho^3\,\chi + 500\,\rho^2\,\chi^2 - 6800\,\rho^3\,\chi^3\right)\}$$

In[448]:=
```
Expand[denPolar]
```

Out[448]=
$$21\,445\,828\,608 - 60\,440\,576\,\rho^2 - 5\,087\,232\,\rho^4 + 2\,785\,280\,\rho\,\chi - 41\,779\,200\,\rho^3\,\chi - 4\,096\,000\,\rho^2\,\chi^2 + 55\,705\,600\,\rho^3\,\chi^3$$

```
(* lower bound on denPolar  *)
```

In[449]:=
```
21 445 828 608 - 60 440 576 (3^2) - 5 087 232 (3^4) +
 2 785 280 (-3) - 41 779 200 (3^3) - 4 096 000 (3^2) + 55 705 600 (-3^3)
```

Out[449]=
$$17\,812\,488\,192$$

```
(* upper bound on denPolar  *)
```

In[450]:=
```
21 445 828 608 - 60 440 576 (0) - 5 087 232 (0) +
 2 785 280 (3) - 41 779 200 (-3^3) - 4 096 000 (0) + 55 705 600 (3^3)
```

Out[450]=
$$24\,086\,274\,048$$

In[451]:=
```
Expand[numPolar]
```

Out[451]=
$$6\,197\,841\,846\,567 - 270\,144\,022\,668\,\rho^2 + 585\,260\,050\,\rho^4 + 55\,321\,476\,\rho^6 - 330\,625\,\rho^8 + 807\,334\,704\,\rho\,\chi + 6\,949\,989\,152\,\rho^3\,\chi + 439\,735\,024\,\rho^5\,\chi - 7\,176\,000\,\rho^7\,\chi - 1\,186\,409\,352\,\rho^2\,\chi^2 + 50\,495\,184\,\rho^4\,\chi^2 - 37\,399\,400\,\rho^6\,\chi^2 - 9\,310\,310\,016\,\rho^3\,\chi^3 - 589\,707\,072\,\rho^5\,\chi^3 + 9\,568\,000\,\rho^7\,\chi^3 - 3\,292\,080\,\rho^4\,\chi^4 + 99\,026\,400\,\rho^6\,\chi^4 + 4\,840\,000\,\rho^5\,\chi^5 - 66\,017\,600\,\rho^6\,\chi^6$$

```
(* lower bound on numPolar *)
```

In[452]:=
```
6 197 841 846 567 - 270 144 022 668 (3^2) + 585 260 050 (0) +
 55 321 476 (0) - 330 625 (3^8) + 807 334 704 (-3) + 6 949 989 152 (-3^3) +
 439 735 024 (-3^5) - 7 176 000 (3^7) - 1 186 409 352 (3^2) + 50 495 184 (0) -
 37 399 400 (3^6) - 9 310 310 016 (3^3) - 589 707 072 (3^5) + 9 568 000 (-3^7) -
 3 292 080 (3^4) + 99 026 400 (0) + 4 840 000 (-3^5) - 66 017 600 (3^6)
```

Out[452]=
```
2 948 641 317 306
```

```
(* lower bound on numPolar/denPolar *)
```

In[453]:=
```
N[ 2 948 641 317 306 / 24 086 274 048 ]
```

Out[453]=
```
122.42
```

```
(*
   So r[XH,YH,x,y,9] is at least this big, and so too is
   r[XH,YH,x,y,Z] for all Z ≥ 9.
*)
```